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ARITHMETIC

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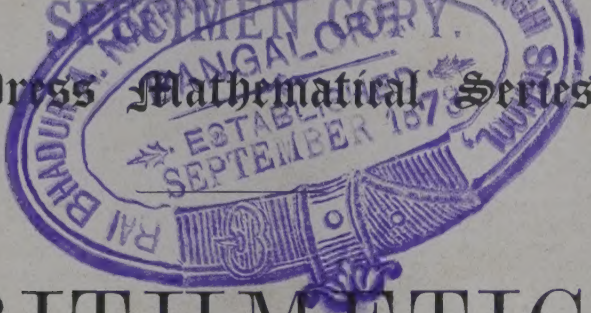

ARITHMETIC

FOR SCHOOLS

Ed. m 75

BY

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PREFACE.

IN the following work I have endeavoured to put the science of Arithmetic on a sound basis, and to give clear and complete explanations of all the fundamental principles and processes. It has not been my aim to introduce novelties, but to promote accuracy and clearness of conception, so as to make the study of Arithmetic not only of practical utility but also of great educational value.

The order in which the different Chapters may be read is to some extent optional. I have considered Compound Quantities before the Chapters on Fractions and Decimals, and I have discussed all the Compound Rules with reference to Money before considering the less familiar Weights and Measures. Decimals are defined independently of Vulgar Fractions, as a simple extension of the ordinary mode of notation for whole numbers. The Chapter on Decimals is, however, placed after that on Vulgar Fractions: this will doubtless continue to be the usual order, at any rate until we have adopted the Decimal System of Weights and Measures.

The Unitary method of solving problems involving Proportion is adopted.

Shares and Stocks and other applications of Arithmetic have been made to approximate more closely to actual transactions than appears to be usual in books on Arithmetic. All questions with reference to Consols have been framed in accordance with the recent changes.

A Chapter on Foreign Exchanges has been inserted, principally for the benefit of those preparing for examinations in Commercial Arithmetic.

Sets of Miscellaneous Examples are given at intervals throughout the book, to be used for revision. To ensure ample variety in the examples I have consulted many recent examination papers; and I am indebted to the kindness of the Secretaries of the Cambridge Local Examinations Syndicate and the Oxford and Cambridge Schools Examination Board for opportunities of seeing and selecting questions from all the papers in Arithmetic given in examinations conducted by those bodies.

I am indebted to many friends for their kindness in looking over the proof sheets, for help in the verification of the answers, and for valuable criticisms and suggestions. My special thanks are due to Mr J. Barnard, M.A., Head Mathematical Master at Christ's Hospital.

CHARLES SMITH.

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ARITHMETIC.

CHAPTER I.

NUMERATION AND NOTATION.

1. THE idea of **number** is first acquired from the observation of groups or repetitions of distinct objects: we thus learn to speak of *two* boys, *three* balls, *four* strokes of a clock, &c.

Arithmetic is the science which treats of numbers and of the different operations to which they are subject.

2. A single object of any kind, or any group of objects considered as a whole, is called a **unit**.

Thus *a ball*, *an inch*, *a dozen* are units.

3. Anything which can be increased or diminished is called a **magnitude**.

Lengths, areas, weights, &c., are magnitudes.

To **measure** a magnitude is to compare it with some known magnitude of the *same kind* which is taken as a unit, and to say how many times the unit must be repeated in order to make up the magnitude in question.

For example, to measure any given length of string is to find how many times some known length, say a foot, must be repeated to make up the given length; and this number of times is called the **measure** of the length.

A measured magnitude is called a **quantity**.

Thus any quantity is expressed by a *number* and *a unit of the same kind as itself.*

4. When numbers are used without reference to any particular units they are called **abstract numbers**.

Thus *two* and *five* are abstract numbers. *Two feet* and *five tons* are called **concrete quantities.**

5. The first few numbers in order are, *one, two, three, four, five, six, seven, eight, nine* and *ten;* each successive number being *one more* than the preceding.

Thus *two* is *one* and *one* more, *three* is *two* and *one* more, and so on.

6. It will be observed that the names of the first ten numbers are in no way connected with one another.

Now it is obvious that the knowledge of numbers and of their relations to one another must always have remained very limited if every successive number had had a special name given to it independent of the names of the preceding numbers; for it would be impossible merely to remember, in their order, very many such names.

7. Successive numbers have therefore been named according to a systematic plan which requires the use of as few independent names as possible.

A method by which numbers are expressed in words according to some systematic plan is called a system of **Numeration**.

NUMERATION.

8. To shew how all numbers can be named by means of a few special words, imagine a collection of objects of the same kind, for example a heap of pebbles; and suppose that we wish to know how many pebbles there are, and to give a name to this number.

If there are not more than ten pebbles altogether, we find the number at once by **counting** them, that is

by saying in order the names one, two, three, &c., each time separating one of the pebbles from the original heap; and the name which is said with the last of the heap gives the number of the pebbles.

If there are more than ten pebbles in the heap, count off ten and put them apart, and go on making groups of ten until there are fewer than ten pebbles left. Suppose there are seven groups of ten each and five pebbles over, then we could call the number seven tens and five.

By separating the whole heap into groups of ten in this way we at once find, and can give a name to, the number of the pebbles, provided there are not more than ten of the groups. Thus our original ten names suffice to name all numbers up to that which is made up of ten groups each containing ten pebbles, and we have a new name, namely one *hundred*, for the number which consists of ten tens.

It will be seen at once that the names in actual use are only slightly modified forms of the names which naturally arise from the above method of division into groups of ten. Instead of saying two tens, three tens, four tens, &c. we say twenty, thirty, forty, &c., and we say seventy-five instead of seventy *and* five. Also instead of the names ten and one, ten and two, ten and three, ten and four, &c. we use the names *eleven* [Gothic *ainlif*, *ain* one and *lif* ten] *twelve* [Gothic *twalif*, *twa* two and *lif* ten], *thirteen*, *fourteen*, &c.

If there are more than ten of the groups each of which contains ten pebbles, the groups can be arranged in sets of ten, so that there will be one hundred pebbles in each of these sets. Suppose that there are five of these sets and six groups over and four single pebbles besides, then the number is made up of five hundreds, six tens and four, and is called five hundred and sixty-four. Thus no new name is necessary until we come to the number which consists of ten hundreds, and this number is called a *thousand*.

The principle of the ordinary system of numeration will now be apparent.

9. The English names which are employed in the system of Numeration which is universally used are the following:

one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.

a *hundred*	which	is	ten tens,
a *thousand*	„	„	ten hundreds,
a *million*	„	„	a thousand thousands,
a *billion**	„	„	a million millions,
a *trillion*	„	„	a million billions,
a *quadrillion*	„	„	a million trillions,

and so on.

The names *billion, trillion,* &c. are very rarely used.

10. The numbers *one, ten, a hundred, a thousand, ten thousand, a hundred thousand, a million,* &c. are often called *units* of the *first order,* of the *second order,* of the *third order,* &c.; and ten units of any order are required to make one unit of the next higher order.

To give a name to any number it is sufficient to state the number of units of each different order that the number contains.

Thus the number which is made up of three *millions* two *hundreds-of-thousands* three *tens-of-thousands* four *thousands* five *hundreds* seven *tens* and six *units* is called *three million two hundred and thirty-four thousand five hundred and seventy-six;* also the number which is made up of five *thousands-of-millions* two *hundreds-of-millions* three *tens-of-millions* two *tens-of-thousands* three *thousands* and five *hundreds* is called *five thousand two hundred and thirty million twenty-three thousand five hundred.*

It should be remarked that the parts of a number are mentioned in the order of their magnitude, the largest being given first; in English, however, this

* In the United States and on the continent of Europe one *thousand* millions is called a billion, and one *thousand* billions is called a trillion, and so on.

order is reversed for the numbers between ten and twenty. It should also be noticed that all the thousands, all the millions, all the trillions, &c. are grouped together, as in the above two cases.

11. The system of numeration described above is called the **decimal** system, since *ten* units of any order are required to make up one unit of the next higher order.

The decimal system of numeration is employed by all people who have any names at all for numbers greater than ten, and the origin of the system was doubtless the natural habit of counting on the fingers.

NOTATION.

12. We have now to shew how numbers can be represented in a simple manner by means of a few symbols called *figures* or *digits*.

A method by which numbers are expressed by means of symbols according to some systematic plan is called a system of **Notation**.

13. The first nine numbers in order are represented by the symbols

1, 2, 3, 4, 5, 6, 7, 8, 9.

The same figures are also employed to represent the first nine collections of *tens*, of *hundreds*, of *thousands*, &c. but on the understanding that the figures are written in a row and that any figure placed first on the right represents so many *units* or *ones*, the same figure in the second place represents as many *tens*, the same figure in the third place represents as many *hundreds*, and so on; so that *any figure placed to the left of another represents units of the order next above those represented by the other.*

Thus in 45 the 5 represents five *ones*, in 456 the 5 represents five *tens*, in 4567 the 5 represents five *hundreds*, and so on. Again 53462 represents two *ones* six *tens* four *hundreds* three *thousands* and five *tens-of-thousands*, that is the number *fifty-three thousand four hundred and sixty-two.*

14. Any figure, say 5, has then *two* values, namely (1) its **digit value** which is indicated by its shape and in virtue of which it always represents *five* units of some kind, and (2) its **local value** which depends only on its place and in virtue of which it represents units of the order which corresponds to its position in the horizontal row, so that if 5 be in the first place from the right it represents five units, if in the second place it represents five tens, and so on.

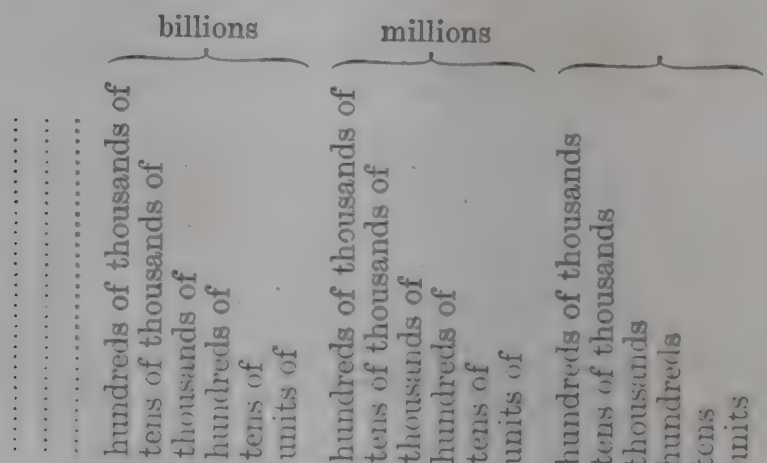
15. Besides the nine symbols already specified it is necessary to have an additional symbol to meet the case when units of one or more of the different orders are absent. This symbol is 0, which is read *nought* or *cipher*. The symbol 0 has no value by itself, it is simply used to indicate that there are no units of the particular order corresponding to the place in which it occurs.

The other figures are sometimes distinguished from the nought by being called *significant figures* or *digits*.

Thus 20 represents two *tens* and no *ones*, that is the number *twenty*. Again 2005 represents two *thousands* no *hundreds* no *tens* and five *ones*, that is the number *two thousand and five*, the noughts serving to bring the figures into the places intended for them.

It should be noticed that a nought placed at the *beginning* of a number would be meaningless, thus 056, for example, is the same as 56.

16. The names of the units of all the different orders and the positions which correspond to them are shewn below:



It will be seen that the names are repeated in groups of six; the second six being units, tens, &c. of *millions;* the third six being units, tens, &c. of *billions;* the next being units, tens, &c. of *trillions;* and so on.

17. To write in figures any number expressed in words it is only necessary to write down the figures which represent the number of the units of the different orders in their proper places as shewn above, filling up the vacant places, if any, with noughts.

Thus to write in figures the number two hundred and forty-three. We must have 2 in the place for *hundreds*, that is in the *third* place (counting from right to left), 4 must be put in the place for *tens*, that is in the *second* place, and 3 in the place for *units*, that is in the *first* place. Hence the number is written 243.

Again, to write five hundred and six in figures. We must have 5 in the third place and 6 in the first, 0 being put in the second place, since there are no tens. Hence the number is written 506.

So also, seven hundred and forty is written 740.

Having learnt to write down readily numbers containing only hundreds, tens and units, that is numbers of *three* figures, larger numbers can be considered.

For example, to write in figures the number five hundred and twenty-four thousand six hundred and seven. The number is 5 hundreds-of-thousands, 2 tens-of-thousands, 4 thousands, 6 hundreds, 0 tens and 7. It is therefore written 524607.

It should be carefully noted that to write down any number of *thousands*, we have only to write down the figures which stand for so many *units*, provided these are followed by *three* other figures.

Also to write down any number of *millions*, we write down the figures which stand for so many *units*, taking care that these figures are followed on the right by *six* other figures.

Thus sixteen millions is expressed by 16 followed by six noughts, that is by 16000000. Again, one thousand two hundred millions is expressed by 1200000000.

18. To express in words any number given in figures, first divide the figures into groups of six beginning at the *right*, so that the last group, namely that on the left, may be incomplete; then read off each group beginning at the left as if it were alone but remembering that in the second group from the *right* the numbers are *millions*, in the third group *billions*, &c.

For example, to express in words the number represented by 24160523. We can separate off one complete group of six figures, and 24,160523 is then read off twenty-four *million*, one hundred and sixty thousand five hundred and twenty-three.

Again 34120670532035 is first written 34,120670,532035 and then read off thirty-four *billion*, one hundred and twenty thousand six hundred and seventy *million*, five hundred and thirty-two thousand and thirty-five.

EXAMPLES I. A.

1. What does 5 stand for in the numbers 15, 157, 514, 352167 and 3561234 respectively?

2. What does 7 stand for in the numbers 70, 37123, 715243, 274126315 and 370001002003 respectively?

3. Underline all the figures which represent their digit value of *hundreds* in

314, 2167, 50412 and 31024.

4. Underline all the figures which represent their digit value of *thousands* in

2314, 56123, 60417 and 3005167.

5. Write out in words the separate value of every figure in 25, in 356, in 4236, in 53461 and in 613425.

6. Write out in words the separate value of every figure in 304, in 2106, in 307509 and in 4030507090.

7. Express in words the numbers 27, 349, 560, 306, 1204 and 5020.

8. Express in words the numbers 2009, 6050, 12345, 10305, 40050 and 120460.

9. Express in words the numbers 518618, 602010, 100010, 504075, 420040 and 107005.

10. Express in words 111111111, 1203405, 2314100, 5040314, 20050060, 30300074.

11. Express in words the numbers 3012004, 110110110, 201201201, 1000040305101 and 604102000300004.

12. Write in figures the numbers fifty-eight, eighty-five, two hundred and eleven, three thousand and twelve, six thousand and forty, and nine thousand three hundred.

13. Write in figures the numbers one thousand and fourteen, seven thousand and thirty-six, twelve thousand two hundred, fourteen thousand and sixteen, sixty thousand and seventy-one, and eighty-five thousand one hundred and six.

14. Write in figures the numbers one hundred thousand four hundred and four, three hundred and five thousand three hundred

and five, six hundred and twenty thousand and seven, six million six thousand and sixty, five million three hundred thousand, and seven million eleven thousand and fourteen.

15. Write in figures twenty million twenty thousand, seventeen million fifty thousand and nineteen, one hundred and four million six hundred and two thousand and eleven, six thousand three hundred and seven million two thousand and fifty six, and five hundred and twenty thousand three hundred and fourteen million one hundred and five thousand two hundred and eighteen.

16. Write in figures the numbers one hundred and fifteen thousand million two hundred and forty thousand and five, six hundred thousand million three hundred thousand, eighty-four thousand million five hundred and two, two billion six million and forty-five, and nine billion fifty thousand million two hundred thousand five hundred.

17. Write in figures the numbers eleven hundred and eleven, fourteen hundred and sixty, twelve hundred thousand sixteen hundred, six million twelve hundred and sixteen, and eleven billion eleven thousand million eleven hundred and eleven.

18. Write down the greatest and least number (1) with two figures, (2) with three figures and (3) with six figures.

19. The ordinary system of notation was introduced into Europe by the Arabians and is still called the **Arabic system** of Notation although it is now known that the Arabians derived their knowledge from the Hindoos.

ROMAN NUMERALS.

20. Besides the Arabic system of notation some use is still made of the cumbrous system employed by the Romans.

The symbols which were used by the Romans, and which are called **Roman Numerals**, are the following

I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, M for 1000.

A horizontal line over any numeral increases its value one thousand fold: thus $\overline{\text{V}}$ stands for 5000, $\overline{\text{X}}$ for 10000, &c.

Roman numerals are arranged in order of magnitude from left to right, and are repeated as often as may be necessary.

Thus 2 is represented by II, 30 by XXX, 233 by CCXXXIII, and 1887 by MDCCCLXXXVII.

21. To avoid some of the troublesome repetitions which are common to the Roman system of notation, a numeral is in certain cases placed *before* another of greater value to denote that the value of the larger is to be *diminished* by the amount of the smaller.

Thus IV denotes one less than five, that is 4; IX denotes one less than ten, that is 9; XL denotes ten less than fifty, that is 40; and XC denotes ten less than one hundred, that is 90; so also CCXC denotes 290.

22. The symbols CIↃ, CCIↃↃ, CCCIↃↃↃ, &c. were anciently employed to denote respectively 1000, 10,000 100,000 &c; also IↃ, IↃↃ, IↃↃↃ, &c. to denote respectively 500, 5000, 50,000, &c. In fact M and D are only modified forms of CIↃ and IↃ respectively.

23. Roman numerals were only used to *register* numbers and were never employed in making numerical calculations. The Romans made their calculations by means of counters and a mechanical apparatus called an *Abacus*. The counters used were often pebbles (Latin, *calculus*), whence our word *calculation*.

EXAMPLES I. B.

1. Express all the numbers from 1 to 20 by means of Roman numerals.

2. Express by means of Roman numerals the numbers 20, 30, 40, 50, 60, 70, 80, 90, 200, 400, 600, 800 and 900.

3. Express by means of Roman numerals the numbers 39, 49, 59, 69, 79, 89, 99, 96, 444, 1294 and 1889.

4. Write the numbers LVIII, XXXIX, XLIV, XCIV XCIX, CXCIX, and MMDCCXCIX, in the Arabic notation.

CHAPTER II.

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

24. To **add** two or more numbers is to find a single number which contains as many units as there are in the given numbers *taken together;* and this single number is called the **sum** or the **total.**

The sum of the numbers of the units in two or more groups would therefore be found by forming a single group containing them all, and then counting the number of the units in this single group.

25. The following fundamental truth is evident:

The number of the things in any group will always be found to be the same in whatever order they may be counted.

From this it follows that the sum of the numbers of the things in any two groups will be found by first counting all the things in the first group and then proceeding to the second, that is *by increasing the number in the first group by as many units as there are in the second;* and that the same sum will also be found *by increasing the number in the second group by as many units as there are in the first.*

Thus the sum of 3 and 5 is found by counting five onwards from three, namely four, five, six, seven, *eight;* or by counting three onwards from five, namely six, seven, *eight.* In the first case we are said to add 5 to 3, and in the second case we are said to add 3 to 5; but by the above we see that the results must be the same.

26. Addition is indicated by the sign +, which is read '**plus**.'

Thus 5 + 4 is read five plus four, and denotes that 5 is to be *increased by* 4, that is, that 4 is to be added to 5; also 5+4+3 denotes that 4 is to be added to 5, and then 3 added to the result.

27. The sign =, which is read '*equals*' or '*is equal to*,' is used to denote the equality of two numbers.

Thus 5+4=9 denotes that the sum of 5 and 4 is 9.

28. When children first begin to add they make use of their fingers, but all counting on the fingers, or with any other real objects, should be discontinued as soon as possible, and the results of adding numbers not greater than nine should be given instantaneously.

Tables of the results of the addition of any two numbers each not greater than 10 might at first be made by the pupil, arranged in lines, as for example

8 and 1 are 9, 8 and 2 are 10, 8 and 3 are 11, &c.

EXAMPLES II. A.

[These examples should be practised until great rapidity is attained.]

1. Add 1 and 9, 3 and 8, 2 and 6, 4 and 7, 6 and 3, 4 and 4.
2. Add 7 and 8, 7 and 6, 3 and 9, 5 and 4, 3 and 5, 9 and 8.
3. Add 4 and 3, 9 and 9, 8 and 8, 6 and 9, 7 and 2, 3 and 3.
4. Add 5 and 9, 9 and 4, 6 and 8, 5 and 7, 2 and 9, 8 and 5.
5. Add 7 and 7, 5 and 5, 6 and 6, 8 and 4, 6 and 4, 9 and 7.
6. Add 8 to 15, to 25, to 35, to 45, to 65, and to 95.
7. Add 13 and 7, 23 and 7, 43 and 7, 63 and 7, 83 and 7, and 93 and 7.
8. Add 9 to 17, to 27, to 57, to 67, to 87, and to 97.
9. Begin with 7 and add 2 again and again up to 27.
10. Begin with 2 and add 3 again and again up to 35.
11. Begin with 85 and add 4 again and again up to 101.
12. Begin with 50 and keep on adding sevens until the sum exceeds 100.
13. Begin with 15 and keep on adding nines until the sum exceeds 100.

14. Add the following numbers in order first beginning at the right and then at the left

(1) 2, 7, 4, 0, 6, 9, 5, 2, 6, 5, 9, 3, 4, 8.
(2) 7, 9, 5, 4, 0, 8, 6, 7, 3, 5, 9, 8, 2, 6.
(3) 3, 5, 6, 9, 0, 7, 8, 4, 3, 6, 2, 5, 7, 9.
(4) 9, 6, 7, 4, 2, 8, 1, 3, 7, 5, 4, 6, 5, 8.

29. The sum of any two numbers would be found by counting onwards from the first as many units as there are in the second, but this method would obviously be very troublesome except when the second number is very small.

Now numbers are divided, as we have already learnt, into groups of units, tens, hundreds, &c.; and when numbers are to be added the parts into which they are divided may be added in *any order* we please provided they are all counted; hence *we may first add the units together, then the tens, then the hundreds, and so on.*

The following examples will shew how this principle enables us readily to find the sum of any given numbers.

Ex. 1. Add 235 and 524.

Since we may add the units together, the tens together, &c. we write the numbers so that the units of one fall under the units of the other, and so for the tens and hundreds. Thus

235
524

Now 4 units and 5 units make 9 units, 2 tens and 3 tens make 5 tens, and 5 hundreds and 2 hundreds make 7 hundreds. The required sum is therefore 759. The sum is generally placed below the numbers to be added and is separated from the lowest of them by a horizontal line. Thus

235
524
759

Ex. 2. Add 548, 789 and 197.

As before write the numbers so that units of the same order are in the same vertical line, and draw a horizontal line at the bottom. Thus

548
789
197
1534

Now 7 units and 9 units and 8 units make 24 units, that is 2 tens and 4 units. The 4 units can be put in the column for units but the two tens must be counted with the other tens. We then have 2 tens and 9 tens and 8 tens and 4 tens, which make 23 tens, that is 2 hundreds and 3 tens. The 3 is put in the column for tens, and the two hundreds are '**carried**,' as it is called, to the hundreds, so that we now have 2 hundred and 1 hundred and 7 hundred and 5 hundred, which make 15 hundred. The sum required is therefore 1534 which is written below the horizontal line.

Note. Since ten units of *any* order make one unit of the next higher order, the figures in any column may be added without specifying the kind of units they represent, that is without calling them *tens*, or *hundreds*, or *thousands*, &c., as the case may be. Also instead of saying 7 and 9 are 16 and 8 are 24, we say simply the results, namely 7, 16, 24, and so for the other columns.

Thus the only words used in the last example should be

7, 16, 24; 2 (carried), 11, 19, 23; 2 (carried), 3, 10, 15.

The figures which are printed in thick type are to be written down as they are said.

30. **Concrete quantities** can be added together only when *the unit is the same.* For example 3 horses and 4 cows do not make 7 horses or 7 cows; they do however make 7 *animals;* because regarding them as animals the unit is the same. Also the sum of 3 feet and 4 inches is not 7 feet or 7 inches.

Concrete quantities which are expressed in terms of the same unit are added by finding the sum of their measures.

31. To detect mistakes in addition it is well to add up each line of figures *twice,* once from bottom to top and once from top to bottom. An error is much more likely to be detected in this way than by simply repeating the addition in the same order, for the same mistake is very likely to be made again.

EXAMPLES II. B.

1. Add together 3104, 297, 5649 and 989.
2. Add together 3257, 7523, 6134 and 4316.
3. Find the sum of 319, 562, 1230, 857, 4908 and 9087.
4. Find the sum of 235, 796, 804, 987, 359 and 856.

5. Add together the numbers in each column and in each row of the squares.

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

9	16	23	12	5
13	2	10	19	21
20	24	11	3	7
1	8	17	25	14
22	15	4	6	18

1	101	80	59	38	17	117	96	75	54	33
21	121	89	68	47	26	5	105	84	63	42
30	9	109	88	56	35	14	114	93	72	51
39	18	118	97	76	55	23	2	102	81	60
48	27	6	106	85	64	43	22	111	90	69
57	36	15	115	94	73	52	31	10	110	78
77	45	24	3	103	82	61	40	19	119	98
86	65	44	12	112	91	70	49	28	7	107
95	74	53	32	11	100	79	58	37	16	116
104	83	62	41	20	120	99	67	46	25	4
113	92	71	50	29	8	108	87	66	34	13

Perform the additions indicated below :

6.
3157
294
16903
8057
62934
998
30725

7.
58976
3571
84069
39275
156942
35917
358496

8.
412645
39174
246824
49173
387198
129389
38795

9.
50971
8265
13926
78912
34056
19389
8747

10.
314569
73985
387648
930807
186794
389548
153875

11.
842713
9185
38977
796359
246824
135791
924678

12. Find $301+297+3516+1079+8017+10053$.

13. Find $93084+15614+380176+536174+123456+404040$.

14. Find
$218904+37215+199+582163+397157+81429+79163$.

15. Add together six hundred and ninety-five, one thousand and seventy-four, eleven thousand four hundred and eighty-nine, and fifty-four thousand three hundred and seventy.

16. Add together three million four hundred and seventeen thousand and thirty-five, nine hundred and forty-six thousand seven hundred, fifteen million fifteen thousand and fifteen, and sixty million sixteen hundred and twenty-four.

17. Add together six million five hundred and nine thousand seven hundred and six, three hundred and ninety thousand, five million seven hundred and ninety-nine, eighteen million three hundred and seventy-four thousand five hundred and sixty-seven, and one billion one million one thousand one hundred and one.

18. The population of each of the six northern counties of England is as under:—

Cumberland 250647, Durham 867258, Lancashire 3454441, Northumberland 434086, Westmoreland 64191 and Yorkshire 2886564.

What is the total population?

19. The area in acres of each of the six largest counties of England is as under:—

Yorkshire 3882851, Lincolnshire 1767879, Devon 1655208, Norfolk 1356173, Northumberland 1290312, and Lancashire 1208154.

What is the total acreage of these six counties?

20. The area in square miles of each of the five colonies into which Australia is divided is as under:—

New South Wales 310938, Victoria 87884, South Australia 903690, Queensland 670000, and Western Australia 978300.

What is the number of square miles in the whole of Australia?

SUBTRACTION.

32. **Subtraction** is the process of finding how many units are left when one given number is *taken away* from another. The result of subtraction is called the **Remainder** or the **Difference**.

Any two numbers can be added together; it is, however, impossible to subtract one number from a smaller number.

33. It is clear that if what was left were added to what was taken away the sum would be the original number from which the subtraction was made.

Hence *the remainder is that number which when added to the number to be subtracted will give the number from which the subtraction is to be made.*

Thus to subtract 5 from 12 is to find the number which when added to 5 will make 12.

The question involved in subtraction may be put in different ways. Thus it may be asked:

(1) What must be added to 5 to make 12?
(2) What is the remainder when 5 is taken from 12?
(3) By how many is 12 greater than 5?

34. Subtraction is indicated by the sign −, which is read "**minus.**"

Thus $9-4$ is read nine minus four, and denotes that 9 is to be diminished by 4, that is that 4 is to be subtracted from 9; also $5-4+3$ denotes that 4 is to be taken from 5, and then 3 added to the result.

35. The knowledge of the results of the addition of numbers not greater than ten will furnish us with the results of the subtraction of small numbers. Examples of subtractions of this kind should be practised until great rapidity is attained.

EXAMPLES II. C.

1. What must be added to 5 to make 8, to make 13, to make 10, to make 12?

2. What must be added to 7 to make 9, to make 12, to make 10, to make 15?

3. What must be added to 8 to make 10, to make 12, to make 14, to make 16?

Fill up the blanks below

4.	9 and	make 10,	3 and	make 11,	2 and	make 8,
	4 and	make 11,	6 and	make 9,	4 and	make 8.
5.	7 and	make 15,	6 and	make 13,	9 and	make 12,
	4 and	make 9,	3 and	make 8,	8 and	make 17.
6.	3 and	make 7,	9 and	make 18,	8 and	make 16,
	6 and	make 15,	7 and	make 9,	3 and	make 6.

7. How many are left when we take 7 from 14, 5 from 10, 6 from 12, 8 from 12, 4 from 10, and 7 from 16 respectively?

8. How many are left when we take 5 from 14, 4 from 13, 8 from 14, 7 from 12, 9 from 11, and 5 from 13 respectively?

Find the difference between each of the following pairs of numbers:

9. 5 and 12, 7 and 16, 9 and 18, 3 and 11, 6 and 14, 8 and 15.

10. 3 and 8, 5 and 11, 6 and 13, 8 and 14, 7 and 15, 9 and 16.

11. Begin with 50 and go on diminishing by fours as many times as possible.

12. Begin with 53 and go on diminishing by fives as many times as possible.

13. Begin with 70 and go on diminishing by sixes as many times as possible.

36. The consideration of the following examples will shew how the difference between any two numbers can be found.

Ex. 1. Subtract 524 from 759.

The smaller number is placed below the greater so that the units of the same order are in the same vertical column.

$$\begin{array}{r} 759 \\ 524 \\ \hline \end{array}$$

We have to see what must be added to 524 to make 759. Begin with the units: **5** must be added to 4 to make 9; **3** must be added to 2 to make 5; and **2** must be added to 5 to make 7. Thus the required difference is 235.

$$\begin{array}{rll} 759 & & \\ 524 & \text{mental work} & \begin{cases} \textbf{4 and 5 are 9,} \\ \textbf{2 and 3 are 5,} \\ \textbf{5 and 2 are 7.} \end{cases} \\ \hline 235 & & \end{array}$$

The figures which are printed in thick type are to be written down as they are said.

Ex. 2. Subtract 357 from 783.

$$\begin{array}{r} 783 \\ 357 \\ \hline \end{array}$$

We have to find what must be added to 357 to make 783.

Now 7 is greater than 3, and therefore no number can be added to 7 to make 3; we can, however, find a number, namely 6, which when added to 7 will make 13, *and 6 is the only number less than* 10 *which when added to* 357 *gives a sum which ends with* 3. The units figure of the required difference must therefore be 6. Then, as the 7 and 6 together exceed 3 units by 1 ten,

we must count this ten with the 5 tens, and therefore only 2 more tens are required to make up the 8. Finally 3 and 4 are 7. Thus the required difference is 426.

783
357 mental work { 7 and 6 are 13,
426 { 1 (carried) and 5 and 2 are 8,
{ 3 and 4 are 7.

In dealing with the figures of any column it is not necessary to specify the kind of units they represent. [See Note, page 14.]

From the above examples it will be seen that one number can be subtracted from another by the following method.

Rule:—*Place the numbers so that the smaller is below the larger and with units under units, tens under tens, &c. Then, beginning at the units column, write down what is required to make the lower digit equal to the higher; but, if the lower digit is already greater than the higher, write down what is required to make the lower digit exceed the higher by ten, and in this case 'carry' one to the lower digit of the next column; and proceed in this way to the end.*

Subtract 4973 from 6248.

6248
4973 mental work { 3 and 5 are 8,
1275 { 7 and 7 are 14,
{ 1 (carried) and 9 and 2 are 12,
{ 1 (carried) and 4 and 1 are 6.

37. One concrete quantity cannot be subtracted from another unless they are both expressed in terms of the same unit. For example, we cannot take 5 tons from 7 miles; nor can we take 3 feet from 60 inches, unless either 3 feet is expressed in inches or 60 inches expressed in feet.

38. It is easily seen that if from a given number several numbers be taken in succession the result will be the same as if the *sum* of those numbers were subtracted all at once from the given number.

The following example will shew how the sum of several numbers can be taken from a given number by one operation.

Ex. Subtract the sum of 365, 648 and 759 from 2314.

2314
365
648
759
542

Add 9, 8 and 5 and then 'make up' to the next greater number which ends in 4. Carry as in addition, and so proceed.

Mental work { 9, 17, 22 and 2 make 24;
2, 7, 11, 17 and 4 make 21;
2, 9, 15, 18 and 5 make 23.

39. When several operations of addition and subtraction have to be performed in succession the result is the same *in whatever order the operations are performed.*

For example, to take away 5 from any given number (which must, however, be not less than 5) and then add 3 gives the same result as to add 3 and then take away 5.

Hence, to find $28-15+26-17-14+12$, it is not necessary actually to take 15 from 28, and then add 26, and then subtract 17, and then subtract 14, and finally add 12. The result will be found equally well by first finding the sum of 28, 26 and 12, the numbers to be added; then the sum of 15, 17 and 14, the numbers to be subtracted; and finally taking the difference of these two sums.

EXAMPLES II. D.

1. Subtract 1296 from 3145, 817 from 3002, and 1234 from 4321.

2. Subtract 1597 from 7915, 18235 from 1000000, and 13579 from 24680.

3. Find the values of $645-378$, $307-149$, $294-208$, $2179-1984$, $3206-1679$, and $120573-98765$.

4. By how many does 384127 exceed 197246?

5. By how many is 569 less than 1000?

6. By how many is 38427 less than 50182?

7. Find the values of

(1) $31+97-23+175-184$. (2) $151-77+94-111$.

(3) $315-127-172+358-265$. (4) $742-329-197+215$.

(5) $9824-1793+4268-8997$. (6) $1897-386+2785-2907$.

8. Subtract from 121097 the sum of 7916, 1214, 1397 and 34162.

9. Subtract from 1000000 the sum of 421654, 127, 31562, 1795 and 123456.

10. Subtract 27 from 80, and then 27 from the remainder, and so on as many times as possible; and find the final remainder.

11. What number must be taken from 81 to leave 37 as remainder?

12. What number must be taken from one hundred thousand to leave five thousand four hundred and eighty-seven as remainder?

13. The difference between two numbers is 145 and the greater is 597; what is the smaller?

14. The sum of two numbers is 1000 and one of them is 594; what is the other?

15. On a man's birthday in 1891 he was 63 years old. In what year was he born?

16. In 1891 a man of 65 was on his birthday just 37 years older than his son. In what year was the son born?

17. Add the sum of 516 and 784 to the difference between 314 and 176.

18. Add the difference between 1925 and 1789 to the difference between 3421 and 1679.

19. In an orchard there are 1572 fruit trees; of these 352 are apple trees, 275 are pear trees, and 187 are plum trees. How many other trees are there?

MULTIPLICATION.

40. To **multiply** one number, called the **multiplicand**, by another number, called the **multiplier**, is to find the sum of as many repetitions of the multiplicand as there are units in the multiplier.

Thus multiplication is an *addition* of numbers which are all equal.

The result of the operation of multiplication is called the **product** of the multiplicand and multiplier, and they are called **factors** of the product.

For example, to multiply 5 by 4 is to find the sum of 5 repeated 4 times, that is $5+5+5+5$. The product is 20, and 5 and 4 are factors of 20.

41. The multiplication of any two numbers not greater than nine is easily found by actual addition. It will be shewn that every case of multiplication can be reduced to a series of cases of multiplications of numbers not greater than ten; it is therefore essential to learn by heart all the products of such numbers.

These products are given in the following table, called the **Multiplication Table***.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Every horizontal line in the table gives the products of the number which begins the line by the first twelve numbers in order. Thus the fourth line can be read 1 four is 4, 2 fours are 8, 3 fours are 12, 4 fours are 16, &c.

It would be good practice for the student to write down rows of figures, for example

3, 5, 7, 2, 0, 9, 7, 6, 4, 3, 5, 8, 7;

and then give, *as rapidly as possible*, the products of each figure and that which comes next. Thus

15, 35, 14, 0, 0, 63, 42, 24, 12, 15, 40, 56.

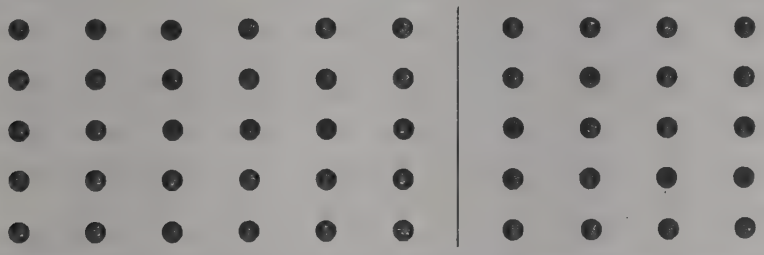
* It is usual and desirable, though not absolutely necessary, to learn the Multiplication Table as far as 12 times 12. This table should in the first instance be made again and again by the pupil himself by actual addition.

42. Multiplication is indicated by the sign $\times$, which is read '**multiplied by**' or '**into**'.

Thus 5×4 is read 5 multiplied by 4, or 5 into 4, and denotes that 5 is to be multiplied by 4; also $5 \times 4 \times 3$ denotes that 5 is to be multiplied by 4 and this product multiplied by 3.

When one number is multiplied by two or more other numbers *in succession* the result is called the **continued product**.

43. Before considering how to find the product of any two numbers certain general truths, which hold good for all numbers whatever, must be investigated. For this purpose consider the following arrangement of dots:



The total number of the dots is independent of the way in which they are counted.

Now there are 10 dots in each row and 5 rows; the whole number of the dots can therefore be counted as 10 repeated 5 times, that is 10×5. Also there are 5 dots in each vertical column and 10 columns; the whole number of the dots can therefore be counted as 5 repeated 10 times, that is 5×10. Hence 10×5 is the same as 5×10. It is clear that this result would hold good however many rows and columns there might be; thus we are led to

Theorem I. *The product of any number by any second number is the same as the product of the second by the first.*

Again, if we consider separately the two parts divided by the vertical line, we see that the whole number of dots is the sum of 6 repeated 5 times and

4 repeated 5 times, the 6 and 4 together making 10, so that 10×5 is the same as $6 \times 5 + 4 \times 5$. It is clear that the same result would hold good wherever the vertical line were drawn, and would still be true if there were two or more vertical lines: thus we are led to

Theorem II. *The product of any two numbers is the same as the sum of the products of the multiplier and any two or more numbers which together make up the multiplicand.*

Again, the whole number instead of being considered as 5 repeated 10 times may be taken as the sum of 5 repeated 6 times and 5 repeated 4 times. And here again the result would hold good in whatever way the number of columns was subdivided: thus we are led to

Theorem III. *The product of any two numbers is the same as the sum of the products of the multiplicand and any two or more numbers which together make up the multiplier.*

44. Now consider the following arrangement

5 5 5 5 5 5
5 5 5 5 5 5
5 5 5 5 5 5

Here the sum of all the numbers consists of 5 repeated 6×3 times. But in each row 5 is repeated 6 times and the rows are repeated 3 times; hence the sum of all the numbers is $5 \times 6 \times 3$. Again, in each column 5 is repeated 3 times and the columns are repeated 6 times; hence the whole sum is $5 \times 3 \times 6$. Thus to multiply by 6 and 3 in succession, in any order, gives the same result as to multiply at once by 6×3, that is by 18. It is clear that the same would be true for any other numbers whatever: thus we have

Theorem IV. *To multiply by two or more numbers in succession gives the same result as to multiply at once by their product.*

45. To multiply by 10, 100, &c. Numbers are divided into groups of units, tens, hundreds, &c.; and a number is multiplied by 10 when each of its parts is multiplied by 10 [Theorem II.]. Now, by putting a nought (or cipher as it is sometimes called) at the right of any number expressed in figures, each figure is moved one place to the left, and therefore the units are turned into tens, the tens into hundreds, &c., so that each of the parts of the number, and therefore the whole number, is multiplied by 10.

Hence *any number expressed in figures is multiplied by* 10 *by annexing a nought at the right.*

By affixing a second nought the number is again multiplied by 10, and therefore the addition of two noughts at the right of a number multiplies the number by 10×10, that is by 100.

So also, a number is multiplied by 1000 by adding three noughts to the right; and so on.

For example

$79 \times 10 = 790$, $534 \times 100 = 53400$, and $370 \times 1000 = 370000$.

46. The following examples will shew how to find the product of any two numbers.

Ex. 1. Multiply 423 by 7.

In multiplication the multiplier is placed under the multiplicand so that units are under units, tens under tens, &c.

$$\begin{array}{r} 423 \\ 7 \\ \hline 2961 \end{array}$$

By Theorem II, we may multiply the units, tens, &c. *separately* by 7 and add the results. As in addition, it is best to begin with the units.

We know from the Table that 3 units multiplied by 7 is 21 units, that is 2 tens and 1 unit. The 1 unit is put in the units column, and the 2 tens are to be counted with other tens. Next 2 tens multiplied by 7 are 14 tens, and 14 tens and the 2 tens carried make 16 tens, that is 6 tens and 1 hundred. The 6 tens are put in the column for tens and the 1 hundred is 'carried'. Finally 4 hundred multiplied by 7 are 28 hundred, and 28 hundred and the 1 hundred carried make 29 hundred. The product required is therefore 2961.

It is not necessary to specify, as we have done above, the kind of units which are being multiplied at any stage [see Note, page 14]. Thus the only words used in the above example should be 7 threes are 21; 7 twos are 14 and 2 are 16; 7 fours are 28 and 1 are 29.

After some practice it would only be necessary to say 21; 14, 16; 28, 29.

Ex. 2. Multiply 564 by 2037.

By Theorem III. we may multiply 564 separately by 7, by 30 and by 2000, and then add these partial products.

To multiply 564 by 7 we proceed as in the preceding example, and thus obtain 3948.

In order to multiply 564 by 30, we may [Theorem IV.] multiply by 3 and 10 in succession. Now 564×3 is found as above to be 1692, and 1692×10 is 16920.

Again, to multiply 564 by 2000, we may multiply by 2 and by 1000 in succession. Now 564×2 is found as above to be 1128, and 1128×1000 is 1128000.

The whole work is written as under:

$$\begin{array}{r} 564 \\ 2037 \\ \hline 3948 \\ 16920 \\ 1128000 \\ \hline 1148868 \end{array}$$

The noughts at the ends of the lines, which are due to multiplication by so many tens, hundreds, thousands, &c., may be omitted provided care be taken to **put the first figure of each line of the multiplication under the figure of the multiplier from which that line arises.** [Examples in multiplication should however be worked in full until the process is firmly fixed in the mind of the student.]

Thus the work in finding the product of 31642 by 100506 would be written

$$\begin{array}{r} 31642 \\ 100506 \\ \hline 189852 \\ 158210 \\ 31642 \\ \hline 3180210852 \end{array}$$

47. In all cases of multiplication **the multiplier must be an abstract number**, for to repeat anything 5 shillings times or 3 tons times is absurd; the multiplicand, however, may be either a concrete quantity or an abstract number.

Thus we can multiply 3 feet by 5, but not 3 feet by 5 feet, nor 5 by 3 feet.

48. The continued product of the same number repeated two or more times is called a **power** of that number.

Thus 5×5 is called the *second power* or the **square** of 5, $5 \times 5 \times 5$ is called the *third power* or the **cube** of 5, $5 \times 5 \times 5 \times 5$ is called the *fourth power* of 5, and so on.

Thus the squares of the first nine numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81. Also the cubes are 1, 8, 27, 64, 125, 216, 343, 512, 729.

A small figure placed above and to the right of a number is used to denote a power of that number; for example 5^4 denotes $5 \times 5 \times 5 \times 5$. Also $10^2 = 10 \times 10 = 100$, $10^3 = 10 \times 10 \times 10 = 1000$, and $10^6 = 1000000$.

It should be noticed that the second power of 10 contains *two* noughts, the third power *three* noughts, and so on.

49. We have seen that to multiply by two or more numbers in succession gives the same result as to multiply at once by their product; for example to multiply by 7 and then by 5 gives the same result as to multiply at once by 7×5, that is by 35.

We shall hereafter meet with cases where this principle can be applied with advantage, but in the multiplication of abstract numbers, or of concrete quantities expressed in terms of a single unit, there is no great advantage in multiplying by factors in succession rather than by their continued product.

EXAMPLES II. E.

Multiply

1. 37 by 3.	**2.** 65 by 5.	**3.** 253 by 9.
4. 197 by 8.	**5.** 384 by 7.	**6.** 909 by 6.
7. 1083 by 11.	**8.** 3408 by 12.	**9.** 597 by 11.
10. 142857 by 7.	**11.** 12345679 by 9.	
12. 25 by 20.	**13.** 27 by 30.	**14.** 36 by 70.
15. 318 by 50.	**16.** 527 by 60.	**17.** 894 by 80.
18. 125 by 700.	**19.** 389 by 600.	**20.** 239 by 900.
21. 21670 by 4000.	**22.** 5790 by 8000.	
23. 6175 by 8000.	**24.** 821400 by 5000.	
25. 25 by 25.	**26.** 27 by 39.	**27.** 79 by 97.

28. 38 by 56. 29. 79 by 87. 30. 98 by 39.
31. 125 by 47. 32. 384 by 65. 33. 908 by 73.
34. 18 by 12345679. 35. 63 by 12345679.
36. 697 by 123. 37. 587 by 358. 38. 399 by 586.
39. 2809 by 702. 40. 1973 by 904. 41. 3097 by 807.
42. 3156 by 2065. 43. 2178 by 5506.
44. 7802 by 2005. 45. 3008 by 2345.
46. 3251 by 3579. 47. 6874 by 2468.
48. 13579 by 21695. 49. 12837 by 56294.
50. 310972 by 90706. 51. 305009 by 72809.
52. 823156 by 753698. 53. 123456 by 654321.
54. 826075 by 1509607. 55. 5009826 by 7090068.
56. 8257314 by 78167094. 57. 21840376 by 9287915.
58. 20 by 125. 59. 50 by 350. 60. 800 by 125.
61. 20 by 315. 62. 400 by 125. 63. 600 by 8745.
64. 8000 by 1250. 65. 9000 by 81472.
66. 12000 by 28971.

Find the continued products of

67. 12, 18 and 15. 68. 17, 18 and 19.
69. 3, 4, 5, 6, 7, 8 and 9. 70. 10, 11, 12, 13 and 14.

Find

71. $333667 \times 27 \times 37$. 72. $3 \times 7 \times 9 \times 11 \times 13 \times 37$.

Find the squares of

73. 170. 74. 220. 75. 360. 76. 430.
77. 125. 78. 537. 79. 4690. 80. 64700.
81. 17495. 82. 38764. 83. 215729. 84. 983459.

Find the cubes of

85. 80. 86. 160. 87. 800. 88. 1600.
89. 425. 90. 1608. 91. 3507. 92. 16730.

93. There are 20 shillings in one pound; how many shillings are there in 37 pounds?

94. There are 2240 pounds in one ton; how many pounds are there in 517 tons?

95. There are 168 hours in one week; how many are there in 506 weeks?

96. There are 24 sheets in a quire of paper, and 20 quires in a ream; how many sheets are there in 524 reams?

97. There are 86400 seconds in a day; how many are there in 365 days?

98. There are 4840 yards in an acre; how many yards are there in 8567 acres?

99. In an orchard there are 57 rows of gooseberry bushes, and there are 256 bushes in each row; how many are there altogether?

100. A book has 312 pages, on each page there are 32 lines, and in each line there are 42 letters. How many letters are there altogether?

DIVISION.

50. To **divide** one number, called the **dividend**, by another number called the **divisor**, is to find how many times the divisor is contained in the dividend; and this number of times is called the **quotient.**

For example, to divide 12 by 4 is to find how many fours there are in 12. The simplest method of finding the required number of fours is, take 4 from 12, and then 4 from the remainder, and so on as many times as possible. It will be found that nothing is left after taking away 3 fours. Hence 3 is the required quotient.

51. Since the dividend contains the divisor as many times as there are units in the quotient, *the dividend is equal to the product of the divisor and the quotient.*

We may then say that *division is the process by which the remaining factor is found when the product of two factors is given and also one of the factors.*

Thus division is the inverse or undoing of multiplication.

52. Division can be looked upon from two different points of view, the distinction between which is best seen by taking as an example the division of a concrete quantity.

We have 5 feet $\times 7 = 35$ feet; and in connection with the undoing of this multiplication, there are the two distinct questions

(1) How many times is 5 feet contained in 35 feet? The answer to which is 7 *times.*

(2) If 35 feet be divided up into 7 equal parts, what will each part be? Or, what length is contained 7 times in 35 feet?

The answer to which is 5 *feet.*

Thus in division, either the divisor is an abstract number and the quotient a quantity of the same kind as the dividend, or else the divisor is a quantity of the same nature as the dividend and the quotient is an abstract number.

53. Division is indicated by the sign ÷, which is read '**divided by**' or '**by**'.

Thus $24 \div 4$ is read 24 by 4, and denotes that 24 is to be divided by 4; also $24 \div 4 \div 3$ denotes that 24 is to be divided by 4 and the result divided by 3, and $24 \div 4 \times 3$ denotes that 24 is to be divided by 4 and the result multiplied by 3.

54. Inexact Division. If we try to divide 14 by 4, we find that after subtracting 3 fours there is only 2 left. Thus 14 contains 3 fours and there is 2 left over.

The number left over is called the **Remainder.**

One number is said to be **exactly divisible** by another when it is divisible without remainder.

55. It follows from the definition of division that *the product of the divisor and the quotient plus the remainder is equal to the dividend,* that is

$$\text{Divisor} \times \text{Quotient} + \text{Remainder} = \text{Dividend}.$$

Hence, if any three of these four numbers be given the remaining one can be found.

Ex. 1. The divisor is 5, the quotient is 20, and the remainder is 2; what is the dividend?

The dividend must exceed the product of the divisor and quotient by 2. Hence

$$\text{Dividend} = 5 \times 20 + 2 = 102.$$

Ex. 2. The dividend is 59, the quotient 7, and the remainder 3. What is the divisor?

The dividend must exceed the product of the quotient and divisor by 3. Hence the product of the quotient and the divisor is $59 - 3 = 56$. Hence the divisor $= 56 \div 7 = 8$.

56. Division could always be performed by successive subtractions of the divisor, as in Art. 50; but, except in the case of very small numbers, the process would be extremely tedious, and the necessity for these successive subtractions is obviated by a knowledge of the results of multiplication.

For example, to divide 75 by 9.

Since we know that 8 nines are 72 and that 9 nines are 81, we see that $75 \div 9$ gives 8 for quotient and 3 for remainder.

EXAMPLES II. F.

Give the quotient in each of the following cases, and the remainder whenever the division is not exact.

1.	$12 \div 4$.	**2.**	$18 \div 9$.	**3.**	$35 \div 7$.	**4.**	$56 \div 8$.
5.	$60 \div 10$.	**6.**	$49 \div 7$.	**7.**	$81 \div 9$.	**8.**	$72 \div 8$.
9.	$56 \div 7$.	**10.**	$36 \div 6$.	**11.**	$64 \div 8$.	**12.**	$45 \div 9$.
13.	$15 \div 4$.	**14.**	$17 \div 5$.	**15.**	$18 \div 7$.	**16.**	$17 \div 3$.
17.	$37 \div 9$.	**18.**	$43 \div 5$.	**19.**	$68 \div 7$.	**20.**	$70 \div 8$.
21.	$80 \div 9$.	**22.**	$55 \div 9$.	**23.**	$53 \div 7$.	**24.**	$48 \div 5$.
25.	$92 \div 9$.	**26.**	$87 \div 8$.	**27.**	$80 \div 7$.	**28.**	$63 \div 5$.
29.	$70 \div 6$.	**30.**	$100 \div 9$.				

57. The following example will shew how one number can be divided by another.

Ex. To divide 102739 by 29.

We have to find what number multiplied by 29 will produce 102739, or if there be no such number, then what number when multiplied by 29 will produce a product which is smaller than 102739 but which differs from it by less than 29.

We shall find the number of units of the different orders *separately*, beginning at the highest. To do this, separate off from the left of the dividend the least number of figures such that the number represented by these figures alone is not less than the dividend: in the present case this would be 102. Since 102 actually represents 102 *thousands*, the quotient required must be greater than *one* thousand and less than *ten* thousand. It will be readily seen that 102 is greater than 29×3 but less than 29×4: the quotient must therefore lie between 3000 and 4000. Multiply 29 by 3000 and subtract the product from the dividend; the remainder will then be 15739. The first stage of the process being as under

$$\begin{array}{r} 29 \,)\, 102739 \,(\, 3000 \\ \underline{87000} \\ 15739 \end{array}$$

It now remains to find how many times 15739 contains 29. Proceed in the same manner as before, and we see that 15739 when divided by 29 will give a quotient greater than 5 *hundred* but less than 6 *hundred*. Multiply 29 by 500 and subtract from 15739; the remainder will then be found to be 1239. Repeat this process until we have found the greatest number of *tens* in the quotient of 1239 divided by 29 and also the remainder; and then the greatest number of *units* in the quotient of this last remainder when divided by 29; what is left over from this last division being called *the* remainder. The whole process would be

```
29 ) 102739 ( 3000 + 500 + 40 + 2
       87000
      -------
       15739
       14500
      -------
        1239
        1160
      -------
          79
          58
      -------
          21 remainder.
```

Thus the required quotient is $3000+500+40+2$, that is 3542, and the remainder is 21.

It is not necessary to write down all the zeros provided the figures are all put in their proper places without this aid. Also the figures on the right of the dividend can be brought down *one at a time*, and there will be a figure of the quotient corresponding to each one of them; if however the number formed by the remainder at any stage with the next figure of the dividend placed on its right be less than the divisor, the corresponding figure of the quotient will be zero.

The process of division may therefore, after a little practice, be written in the shortened form:

```
29 ) 102739 ( 3542
      87
     ---
     157
     145
     ---
      123
      116
     ---
       79
       58
     ---
       21 remainder.
```

```
145 ) 44393 ( 306
      435
     ----
      893
      870
     ----
       23 remainder.
```

In the example on the right, the remainder 8 with the next figure, 9, of the dividend affixed is less than 145; we therefore put 0 after the 3 in the quotient, and bring down 3, the next figure of the dividend.

The whole difficulty of the process consists in finding how many times the divisor is contained in the part of the dividend under consideration at any stage. Beginners often make mistakes, which however are at once obvious; for the product of the divisor and the supposed quotient must never be *greater* than the part of the dividend under consideration, and the apparent remainder at any stage must be *less* than the divisor.

EXAMPLES II. G.

Divide

1. 182 by 13.	**2.** 204 by 17.	**3.** 221 by 17.
4. 399 by 19.	**5.** 575 by 23.	**6.** 899 by 29.
7. 702 by 26.	**8.** 1054 by 34.	**9.** 4185 by 31.
10. 1591 by 37.	**11.** 6016 by 94.	**12.** 710007 by 87.

13. 435435 by 65.
14. 430686 by 71.
15. 415242 by 59.
16. 426713 by 47.
17. 562171 by 53.
18. 850902 by 78.
19. 1173021 by 97.
20. 1034550 by 95.
21. 2706420 by 86.
22. 11336 by 109.
23. 22563 by 207.
24. 160335 by 315.
25. 39483 by 123.
26. 21112 by 104.
27. 185745 by 305.
28. 801738 by 567.
29. 8035370 by 2674.
30. 9570744 by 1593.
31. 407514744 by 6724.
32. 31587678 by 5067.
33. 1828740450 by 3075.
34. 245260581 by 81509.
35. 403755770 by 64046.
36. 1326578163 by 32507.
37. 81819924 by 52348.
38. 3980946216 by 56209.
39. 1840474701 by 56789.
40. 322828856 by 686.
41. 191810713444 by 437962.
42. 122615327232 by 4968.
43. 807051 by 916.
44. 900675 by 765.
45. 3156024 by 1357.
46. 8124016 by 2894.
47. 3172150 by 3467.
48. 3452164 by 1858.

49. The trees in an orchard are arranged in 153 rows, with the same number of trees in each row, and there are 16371 trees altogether. How many trees are there in each row?

50. There are 86400 seconds in a day; in how many days are there 13564800 seconds?

58. Short Division. When the divisor is not greater than 12, the process of division can be written down in a very compact form. The method will be seen from the following example.

Ex. Divide 43251 by 8.

The operation is set down in the following form,

$$8\overline{)43251}$$

5406, remainder 3.

Explanation. First $43 \div 8$ gives quotient 5 and remainder 3; we put 5 under the 3 of the dividend, as the 5 represents units of the same order as the 3 (namely *thousands*, in the present case). Then the remainder 3 is equal to 30 units of the next lower order, and taking into account the next figure of the dividend, namely 2, we have 32 which when divided by 8 gives quotient 4 and 0 remainder; we put down 4 next to 5, and have nothing to 'carry'. Then $5 \div 8$ gives quotient 0 and remainder 5; we put down 0 next to 4 and 'carry' 5. The 5 carried and 1, the next figure of the dividend, make 51 which when divided by 8 gives quotient 6 and remainder 3. Thus the complete quotient is 5406 with remainder 3.

EXAMPLES II. H.

Divide

1.	92 by 4.	**2.**	87 by 3.	**3.**	75 by 5.
4.	234 by 6.	**5.**	785 by 5.	**6.**	918 by 9.
7.	7215 by 5.	**8.**	6402 by 6.	**9.**	3564 by 9.
10.	6822 by 12.	**11.**	7568 by 11.	**12.**	35628 by 12.
13.	72156 by 9.	**14.**	346089 by 7.	**15.**	823507 by 8.
16.	2104316 by 6.	**17.**	123456 by 7.	**18.**	987654 by 9.

19. 563753696 by 11. **20.** 1374819756 by 12.

59. Division by Factors. We have seen that to multiply by two or more numbers in succession gives the same result as to multiply at once by their product. It therefore follows conversely that to divide by two or more numbers in succession gives the same result as to divide at once by the product of the numbers.

Ex. Divide 11445 by 35.

Since $35 = 7 \times 5$, we may divide by 7 and 5 in succession. The process is written as under

$$\begin{array}{r} 7\overline{)11445} \\ 5\overline{)1635} \\ 327 \end{array}$$

60. When one number is divided by several others in succession, the method of finding the remainder will be seen from the following example.

Ex. 1. Divide 11467 by 35.

7)11467

5)1638 *sevens* and 1 unit over.

327 *thirty-fives* and 3 *sevens* over.

The whole remainder is therefore 3 sevens and 1 unit, that is 22.

From the above it will be seen that the whole remainder is found by *multiplying the remainder after the second division by the first divisor and then adding the remainder after the first division.*

Ex. 2. Divide 251633 by $3 \times 5 \times 7$.

3) 251633

5) 83877 groups of 3 each and 2 units over.

7) 16775 groups of 3×5 each and 2 groups of 3 each over.

2396 groups of $3 \times 5 \times 7$ each and 3 groups of 3×5 each over.

The whole remainder is therefore 3 groups of 3×5 each $+ 2$ groups of 3 each $+ 2 = 3 \times 3 \times 5 + 2 \times 3 + 2 = 45 + 6 + 2 = 53$.

Thus if there are more than two successive divisions the whole remainder is found by multiplying each remainder by all the divisors preceding that from which the remainder arises, and then adding these results to the first remainder.

61. Division by 10, 100, &c. To divide any number by 10 it is only necessary to cut off the last figure on the right.

For example, the number 314625 consists of 31462 *tens* and 5 *units;* hence when the number is divided by 10 the quotient is 31462 and the remainder 5. Again 314625 consists of 3146 *hundreds* and 25; hence when the number is divided by 100 the quotient is 3146 and the remainder 25.

Ex. To divide 315637 by 20.

20)315637

15781 remainder 17.

Explanation. We divide by 20 by dividing by 10 and then by 2. The division by 10 is performed by cutting off the last figure, namely 7. We then divide 31563 by 2 and find the quotient 15781 and remainder 1, that is 1 *ten*, which with 7, the previous remainder, makes 17 for the whole remainder.

62. The work of finding some products may be shortened by making use of the fact that multiplication or division by 10, or by any power of 10, can be written down at once.

Ex. 1. Multiply 6174 by 25.

Since $25 = 100 \div 4$, we shall multiply by 25 if we first multiply by 100 and then divide by 4. For by multiplying by 100 we get 4 times too much, which is put right when we divide by 4. To multiply by 25 we may therefore affix two noughts and divide by 4. Thus

$$4 \overline{)\, 617400}$$
$$154350$$

Ex. 2. Multiply 6174 by 125.

Since $125 = 1000 \div 8$, we multiply by 1000 and then divide by 8, that is, we affix three noughts and divide by 8. Thus

$$8 \overline{)\, 6174000}$$
$$771750$$

The methods adopted in the following examples are also worth notice.

Ex. 3. Multiply 617425 by 10812.

Since $108 = 12 \times 9$, $10812 = 12 + 12 \times 900$.

Hence if we multiply first by 12, and then multiply *that result* by 900, which we do by multiplying by 9 and shifting the product two places to the left, the whole product will be the sum of these two partial products.

$$\begin{array}{r} 617425 \\ 10812 \\ \hline 7409100 \\ 66681900 \\ \hline 6675599100 \end{array}$$

Ex. 4. Multiply 7964 by 9998.

Since $9998 = 10000 - 2$, we can multiply by 10000 and by 2, and take the difference of these products.

$$\begin{array}{r} 7964 \\ 9998 \\ \hline 79640000 \\ 15928 \\ \hline 79624072 \end{array}$$

EXAMPLES II. I.

Divide, using factors not greater than 12.

1. 396 by 18. **2.** 816 by 24. **3.** 625 by 25.
4. 3753 by 27. **5.** 8820 by 36. **6.** 15750 by 42.
7. 1958528 by 64. **8.** 59081805 by 81.
9. 13339728 by 108. **10.** 10654069140 by 132.
11. 316794 by 45. **12.** 7196243 by 35.
13. 2106935 by 36. **14.** 9172143 by 72.
15. 22222222 by 99. **16.** 123456789 by 132.
17. 32163 by 20, 30 and 40. **18.** 21574 by 20, 40 and 60.
19. 123456 by 20, 30 and 40. **20.** 158937 by 20, 50 and 70.
21. 2167 by 30 and 50. **22.** 16819 by 30 and 80.
23. 17943 by 40 and 60. **24.** 21985 by 50 and 90.
25. 217943 by 500. **26.** 712415 by 700.
27. 217643 by 216. **28.** 1234567 by 242.

Multiply as shortly as you can

29. 74562 by 25. **30.** 4162 by 25.
31. 12678 by 125. **32.** 3874 by 125.
33. 79624 by 99. **34.** 1897 by 999.
35. 29075 by 998. **36.** 79184 by 99997.
37. 2841 by 427. **38.** 189743 by 819.
39. 982406 by 14412. **40.** 87425 by 132012.
41. 24687 by 48164. **42.** 791235 by 72366.

MISCELLANEOUS EXAMPLES I.

1. Express in words 3015602, and in figures eleven million five hundred thousand two hundred and fourteen.

2. Find the sum of 30157, 12468, 31947 and 36539.

3. By how many is 13018 greater than 12997?

4. Multiply 8000 by 1250 and 3200 by 12345.

5. How many times can 317 be subtracted from 1389, and what is the remainder?

6. Express MDCCCLXXIX in the Arabic notation, and 1449 by means of Roman numerals.

7. Find $1325+3016+79+90167$.

8. Find $316-179+257-89-185+398-485$.

9. Multiply 1234 by 4321 and 9009 by 31562.

10. How many nineteens are there in five thousand, and how many are over?

---

11. Express in words 1632057 and 3004167201500.

12. Subtract the sum of 3158, 2016 and 5143 from 11111.

13. Multiply the difference between seventy-six million and seventy-six and four hundred and forty thousand four hundred and forty by eleven hundred and fourteen.

14. A farmer has 197 sheep and three times as many lambs. How many sheep and lambs has he altogether?

15. Find by short divisions how many thirty-fives there are in 31578, and how many are over.

---

16. By how many is one million eight thousand nine hundred and seventy-four less than two million eleven hundred and twelve?

17. Find $3142-1250-989+6217-3587-1924$.

18. A farmer had 2000 quarters of wheat. He sold 527 quarters to one man and 255 quarters to each of three others. How many quarters were left unsold?

19. How many letters are there in a book of 375 pages, each page of which contains 32 lines and each line 45 letters?

20. What number must 59755 be divided by in order that the quotient may be 19?

---

21. In one school there are one hundred and seventy-six boys and one hundred and twelve girls; and in another school there are half as many boys and twice as many girls. How many scholars altogether are there in the two schools?

22. The sum of two numbers is 317205 and one of them is 185964; what is the other?

23. A farmer sold 75 cattle at 24 pounds a head and bought with the money sheep at 2 pounds each. How many sheep did he buy?

24. Divide 358 by 15 by short divisions.

25. What is the least number which must be added to 57914 in order that the sum may be exactly divisible by 315?

---

26. Express MDCCCXCIV in the Arabic notation, and 2875 by means of Roman numerals.

27. In a school of four hundred and ninety children there are two hundred and seventy-six girls. How many more girls than boys are there?

28. In a train there are 37 carriages each having seats for 36 people, and there are 375 passengers in the train; how many seats are empty?

29. Shew that the sum of the squares of three thousand and nine, and four thousand and twelve, is equal to the square of five thousand and fifteen.

30. What is the least number which must be subtracted from 2146537 in order that the remainder may be exactly divisible by 4275?

---

31. Subtract nine hundred and five million eight thousand nine hundred and sixty-five from eleven hundred million two thousand three hundred, and express the result in words.

32. The successful candidate at an election, who obtained 12597 votes, had a majority of 1479 over the unsuccessful candidate. How many votes were given altogether?

33. Find $2197 - 1982 + 374 + 10085 - 8216 + 11597 - 7986$.

34. Multiply 16777216 by 131072, also divide 16777216 by 131072, and express the results in words.

35. Find the least number of repetitions of 3745 whose sum is greater than a million.

---

36. Express the numbers 29, 47, 158, 679, 1464 and 10385 by means of Roman numerals.

37. How many figures are there in all the numbers from 1 to 100? How many in the numbers from 1 to 1000?

38. A certain number when divided by 3008 gives a quotient 3875 and a remainder 2794. What is the number?

39. Divide 999999 by the continued product of 3, 7, 11 and 13.

40. The sum of two numbers is 315642, and one of the numbers is twice the other: find them.

CHAPTER III.

COMPOUND QUANTITIES.

63. NUMBERS are first used in connection with distinct objects, and are afterwards applied to measure continuous magnitudes of any kind by means of some fixed magnitude of the same kind which is taken as a unit. If the magnitude does not contain the unit an exact number of times, the part left over can be measured by taking some smaller unit and expressing this remainder in terms of that unit; again taking a still smaller unit, if necessary.

For example, to measure a certain length. Some definite length, say a yard, is fixed on as a standard unit. Suppose the given length contains 6 yards with a remainder. This remainder is then expressed in terms of a unit smaller than a yard, and the next smaller unit in use is a *foot*, three of which go to make a yard. Suppose that the remainder contains 2 feet, and that there is still something left over. This remainder is expressed in terms of an *inch*, twelve of which go to make a foot. Suppose that the remainder contains 9 of these inches, either exactly or so nearly that the difference is too unimportant to be considered, then the whole length is said to be 6 yards 2 feet 9 inches, and is expressed in terms of the three units, a yard, a foot and an inch.

Quantities expressed in terms of a single unit are called **Simple quantities**, and quantities which are expressed in terms of more than one unit are called **Compound quantities**.

To measure every different kind of quantity some standard unit is employed, and also other units which are obtained by subdivisions and repetitions of the standard unit.

The relations between the different units in use in England are given in the following tables.

Money.

4 Farthings	make	1 Penny (*d.*)
12 Pence	...	1 Shilling (*s.*)
20 Shillings	...	1 **Pound** (£.)

One farthing is written $\frac{1}{4}d.$, two farthings or one half-penny is written $\frac{1}{2}d.$, and three farthings are written $\frac{3}{4}d.$ Thus eightpence farthing is written $8\frac{1}{4}d.$

The coins in use in England are as follows.

Gold coins :—The Sovereign (20*s.*), and Half-sovereign (10*s.*).

Silver coins :—Crown (5*s.*), Half-crown (2*s.* 6*d.*), Florin (2*s.*), Double Florin (4*s.*), Shilling, Sixpence, and Threepence.

Copper coins :—Penny, Half-penny, and Farthing.

The Guinea (21*s.*) is not now coined.

Time.

60 Seconds (*secs.*)	make	1 Minute (*min.*)
60 Minutes	...	1 Hour (*hr.*)
24 Hours	...	1 **Day** (*d.*)
7 Days	...	1 Week (*wk.*)
4 Weeks	...	1 *Lunar* Month.
365 Days	...	1 Year.

The year is divided into 12 months, called Calendar Months, which contain an unequal number of days, namely :—January 31, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December 31.

Every fourth year contains 366 days, and is called Leap Year, and in these years February has 29 days. It is a Leap Year when the number of the year is exactly divisible by 4; thus 1892 and 1896 will be leap years.

The *Solar Year* contains 365 d. 5 h. 56 min. 49 secs. very nearly. Now it would clearly be very inconvenient to reckon by years which did not contain an exact number of days; hence as the Solar Year contains *very nearly* $365\frac{1}{4}$ days, we have 3 years (called **Civil Years**) of 365 days each and then 1 year of 366 days. The Solar Year is however somewhat *less* than $365\frac{1}{4}$ days, and the necessary correction is made by omitting three Leap Years in every 400 years, the years which are not counted as Leap Years (although divisible by 4) are the years which end the Centuries, and are such that the number of the Century is not divisible by 4. Thus 1800 was not a Leap Year, and 1900 will not be a Leap Year; the year 2000 will, however, be a Leap Year.

Avoirdupois Weight.

16 Drams (*dr.*)	make	1 Ounce (*oz.*)
16 Ounces	...	1 **Pound** (*lb.*)
14 Pounds	...	1 Stone (*st.*)
28 Pounds	...	1 Quarter (*qr.*)
4 Quarters (112 lbs.)	...	1 Hundredweight (*cwt.*)
20 Hundredweights	...	1 Ton.

Avoirdupois weight is used for all ordinary substances.

Troy Weight.

24 Grains (*grs.*)	make	1 Pennyweight (*dwt.*)
20 Pennyweights	...	1 Ounce (*oz.*)
12 Ounces	...	1 **Pound Troy**.

Troy weight is used for weighing gold, silver, diamonds, and other costly articles.

1 Pound Troy contains $(12 \times 20 \times 24)$ grains = 5760 grains.
1 Pound Avoir. ... 7000 grains.

Length.

12 Inches (*in.*)	make	1 Foot (*ft.*)
3 Feet	...	1 **Yard** (*yd.*)
$5\frac{1}{2}$ Yards	...	1 Pole (*p.*), Rod or Perch
40 Poles (220 *yds.*)	...	1 Furlong (*fur.*)
8 Furlongs (1760 *yds.*)	...	1 Mile (*m.*)

In Land Surveying the Chain (22 yds.) is used, and this is divided into 100 parts, called Links, so that

22 yds. = 1 Chain (*ch.*) = 100 Links (*lks.*).

Square Measure.

144 Square Inches (*sq. in.*)	make	1 Square Foot (*sq. ft.*)
9 Square Feet	...	1 Square Yard (*sq. yd.*)
$30\frac{1}{4}$ Square Yards	...	1 Square Pole (*P.*)
40 Square Poles	...	1 Rood (*R.*)
4 Roods	...	1 Acre (*A.*)
4840 Square Yards	...	1 Acre.
640 Acres	...	1 Square Mile.
10 Square Chains	...	1 Acre.

Square Measure is used for measuring land, flooring, and in fact everything in which length and breadth have to be taken into account.

Solid Measure.

1728 Cubic Inches (*c. in.*) make 1 Cubic Foot (*c. ft.*)
27 Cubic Feet ... 1 Cubic Yard (*c. yd.*)

Used for measuring solid bodies in which length breadth and thickness have to be taken into account.

Capacity.

2 Pints (*pts.*)	make	1 Quart (*qt.*)
4 Quarts (*qts.*)	...	1 **Gallon** (*gal.*)
2 Gallons	...	1 Peck (*pk.*)
4 Pecks	...	1 Bushel (*bus.*)
3 Bushels	...	1 Sack.
8 Bushels	...	1 Quarter (*qr.*)

Liquids are measured by means of pints, quarts and gallons.

Corn is measured in terms of pecks, bushels and quarters.

Fruit, potatoes, etc. are sometimes measured in terms of pecks, bushels, and sacks; and are sometimes measured by weight.

Standard Units.

64. The Standard Units, from which all the other units are derived, are defined by Act of Parliament. These are the following:

The Standard unit of Length is the **Imperial Yard** fixed by Act of Parliament to be the distance between two marks on a bar of metal kept in the Exchequer Office.

The Standard unit of Weight is the **Imperial Pound** (Avoirdupois), and is the weight of a certain piece of platinum kept in the Exchequer Office.

The Standard unit of Capacity is the **Imperial Gallon**, and is the volume occupied by 10 lbs. of pure water. Hence,

'a pint of pure water weighs a pound and a quarter.'

The Standard unit of Time is the **Mean Solar Day**, that is, the mean interval between two successive passages of the sun across the meridian of any place. A day is supposed to begin at midnight.

Reduction.

65. Changing the units in terms of which any quantity is expressed is called **Reduction**.

The method by which a compound quantity can be expressed as a simple quantity will be seen from the following examples.

Ex. 1. Reduce $9\frac{1}{4}d.$ to farthings, $8s.\ 4d.$ to pence, and £5. 12*s.* to shillings.

Since $1d. = 4$ farthings, $9d. = 4$ farthings $\times 9$, and therefore
$9\frac{1}{4}d. = 4f. \times 9 + 1f. = 37$ farthings.

Since $1s. = 12d.$, $8s. = 12d. \times 8$, and therefore
$8s.\ 4d. = 12d. \times 8 + 4d. = 100d.$

Since £1 = 20*s.*, £5 = 20*s.* × 5, and therefore
£5. 12*s.* = 20*s.* × 5 + 12*s.* = 112 shillings.

Ex. 2. Reduce £7. 12*s.* $6\frac{3}{4}d.$ to farthings.

The process is written compactly as follows:

£.	*s.*	*d.*
7	12	$6\frac{3}{4}$
20		
152 *s.*		
12		
1830 *d.*		
4		
7323 *f.*		

Explanation. £7 = (7 × 20) *s.* = 140*s.*
Hence £7. 12*s.* = 140*s.* + 12*s.* = 152*s.*
Then $152s. = (152 \times 12)d.$; hence $152s.\ 6d. = (152 \times 12)d. + 6d. = 1830d.$
Again $1830d. = (1830 \times 4)f.$; hence $1830\frac{3}{4}d. = (1830 \times 4)f. + 3f. = 7323f.$

EXAMPLES III. A.

Reduce to farthings

1.	$4\frac{1}{2}d.$	2.	$5\frac{1}{4}d.$	3.	$7\frac{3}{4}d.$	4.	$10\frac{1}{2}d.$
5.	$9\frac{3}{4}d.$	6.	$11d.$	7.	$8\frac{3}{4}d.$	8.	$11\frac{1}{2}d.$

Reduce to pence

9.	2*s.* 4*d.*	10.	3*s.* 2*d.*	11.	1*s.* 11*d.*	12.	4*s.* 1*d.*
13.	5*s.* 7*d.*	14.	6*s.* 7*d.*	15.	7*s.* 9*d.*	16.	8*s.* 2*d.*
17.	9*s.* 5*d.*	18.	10*s.* 11*d.*	19.	9*s.* 10*d.*	20.	11*s.* 6*d.*

Reduce to shillings

21.	£2. 10*s.*	22.	£3. 16*s.*	23.	£5. 18*s.*	24.	£4. 12*s.*
25.	£7. 14*s.*	26.	£6. 19*s.*	27.	£8. 14*s.*	28.	£9. 12*s.*

Reduce to pence

29. £3. 4*s*. 6*d*. 30. £5. 12*s*. 9*d*. 31. £7. 14*s*. 8*d*.
32. £6. 17*s*. 11*d*. 33. £9. 15*s*. 10*d*. 34. £8. 18*s*. 9*d*.
35. £12. 14*s*. 9*d*. 36. £25. 17*s*. 11*d*. 37. £32. 18*s*. 10*d*.
38. £59. 17*s*. 11*d*. 39. £157. 11*s*. 10*d*. 40. £217. 19*s*. 11*d*.

Reduce to farthings

41. £1. 2*s*. $4\frac{1}{4}d$. 42. £5. 6*s*. $7\frac{1}{2}d$. 43. £4. 16*s*. 5*d*.
44. £7. 17*s*. $9\frac{3}{4}d$. 45. £17. 15*s*. $11\frac{3}{4}d$. 46. £27. 16*s*. $10\frac{1}{2}d$.
47. £115. 11*s*. $9\frac{1}{4}d$. 48. £217. 19*s*. $7\frac{3}{4}d$. 49. £159. 11*s*. $11\frac{1}{4}d$.
50. 365 guineas. 51. 97 crowns. 52. 1891 half-crowns.

66. The method by which a simple quantity can be expressed as a compound quantity will be seen from the following example.

Reduce 213567 farthings to £. *s*. *d*.

Explanation. Since 4*f*. make 1*d*., if we divide a number of farthings by 4 the quotient will give the equivalent number of pence.

farthings.
4) 213567
12) 53391*d*. + 3*f*.
2,0) 4449*s*. + 3*d*.
222£. + 9*s*.

Hence 213567 farthings = £222. 9*s*. $3\frac{3}{4}d$.

EXAMPLES III. B.

Reduce to pence and farthings

1. 13*f*., 15*f*., 19*f*., 23*f*., 27*f*., 32*f*. 2. 37*f*., 39*f*., 41*f*., 49*f*., 53*f*., 63*f*.

Reduce to shillings and pence

3. 17*d*., 23*d*., 29*d*., 35*d*., 37*d*., 43*d*.
4. 57*d*., 65*d*., 70*d*., 79*d*., 82*d*., 89*d*.
5. 93*d*., 99*d*., 105*d*., 127*d*., 135*d*., 149*d*.

Reduce to pounds and shillings

6. 35*s*., 47*s*., 54*s*., 59*s*., 67*s*., 73*s*.
7. 79*s*., 82*s*., 88*s*., 93*s*., 105*s*., 120*s*.
8. 136*s*., 149*s*., 157*s*., 164*s*., 173*s*., 195*s*.

Reduce to £. *s*. *d*.

9. 167 farthings. 10. 574 farthings. 11. 1000 farthings.
12. a million farthings. 13. a thousand pence.
14. 10000 half-pence. 15. 8257 half-pence.
16. 9125 threepences. 17. 5642 sixpences.
18. 367 half-crowns. 19. 1573 half-crowns. 20. 3157 guineas.

67. We now proceed to consider the addition, subtraction, multiplication and division of compound quantities. It will be seen that no new principle is involved.

Addition.

68. The following example will shew how compound quantities are added.

Ex. Find the sum of £2. 5*s*. 11*d*., £16. 15*s*. 9*d*., £32. 17*s*. 10*d*., and £15. 16*s*. 7*d*.

Write the quantities so that units of the same kind are in a vertical column.

First add the pence which will be found to be 37. Reduce the pence to the next higher denomination, namely shillings; we have 37*d*. = 3*s*. 1*d*. Put 1 in the column for pence, and 'carry' 3 to be counted with the other shillings, which will be found to be 56 altogether. Since 56*s*. = £2. 16*s*., 16 is put in the column for shillings, and 2 is carried to the pounds, which will be found to be 67 altogether. Thus the sum required is £67. 16*s*. 1*d*.

£.	*s*.	*d*.
2	5	11
16	15	9
32	17	10
15	16	7
67	16	1

Wording. 7, 16, 17, 27, 37 pence are 3*s*. **1***d*.
3, 9, 16, 21, 26, 36, 46, 56 shillings are £2. **16***s*.
2, 7, 9, 15, **17**.
1, 2, 5, **6**.

The figures printed in thick type are to be written down as they are said. It is best to add any tens in the columns of pence and shillings after the units are all added, as above.

EXAMPLES III. C.

Add the following:

1.

d.
$4\frac{1}{2}$
$2\frac{1}{2}$
6
$7\frac{3}{4}$

2.

d.
$6\frac{1}{2}$
7
$4\frac{3}{4}$
$10\frac{1}{2}$

3.

d.
$10\frac{3}{4}$
$4\frac{1}{4}$
$6\frac{1}{4}$
$5\frac{1}{2}$

4.

d.
$5\frac{1}{2}$
$6\frac{3}{4}$
$7\frac{1}{4}$
$9\frac{1}{4}$

5.

s.	*d*.
6	4
5	2
3	9
13	10

6.

s.	*d*.
12	4
5	11
13	6
4	10

7.

s.	*d*.
12	6
9	11
2	7
15	8

8.

s.	*d*.
13	1
14	9
6	10
8	11

9.

£.	s.	d.
3	6	9
5	12	10
8	15	11
4	9	3

10.

£.	s.	d.
5	14	4
10	4	9
15	16	11
35	9	10

11.

£.	s.	d.
18	14	7
25	16	9
34	18	11
6	9	5

12.

£.	s.	d.
14	15	0
5	0	$9\frac{1}{2}$
16	12	4
10	17	$10\frac{1}{2}$

13.

£.	s.	d.
13	9	$4\frac{1}{4}$
25	16	$7\frac{1}{2}$
9	17	10
13	14	$2\frac{1}{4}$

14.

£.	s.	d.
5	0	7
13	12	$0\frac{1}{2}$
17	15	$6\frac{1}{4}$
6	17	$8\frac{3}{4}$

15.

£.	s.	d.
16	17	$4\frac{1}{2}$
23	4	$10\frac{1}{4}$
57	17	$8\frac{3}{4}$
14	15	$11\frac{1}{4}$

16.

£.	s.	d.
17	14	$9\frac{1}{2}$
29	13	$11\frac{3}{4}$
16	12	$8\frac{1}{2}$
125	9	$4\frac{1}{4}$

17.

£.	s.	d.
13	14	$6\frac{1}{2}$
	19	$7\frac{1}{4}$
3	11	$10\frac{1}{2}$
15	0	$7\frac{3}{4}$

18.

£.	s.	d.
256	17	$0\frac{1}{2}$
59	0	$9\frac{3}{4}$
2042	17	10
12	9	$7\frac{1}{4}$
157	12	$10\frac{1}{2}$
99	9	$2\frac{1}{4}$
447	17	10

19.

£.	s.	d.
195	14	9
1257	9	$7\frac{1}{2}$
39	14	$0\frac{1}{4}$
674	0	$7\frac{3}{4}$
569	15	$5\frac{1}{2}$
95	19	$10\frac{3}{4}$
259	19	$11\frac{1}{2}$

20.

£.	s.	d.
215	17	$11\frac{1}{2}$
1397	19	9
2	15	$7\frac{1}{4}$
357	8	$6\frac{3}{4}$
79	14	$8\frac{3}{4}$
895	11	10
2754	17	$4\frac{1}{2}$

21.

£.	s.	d.
13	7	$4\frac{3}{4}$
156	17	$10\frac{1}{2}$
907	11	$8\frac{1}{4}$
19	19	$11\frac{3}{4}$
592	13	$9\frac{1}{2}$
80	15	$8\frac{1}{4}$
172	16	$11\frac{1}{2}$

22.

£.	s.	d.
5	5	$7\frac{1}{2}$
305	12	11
84	16	$10\frac{1}{4}$
	17	$9\frac{3}{4}$
6	10	$7\frac{1}{2}$
156	13	$8\frac{3}{4}$
46	19	$10\frac{1}{2}$

23.

£.	s.	d.
179	2	$2\frac{1}{4}$
19	13	$10\frac{1}{2}$
8	17	$9\frac{3}{4}$
507	11	$11\frac{3}{4}$
97	14	$6\frac{1}{2}$
1013	18	10
74	7	$6\frac{3}{4}$

24.

£.	s.	d.
672	13	$4\frac{1}{4}$
58	19	$11\frac{3}{4}$
107	0	$0\frac{1}{2}$
39	18	$9\frac{3}{4}$
25	14	$10\frac{1}{2}$
379	9	$7\frac{3}{4}$
84	17	$11\frac{1}{2}$
15	11	$0\frac{1}{2}$
1014	8	$9\frac{1}{4}$
695	12	$1\frac{1}{2}$

25.

£.	s.	d.
177	13	6
99	18	$8\frac{3}{4}$
1892	6	$10\frac{1}{2}$
29	15	$8\frac{1}{4}$
603	12	$9\frac{1}{2}$
78	15	$7\frac{3}{4}$
235	0	$10\frac{1}{2}$
253	19	$11\frac{1}{4}$
2804	15	$6\frac{1}{2}$
909	5	$8\frac{3}{4}$

26.

£.	s.	d.
16	15	$4\frac{1}{2}$
293	9	$9\frac{3}{4}$
1704	17	$10\frac{1}{2}$
85	10	0
742	18	$9\frac{3}{4}$
39	5	$10\frac{1}{2}$
7	17	$9\frac{1}{4}$
693	9	$7\frac{3}{4}$
1859	19	$10\frac{1}{2}$
678	8	$4\frac{1}{4}$

SUBTRACTION.

69. The following examples will shew how to subtract one compound quantity from another.

Ex. 1. Subtract £3. 4*s*. $6\frac{1}{2}d$. from £8. 12*s*. $9\frac{3}{4}d$.

The smaller number is placed below the greater so that the units of the same kind are in the same vertical line.

£.	*s*.	*d*.
8 .	12 .	$9\frac{3}{4}$
3 .	4 .	$6\frac{1}{2}$
5 .	8 .	$3\frac{1}{4}$

Wording. $2f. + \mathbf{1}f.$ make $3f.$;
6 and **3** make 9;
4 and **8** make 12;
3 and **5** make 8.

Ex. 2. Subtract £5. 14*s*. $8\frac{1}{2}d$. from £10. 6*s*. $4\frac{1}{4}d$.

We have to see what must be added to £5. 14*s*. $8\frac{1}{2}d$. to make a total of £10. 6*s*. $4\frac{1}{4}d$.

Now $\frac{1}{2}d$. is already greater than $\frac{1}{4}d$., we must therefore add to $\frac{1}{2}d$. what will 'make up' to $1\frac{1}{4}d$., that is $5f.$; and $2f.$ and **3***f*. make $5f.$ We put down $\frac{3}{4}d$., and since $\frac{3}{4}d$. and $\frac{1}{2}d$. exceed $\frac{1}{4}d$. by $1d.$, we count this $1d.$ with the $8d.$, making $9d.$ in all. Then as $9d.$ is already greater than $4d.$, we must find what added to $9d.$ will make 1*s*. $4d.$, that is $16d.$ As $9d.$ and **7***d*. make 1*s*. $4d.$, 7 is put in the column of pence and 1*s*. is counted with the 14*s*., making 15*s*. in all. Then again as 15*s*. is greater than 6*s*. we must find what added to 15*s*. will come to £1. 6*s*., that is 26*s*. As $15s. + \mathbf{11}s. = 26s.$, 11 is put in the column for shillings, and £1 is counted with the £5 making £6, and $£6 + £\mathbf{4} = £10$.

£.	*s*.	*d*.
10 .	6 .	$4\frac{1}{4}$
5 .	14 .	$8\frac{1}{2}$
4 .	11 .	$7\frac{3}{4}$

Mental work. $2f.$ and **3***f*. make $1\frac{1}{4}d.$
1, 9 and **7** pence make 1*s*. $4d.$
1, 15 and **11** shillings make £1. 6*s*.
1, 6 and **4** make 10.

Ex. 3. Subtract from £59. 12*s*. $4\frac{1}{2}d$. the sum of £12. 11*s*. $4\frac{3}{4}d$., £5. 19*s*. $10\frac{1}{2}d$., and £26. 12*s*. $2\frac{1}{2}d$.

Here the sum can be subtracted in one operation, as in Art. 38.

£.	*s*.	*d*.
59 .	12 .	$4\frac{1}{2}$
12 .	11 .	$4\frac{3}{4}$
5 .	19 .	$10\frac{1}{2}$
26 .	12 .	$2\frac{1}{2}$
14 .	8 .	$10\frac{3}{4}$

Add the farthings, and then 'make up' to the next greater number of pence and farthings which ends in $\frac{1}{2}d$. Thus $7f.$ and **3***f*. make $2\frac{1}{2}d.$ Put down **3***f*. and carry $2d.$

Then add the pence, including the $2d.$; the sum is $18d. = 1s.\ 6d.$ 'Make up' to 2*s*. $4d.$ And so on.

Mental work. 2, 4, $7f. = 1\frac{3}{4}d.$ and $\frac{3}{4}d.$ make $2\frac{1}{2}d.$;
2 (carried), 4, 8, $18d. = 1s.\ 6d.$ and **10***d*. make 2*s*. $4d.$;
2 (carried), 4, 13, 14, 24, 34, 44*s*. and **8***s*. make £2. 12*s*.;
2 (carried), 8, 13, 15 and **4** make 19;
1 (carried), 3, 4 and **1** make 5.

EXAMPLES III. D.

1. From £64. 12*s*. 9*d*. take £23. 10*s*. 4*d*.
2. From £36. 17*s*. 10*d*. take £27. 14*s*. 11*d*.
3. From £7. 0*s*. $4\frac{1}{4}d$. take £3. 14*s*. 8*d*.
4. From £19. 17*s*. $6\frac{1}{2}d$. take £6. 14*s*. $9\frac{3}{4}d$.
5. From £115. 7*s*. 0*d*. take £95. 18*s*. $8\frac{1}{2}d$.
6. From £307. 10*s*. $7\frac{1}{4}d$. take £275. 19*s*. $10\frac{3}{4}d$.

Find the difference between

7. £3. 19*s*. $4\frac{1}{4}d$. and £7. 11*s*. $8\frac{3}{4}d$.
8. £219. 11*s*. $6\frac{1}{2}d$. and £149. 17*s*. 8*d*.
9. £375. 0*s*. $8\frac{1}{4}d$. and £574. 12*s*. 6*d*.
10. By how much is £157. 14*s*. 7*d*. greater than £94. 17*s*. $9\frac{1}{4}d$. ?
11. By how much is £7049. 5*s*. $6\frac{1}{4}d$. greater than £3594. 12*s*. $9\frac{3}{4}d$.?
12. By how much is £21745. 12*s*. $4\frac{3}{4}d$. less than £25000 ?
13. Subtract the sum of £2. 10*s*. 6*d*., £8. 14*s*. 6*d*., and £1. 7*s*. 10*d*. from £15.
14. A man had originally £115. 14*s*. 9*d*.; how much had he left after spending £15. 12*s*. $9\frac{1}{4}d$., £25. 13*s*. $4\frac{3}{4}d$., £19. 11*s*. $8\frac{1}{2}d$. and £6. 19*s*. $10\frac{3}{4}d$.?
15. How much was left out of £300 after paying 5 bills amounting to £73. 9*s*. 11*d*., £15. 19*s*. $10\frac{1}{2}d$., £87. 17*s*. $6\frac{3}{4}d$., £13. 11*s*. $7\frac{1}{2}d$. and £59. 17*s*. $4\frac{1}{2}d$. respectively ?
16. By how much does £100 exceed the sum of £17. 12*s*. $9\frac{1}{2}d$., £19. 11*s*. $4\frac{1}{4}d$., £1. 16*s*. $9\frac{3}{4}d$., £3. 11*s*. $4\frac{1}{2}d$. and £20. 10*s*. 6*d*.?
17. From the sum of £5. 17*s*. $8\frac{3}{4}d$. and £26. 14*s*. $9\frac{1}{4}d$. take away the difference between £50. 17*s*. $4\frac{1}{2}d$. and £48. 19*s*. $8\frac{1}{2}d$.
18. By how much does the sum of £19. 17*s*. $6\frac{3}{4}d$. and £14. 11*s*. $9\frac{1}{2}d$. exceed the sum of £15. 12*s*. $7\frac{3}{4}d$. and £10. 18*s*. $8\frac{1}{2}d$.?
19. Find 1*s*. 4*d*. − 10*d*. + 4*s*. 2*d*. − 3*s*. 8*d*.
20. Find 6*s*. $2\frac{1}{2}d$. + 5*s*. $3\frac{1}{4}d$. − 2*s*. $11\frac{1}{2}d$. − 4*s*. $9\frac{1}{4}d$.
21. Find £17. 16*s*. $4\frac{1}{2}d$. + £2. 19*s*. $11\frac{1}{4}d$. − £7. 13*s*. $8\frac{1}{2}d$. + £6. 10*s*. − £15. 18*s*. 11*d*.
22. Find £115. 17*s*. $6\frac{3}{4}d$. − £82. 12*s*. $7\frac{1}{2}d$. + £200. 14*s*. $1\frac{1}{4}d$. − £187. 19*s*. $10\frac{3}{4}d$.
23. Find £900. 18*s*. $7\frac{1}{2}d$. − £115. 16*s*. $7\frac{1}{2}d$. − £675. 19*s*. $2\frac{1}{4}d$. − £19. 11*s*. $6\frac{3}{4}d$. + £112. 17*s*. $11\frac{1}{2}d$. + £62. 13*s*. $8\frac{1}{2}d$.

MULTIPLICATION.

70. We now proceed to shew how to find the sum of any number of repetitions of a compound quantity, that is, the product of a compound quantity and an abstract number.

CASE I. When the multiplier is not greater than 12.

Ex. Multiply £3. 14*s*. $7\frac{3}{4}d$. by 7.

£.	*s*.	*d*.
3	14	$7\frac{3}{4}$
		7
26	2	$6\frac{1}{4}$

Explanation. First $3f. \times 7 = 21f. = 5\frac{1}{4}d.$ Put down $\frac{1}{4}d.$ and carry the $5d.$ to be counted with the other pence. Then $7 \times 7 = 49$, $49d.$ and $5d.$ (carried) $= 54d. = 4s.\ 6d.$ Put down 6 in the column for pence, and carry $4s.$ Then $14 \times 7 = 98$, $98s.$ and $4s.$ (carried) $= 102s. =$ £5. $2s.$ Put down 2 in the column for shillings and carry £5. Finally $3 \times 7 = 21$, £21 and £5 (carried) = £26. Thus the required product is £26. $2s.$ $6\frac{1}{4}d.$

CASE II. When the multiplier can be seen to be the product of factors each not greater than 12.

Ex. Multiply £5. 14*s*. $7\frac{1}{2}d.$ by 35.

Here $35 = 5 \times 7$, and to multiply by 35 gives the same result as to multiply by 5 and 7 in succession. Thus we have two applications of Case I.

£.	*s*.	*d*.	
5	14	$7\frac{1}{2}$	
		5	
28	13	$1\frac{1}{2}$	= £5. 14*s*. $7\frac{1}{2}d. \times 5$.
		7	
200	11	$10\frac{1}{2}$	= £5. 14*s*. $7\frac{1}{2}d. \times 5 \times 7$.

EXAMPLES III. E.

Multiply

1. $2s.\ 4d.$ by 4.
2. $3s.\ 7d.$ by 6.
3. $5s.\ 8d.$ by 8.
4. £2. $4s.\ 8d.$ by 5.
5. £3. $12s.\ 4d.$ by 6.
6. £5. $11s.\ 10d.$ by 7.
7. £12. $11s.\ 8d.$ by 9.
8. £14. $15s.\ 9d.$ by 10.
9. £8. $13s.\ 10d.$ by 11.
10. £2. $3s.\ 6\frac{1}{2}d.$ by 4.
11. £3. $7s.\ 9\frac{1}{4}d.$ by 6.
12. £8. $9s.\ 10\frac{3}{4}d.$ by 8.
13. £12. $14s.\ 7\frac{1}{2}d.$ by 12.
14. £24. $17s.\ 9\frac{1}{4}d.$ by 9.
15. £37. $19s.\ 6\frac{1}{2}d.$ by 3, 5, 7 and 11.

Find the cost of

16. 7 things at $9\frac{1}{4}d.$ each. **17.** 9 things at $1s.\ 7\frac{3}{4}d$ each.

18. 12 things at $6s.\ 10\frac{1}{2}d.$ each.

19. 11 things at £1. $12s.\ 5\frac{1}{4}d.$ each.

20. 10 things at £5. $17s.\ 10\frac{1}{4}d.$ each.

21. 9 things at £7. $18s.\ 11\frac{3}{4}d.$ each.

Multiply

22. £1. $12s.\ 7d.$ by 20. **23.** £2. $9s.\ 7d.$ by 15.

24. £5. $8s.\ 7d.$ by 21. **25.** £6. $12s.\ 9d.$ by 28.

26. £15. $12s.\ 6\frac{3}{4}d.$ by 45. **27.** £8. $17s.\ 4\frac{1}{4}d.$ by 40.

28. £12. $13s.\ 7\frac{1}{4}d.$ by 54. **29.** £18. $13s.\ 7\frac{3}{4}d.$ by 64.

30. £5. $19s.\ 6\frac{1}{2}d.$ by 100. **31.** £17. $14s.\ 8\frac{3}{4}d.$ by 100.

32. £12. $17s.\ 4\frac{1}{4}d.$ by 200. **33.** £5. $19s.\ 2\frac{1}{2}d.$ by 400.

34. £1. $17s.\ 4\frac{1}{4}d.$ by 1000. **35.** £2. $11s.\ 9\frac{3}{4}d.$ by 3000.

CASE III. When the multiplier cannot be seen to be the product of factors each not greater than 12.

The following example will explain the method to be adopted, which will be seen to differ very little from the method adopted in the multiplication of simple quantities, the only apparent difference arising from the fact that we cannot at once write down the result of multiplying by 10, 100, &c.

Ex. Multiply £3. $14s.\ 8\frac{3}{4}d.$ by 257.

	£.	s.	d.	
	3	14	$8\frac{3}{4}$	
			10	
	37	7	$3\frac{1}{2}$	= £3. $14s.\ 8\frac{3}{4}d. \times 10$
			10	
	373	12	11	= ..... $\times 100$
			2	
	747	5	10	= ..... $\times 200$
Multiplying 2nd line by 5	186	16	$5\frac{1}{2}$	= ..... $\times 50$
Multiplying 1st line by 7	26	3	$1\frac{1}{4}$	= ..... $\times 7$
	960	5	$4\frac{3}{4}$	= ..... $\times (200+50+7)$.

Instead of the method shewn above, the different parts of the multiplicand can be multiplied separately by 257, and the results added. Thus

	£.	s.	d.
$3f. \times 257 = 771f. = 192\frac{3}{4}d.$	= 0	16	$0\frac{3}{4}$
$8d. \times 257 = 2056d. = 171s.\ 4d.$	= 8	11	4
$14s. \times 257 = 3598s.$	= 179	18	0
£3 × 257	= 771	0	0
Hence, by addition,			
£3. $14s.\ 8\frac{3}{4}d. \times 257$	= 960	5	$4\frac{3}{4}$

EXAMPLES III. F.

Multiply

1. 3*s.* 9*d.* by 29.
2. 5*s.* 7*d.* by 37.
3. 12*s.* $4\frac{1}{2}d.$ by 31.
4. 7*s.* $9\frac{1}{2}d.$ by 95.
5. £1. 2*s.* 6*d.* by 73.
6. £2. 5*s.* 9*d.* by 67.
7. £5. 12*s.* 10*d.* by 92.
8. £7. 18*s.* 9*d.* by 57.
9. £16. 14*s.* 7*d.* by 83.
10. £27. 11*s.* 11*d.* by 97.
11. £3. 14*s.* $7\frac{3}{4}d.$ by 89.
12. £5. 17*s.* $6\frac{1}{4}d.$ by 78.
13. £81. 19*s.* $8\frac{3}{4}d.$ by 79.
14. £157. 14*s.* $9\frac{1}{2}d.$ by 87.
15. £12. 6*s.* $8\frac{1}{2}d.$ by 123.
16. £6. 17*s.* $9\frac{1}{4}d.$ by 137.
17. £113. 11*s.* $4\frac{1}{4}d.$ by 153.
18. £179. 17*s.* $9\frac{3}{4}d.$ by 174.
19. £15. 16*s.* $4\frac{3}{4}d.$ by 234.
20. £8. 15*s.* $10\frac{1}{2}d.$ by 355.
21. £6. 17*s.* $7\frac{3}{4}d.$ by 529.
22. £9. 8*s.* $11\frac{3}{4}d.$ by 473.
23. £57. 17*s.* $8\frac{3}{4}d.$ by 579.
24. £79. 13*s.* $7\frac{1}{2}d.$ by 758.
25. £8. 14*s.* $6\frac{1}{2}d.$ by 1234.
26. £3. 14*s.* $6\frac{3}{4}d.$ by 4321.
27. £18. 11*s.* $7\frac{1}{4}d.$ by 3524.
28. £8. 17*s.* $9\frac{1}{4}d.$ by 5694.

DIVISION.

71. In division there are two cases to consider according as the divisor is an abstract number or a concrete quantity of the same kind as the dividend. [See Art. 52.]

CASE I. To divide a compound quantity by an abstract number. [This operation is sometimes called PARTITION.]

Ex. 1. Divide £44. 11*s.* 4*d.* by 7.

Here we have to find the quantity which, when multiplied by 7, gives the product £44. 11*s.* 4*d.*

	£.	*s.*	*d.*
7)	44 .	11 .	4
	6 .	7 .	4

First dividing £44 by 7 we have quotient £6 and £2 over. Then £2 + 11*s.* = 51*s.*; and dividing 51*s.* by 7 we have quotient 7*s.* and remainder 2*s.* Finally 2*s.* 4*d.* = 28*d.*; and dividing 28*d.* by 7 we have quotient 4*d.* and no remainder.

Ex. 2. Divide £113. 1*s.* $10\frac{1}{2}d.$ into 35 equal parts.

Since $35 = 5 \times 7$, we may divide by 5 and 7 in succession. Thus

		£.	*s.*	*d.*
35	5)	113 .	1 .	$10\frac{1}{2}$
	7)	22 .	12 .	$4\frac{1}{2}$
		3 .	4 .	$7\frac{1}{2}$

Ex. 3. Divide £675. 9*s*. 7½*d*. by 127.

	£.	*s*.	*d*.	
127)	675	. 9	. 7½	(£5
	635			
	40			
	20			
127)	809	(6*s*.		
	762			
	47			
	12			
127)	571	(4*d*.		
	508			
	63			
	4			
127)	254	(2*f*.		
	254			

Hence the quotient required is £5. 6*s*. 4½*d*.

EXAMPLES III. G.

Divide

1. 3*s*. 4*d*. by 4.
2. 2*s*. 6*d*. by 10.
3. £1. 3*s*. 4*d*. by 7.
4. £2. 5*s*. 10*d*. by 11.
5. £6. 6*s*. 8*d*. by 8.
6. £15. 7*s*. 6*d*. by 9.
7. £1. 19*s*. 4½*d*. by 6.
8. £4. 7*s*. 4¼*d*. by 7.
9. £5. 12*s*. 6*d*. by 8.
10. £71. 3*s*. 8¼*d*. by 9.
11. £95. 9*s*. 0¾*d*. by 15.
12. £323. 7*s*. 1*d*. by 20.
13. £473. 5*s*. 7½*d*. by 30.
14. £238. 12*s*. 9¾*d*. by 27.
15. £6595. 4*s*. 8*d*. by 44.
16. £942. 12*s*. 2*d*. by 56.
17. £1152. 6*s*. 2¼*d*. by 63.
18. £15542. 9*s*. 6*d*. by 72.
19. £15942. 16*s*. 6*d*. by 108.
20. £4312. 17*s*. 6*d*. by 120.
21. £215. 14*s*. 4¼*d*. by 37.
22. £815. 13*s*. 7½*d*. by 51.
23. £816. 15*s*. 7½*d*. by 59.
24. £229. 4*s*. 11¼*d*. by 57.
25. £725. 16*s*. 11¾*d*. by 43.
26. £406. 9*s*. 2½*d*. by 74.
27. £2586. 15*s*. 4¼*d*. by 97.
28. £22912. 9*s*. 0½*d*. by 53.
29. £1220314. 3*s*. 1¾*d*. by 73.
30. £102354. 14*s*. 8¼*d*. by 93.
31. £1741. 4*s*. 5½*d*. by 127.
32. £6022. 10*s*. 3¾*d*. by 357.
33. £119232. 1*s*. 10½*d*. by 693.
34. £1602. 18*s*. 0¼*d*. by 1205.

CASE II. When the divisor is a concrete quantity of the same nature as the dividend. [This operation is sometimes called QUOTITION.]

Ex. How many times is £3. 15*s.* 6*d.* contained in £94. 7*s.* 6*d.*?

First reduce the dividend and divisor to a common denomination, in the case before us, to *pence*.

It will be found that £94. 7*s.* 6*d.* = 22650*d.*, and that £3. 15*s.* 6*d.* = 906*d.*

Now divide 22650 by 906 in the usual manner and the quotient required will be found to be 25.

We may reduce to *sixpences* instead of pence; and we should then have smaller numbers to deal with in the division.

EXAMPLES III. H.

Divide

1. 3*s.* 9*d.* by $3\frac{3}{4}d.$
2. 3*s.* 9*d.* by $4\frac{1}{2}d.$
3. £1. 8*s.* 5*d.* by 2*s.* 7*d.*
4. £1. 1*s.* $1\frac{3}{4}d.$ by 3*s.* $0\frac{1}{4}d.$
5. £29. 3*s.* 4*d.* by £1. 3*s.* 4*d.*
6. £44. 8*s.* 4*d.* by £1. 2*s.* $2\frac{1}{2}d.$
7. £1741. 4*s.* $5\frac{1}{2}d.$ by £13. 14*s.* $2\frac{1}{2}d.$
8. £6022. 10*s.* $3\frac{3}{4}d.$ by £16. 17*s.* $4\frac{3}{4}d.$
9. £1603. 7*s.* $8\frac{1}{2}d.$ by £12. 16*s.* $6\frac{1}{2}d.$
10. £2586. 15*s.* $4\frac{1}{4}d.$ by £26. 13*s.* $4\frac{1}{4}d.$
11. How many fourpences are there in £1. 12*s.* 8*d.*?
12. How many ninepences are there in £2. 5*s.*?
13. To how many children could 7*d.* each be given out of £2. 9*s.*?
14. How many francs (worth $9\frac{1}{2}d.$ each) could be got for £14. 1*s.* 10*d.*?
15. How many guineas (21*s.*) are there in £593. 5*s.*?
16. How many yards of cloth at 2*s.* 9*d.* a yard can be bought for £21. 9*s.*?
17. How many times can £7. 2*s.* $4\frac{1}{2}d.$ be subtracted from £348. 17*s.* $4\frac{1}{2}d.$, and what will be the final remainder?
18. How many articles which cost £3. 4*s.* $5\frac{1}{2}d.$ each can be bought with £5647. 18*s.* $11\frac{1}{4}d.$, and how much money will be left?
19. A length of cloth cost £11. 3*s.* 6*d.* at 6*s.* $2\frac{1}{2}d.$ a yard. How many yards were there?
20. How many yards worth 4*s.* $2\frac{1}{4}d.$ a yard must be given in exchange for 402 yards worth 3*s.* $5\frac{1}{2}d.$ a yard?

21. How many equal sums of £1. 17*s.* $7\frac{1}{2}d.$ can be given away out of a sum of £208. 16*s.* 9*d.*, and how much will be left over?

22. A certain number of articles cost £43. 12*s.* 8*d.* altogether, and each cost 19*s.* 10*d.* How many were bought?

23. How many times is £21. 17*s.* $2\frac{1}{4}d.$ contained in £13268. 12*s.* $9\frac{3}{4}d.$?

24. Among how many persons can £59. 6*s.* $3\frac{3}{4}d.$ be divided, giving to each £3. 2*s.* $5\frac{1}{4}d.$?

25. A bill of £34. 1*s.* 6*d.* was paid with an equal number of sovereigns, half-crowns, and shillings; how many were there of each?

WEIGHTS AND MEASURES.

72. We have hitherto considered the case of money only; we now proceed to consider other compound quantities. These are treated in precisely the same manner, the connection between the different units being given in the Tables.

REDUCTION.

Ex. 1. Reduce 2 tons 4 cwt. 2 qrs. 17 lbs. 8 oz. to ounces.

tons	cwt.	qrs.	lbs.	oz.
2	4	2	17	8
20				
44	cwt.			
4				
178	qrs.			
28				
1441				
356				
5001	lbs.			
16				
30014				
5001				
80024	oz.			

Explanation. 2 tons 4 cwt.
$= (2 \times 20)$ cwt. $+ 4$ cwt. $= 44$ cwt.
Then 44 cwt. 2 qrs. $= (44 \times 4)$ qrs. $+ 2$ qrs.
$= 178$ qrs.
Then 178 qrs. 17 lbs. $= (178 \times 28)$ lbs. $+ 17$ lbs.
(Multiply as usual by 28 and add in the 17 in the course of multiplication by 8.)
Finally 5001 lbs. $+ 8$ oz. $= (5001 \times 16)$ oz. $+ 8$ oz.
(Multiply as usual by 16 and add in the 8 in the course of multiplication by 6.)

Ex. 2. Reduce 31571 grains to lbs. oz. dwts. grs. (Troy).

grains.
24 { 3) 31571
8) 10523 sets of 3 grains each + 2 grains } 11 grains [Art. 60.]
20) 1315 dwts. + 3 sets of 3 grains
12) 65 oz. 15 dwts. 11 grs.
5 lbs. 5 oz. 15 dwts. 11 grs.

EXAMPLES III. I.

Reduce

1. 1 ton 3 qrs. 10 lbs. to pounds.
2. 3 tons 12 cwt. 1 qr. 16 lbs. to pounds.
3. 6 hrs. 12 min. 10 sec. to seconds.
4. 12 hrs. 5 min. 24 sec. to seconds.
5. 13 yds. 2 ft. 11 in. to inches.
6. 17 yds. 2 ft. 7 in. to inches.
7. 12 m. 3 fur. 10 p. to poles.
8. 13 m. 5 fur. 26 p. to poles.
9. 8 bus. 3 pk. 1 gal. to gallons.
10. 5 gal. 3 qts. 1 pt. to pints.
11. 5 A. 3 R. 27 P. to poles.
12. 17 A. 2 R. 35 P. to poles.
13. 13 sq. yds. 6 sq. ft. 100 sq. in. to inches.
14. 8 sq. yds. 7 sq. ft. 90 sq. in. to inches.
15. 6 lbs. 7 oz. 10 dwts. 15 grs. to grains.
16. 18 lbs. 9 oz. 15 dwts. 20 grs. to grains.
17. 3 wks. 5 days 12 hrs. to hours.
18. 16 days 22 hrs. 40 min. 35 secs. to seconds.
19. 12 tons 13 cwt. 3 qrs. 5 lbs. 7 oz. to ounces.
20. 5 tons 17 cwt. 2 qrs. 18 lbs. 14 oz. to ounces.
21. 2 mi. 3 fur. 80 yds. 2 ft. to feet.
22. 12 mi. 1200 yds. 1 ft. 7 in. to inches.

Reduce to tons, cwt., etc.

23. 1462 lbs.
24. 13574 lbs.
25. 11597 lbs.
26. 56214 oz.
27. 57812 oz.
28. 81974 drams.

Reduce to acres, roods and poles

29. 315 sq. po.
30. 5142 sq. po.
31. 1574 sq. po.
32. 3725 sq. po.

Reduce to yards, feet, etc.

33. 156 in.
34. 342 in.
35. 417 in.
36. 1179 in.

Reduce to lbs., oz., dwts., grs.

37. 517 dwts.
38. 574 dwts.
39. 3156 grs.
40. 4215 grs.
41. 13407 grs.
42. 24709 grs.
43. 35937 grs.
44. 51940 grs.

Reduce to bushels, pecks, etc.

45. 156 pts.
46. 1472 pts.
47. 416 qts.
48. 1875 pts.

Reduce to square yards, etc.

49. 1462 sq. in.
50. 2156 sq. in.
51. 3564 sq. in.

Reduce to days, hours, etc.

52. 31572 secs.
53. 257672 secs.
54. 7142169 secs.

73. The following cases are somewhat more difficult than those previously considered because one pole is not equal to an exact number of yards.

Ex. 1. Reduce 4 m. 31 p. 3 yds. 2 ft. 11 in. to inches.

4 m. 31 po. 3 yds. 2 ft. 11 in.
8
32 fur.
40
1311 po.
11
2) 14421 half-yards.
7210 yds. + 1 ft. 6 in.
3 yds. + 2 ft. 11 in.
7214 yds. 1 ft. 5 in.
3
21643 ft.
12
259721 inches

Explanation.
Since 1 pole = $5\frac{1}{2}$ yards = 11 half-yards,
1311 poles = 1311 × 11
= 14421 half-yards
= 7210 yds. + 1 half-yard
= 7210 yds. 1 ft. 6 in.
Now add 3 yds. 2 ft. 11 in. from the given quantity.

Ex. 2. Reduce 279791 inches to miles, furlongs, poles, &c.

12) 279791 inches
3) 23315 ft. 11 in.
7771 yds. 2 ft.
2
11) 15542 half-yards
40) 1412 po. + 10 half-yards
8) 35 fur. 12 p. 5 yards
4 m. 3 fur. 12 p. 5 yds. 2 ft. 11 in.

But 5 yds. 2 ft. 11 in. is greater than 1 pole which is 5 yds. 1 ft. 6 in. Hence we write the answer 4 m. 3 fur. 13 p. 1 ft. 5 in.

N.B. It is best not to introduce *poles* at all, if this can be avoided.

For example, to reduce 7 miles 5 furlongs to yards. Multiply the number of furlongs by 220.

EXAMPLES III. J.

Reduce

1. 10 p. 2 yds. 1 ft. to feet.
2. 5 p. 3 yds. 2 ft. to inches.
3. 1 m. 3 fur. 20 p. 1 yd. to yards.
4. 6 m. 5 fur. 30 p. 3 yds. to yards.
5. 18 m. 11 p. 3 yds. 1 ft. 6 in. to inches.

6. 27 m. 6 fur. 33 p. 2 yds. 2 ft. 7 in. to inches.

7. 6 m. 3 fur. 52 yds. to yards.

8. 18 m. 5 fur. 160 yds. 2 ft. 11 in. to inches.

9. 3 A. 2 R. 16 P. to sq. yds. 10. 15 A. 3 R. 24 P. to sq. yds.

11. 3 A. 2 R. 5 P. 16 sq. yds. 6 sq. ft. to square inches.

12. 16 po. 18 sq. yds. 5 sq. ft. 100 sq. in. to square inches.

Reduce to miles, furlongs, poles, etc.

13. 6974 yards. 14. 21571 yards. 15. 15737 yards.

16. 6315 feet. 17. 51621 inches. 18. 158743 inches.

Reduce to acres, roods, poles, etc.

19. 20812 sq. yds. 20. 38599 sq. yds.

21. 5172400 sq. in. 22. 8156179 sq. in.

74. In some cases of Reduction we cannot pass directly from one denomination to the other.

Ex. 1. Reduce 320 guineas to pounds.

We first reduce guineas to shillings by multiplying by 21, and then divide the number of shillings by 20 to find the number of pounds. Thus

320 guineas
21
320
640
20) 6720 shillings
336 pounds

Ex. 2. How many lbs. Troy are there in 144 lbs. Avoir.?

Since 1 lb. Avoir. = 7000 grains,

144 lbs. Avoir. = 7000 grs. × 144.

These grains are now reduced to lbs. Troy in the usual manner.

144 lbs. Avoir.
7000
24 { 4) 1008000 grains
6) 252000
20) 42000 dwts.
12) 2100 ounces
175 lbs.

EXAMPLES III. K.

Reduce

1. 384 guineas to pounds. **2.** 7350 pounds to guineas.
3. 714 half-crowns to half-guineas.
4. 175 half-guineas to half-crowns.
5. 128 half-crowns to fourpences.
6. 3930 fourpences to half-crowns.
7. 1 qr. 8 lbs. to Troy. **8.** 720 lbs. Avoir. to Troy.
9. 1 cwt. 1 qr. to Troy weight. **10.** 11 lbs. 8 oz. Av. to Tr.
11. 350 oz. Troy to Avoir. **12.** 4 lbs. 3 oz. 20 grs. to Avoir.

75. We now give examples of the addition, subtraction, multiplication and division of compound quantities other than sums of money; these operations are performed in precisely the same way as in the case of money and will therefore need no further explanation.

EXAMPLES III. L.

Add together:

1.

d.	hr.	min.
5	17	42
3	11	53
7	19	37
11	7	21

2.

hrs.	min.	secs.
1	41	15
6	17	39
7	35	42
5	16	13

3.

d.	hr.	min.	secs.
5	17	27	45
6	11	39	56
17	21	49	40
6	11	11	31

4.

cwt.	qr.	lbs.
5	1	16
3	3	9
7	2	17
1	1	25

5.

lbs.	oz.	dr.
5	12	8
4	13	12
7	9	15
3	11	14

6.

tons	cwt.	qrs.	lbs.
5	15	3	7
1	12	1	17
15	17	1	20
3	9	2	21

7.

lbs.	ozs.	drs.
5	9	13
7	14	12
18	6	9
3	11	11

8.

tons	cwt.	qrs.	lbs.
16	17	2	19
119	16	1	17
72	12	2	7
65	15	3	24

9.

cwt.	qr.	lbs.	oz.
6	2	14	10
17	1	18	12
14	3	7	14
11	3	11	2

10.

lbs.	oz.	dwts.
6	4	19
13	9	7
2	11	17
7	10	13

11.

oz.	dwts.	grs.
1	17	23
2	8	11
5	15	7
7	4	21

12.

lbs.	oz.	dwts.	grs.
5	11	16	18
2	9	11	13
7	10	15	21
3	7	9	16

13.

yds.	ft.	in.
5	1	9
3	2	11
7	0	7
13	2	6

14.

yds.	ft.	in.
16	1	7
23	2	11
9	0	9
13	2	10

15.

m.	fur.	po.
6	5	30
8	3	17
71	6	32
18	0	27

16.

sq. yds.	sq. ft.	sq. in.
17	5	71
6	7	62
37	6	100
29	4	91

17.

sq. yds.	sq. ft.	sq. in.
27	4	72
3	0	37
35	8	84
154	6	16

18.

sq. yds.	sq. ft.	sq. in.
62	0	28
9	8	64
134	7	71
51	5	130

19.

A.	R.	P.
6	3	25
16	1	37
92	2	25
19	3	30

20.

A.	R.	P.
17	1	17
63	3	34
7	3	27
32	2	19

21.

A.	R.	P.
77	2	4
3	1	39
8	3	27
64	1	33

22.

gal.	qt.	pt.
3	3	1
11	2	0
37	1	1
19	2	1

23.

qrs.	bus.	pk.
19	6	3
294	5	2
107	7	1
29	4	3

24.

qrs.	bus.	pk.	gal.
57	3	2	1
9	5	3	0
67	7	0	1
143	4	3	1

25.

m.	fur.	po.	yd.	ft.	in.
7	3	25	3	2	4
18	6	17	4	1	7
6	2	11	3	2	10
27	5	32	2	1	7

26.

ac.	r.	po.	sq. yd.
17	2	35	20
9	1	27	19
143	3	19	14
169	2	11	29

EXAMPLES III. M.

Subtract

1. 5 d. 16 hrs. 22 min. from 11 d. 18 hrs. 10 min.
2. 15 d. 17 hrs. 13 min. 42 secs. from 31 d. 9 hrs. 11 min. 40 secs.
3. 5 cwt. 1 qr. 13 lbs. from 7 cwt. 2 qrs. 11 lbs.
4. 6 lbs. 10 oz. 11 drs. from 16 lbs. 9 oz. 5 drs.
5. 7 tons 13 cwt. 1 qr. 15 lbs. 12 oz. from 10 tons 11 cwt. 2 qrs. 7 lbs. 10 oz.
6. 3 lbs. 4 oz. 10 dwts. from 9 lbs. 1 oz. 5 dwts.
7. 11 lbs. 9 oz. 16 dwts. 21 grs. from 24 lbs. 5 oz. 10 dwts.
8. 5 yds. 1 ft. 10 in. from 10 yds.
9. 17 yds. 2 ft. 11 in. from 29 yds. 1 ft. 4 in.
10. 6 miles 3 fur. 164 yds. from 17 miles 1 fur. 150 yds.
11. 6 sq. yds. 7 sq. ft. 100 sq. in. from 18 sq. yds. 3 sq. ft. 17 sq. in.
12. 118 sq. yds. 6 sq. ft. 112 sq. in. from 215 sq. yds. 3 sq. ft. 84 sq. in.

13. 3 A. 2 R. 15 P. from 17 A. 1 R. 10 P.
14. 117 A. 1 R. 35 P. from 215 A. 29 P.
15. 5 gals. 2 qts. 1 pt. from 9 gals. 1 qt.
16. 23 qrs. 4 bus. 2 pk. from 71 qrs. 3 bus. 1 pk.
17. 8 miles 6 fur. 24 po. 3 yds. from 19 miles 2 fur. 11 po. 1 yd.
18. 2 mi. 4 fur. 17 po. 4 yds. 2 ft. from 7 miles 3 fur. 5 po. 1 yd. 1 ft.
19. 3 A. 2 R. 17 P. 15 Sq. Yds. from 16 A. 1 R. 10 P. 7 Sq. Yds.
20. 17 A. 2 R. 31 P. 25 Sq. Yds. from 31 A. 3 R. 15 P. 10 Sq. Yds.

EXAMPLES III. N.

Multiply

1. 5 hrs. 10 min. 33 secs., (i) by 5, (ii) by 7, (iii) by 9.
2. 5 cwt. 1 qr. 14 lbs., (i) by 7, (ii) by 8, (iii) by 9.
3. 6 tons 17 cwt. 2 qrs. 8 lbs. 6 oz. 5 drs., (i) by 4, (ii) by 6, (iii) by 9.
4. 8 lbs. 10 oz. 15 dwts. 20 grs., (i) by 5, (ii) by 7, (iii) by 12.
5. 10 yds. 1 ft. 7 in., (i) by 8, (ii) by 11, (iii) by 12.
6. 8 miles 3 fur. 215 yds., (i) by 5, (ii) by 8, (iii) by 12.
7. 15 sq. yds. 7 sq. ft. 100 sq. in., (i) by 6, (ii) by 11.
8. 13 A. 2 R. 17 P., (i) by 10, (ii) by 12.
9. 16 qrs. 3 bus. 2 pks., (i) by 5, (ii) by 11.
10. 3 gals. 2 qts. 1 pt., (i) by 5, (ii) by 7.
11. 3 d. 17 hr. 10 min. 15 sec., (i) by 35, (ii) by 45.
12. 15 tons 12 cwt. 1 qr. 16 lbs., (i) by 42, (ii) by 72.
13. 8 lbs. 11 oz. 15 dwt. 18 grs., (i) by 49, (ii) by 84.
14. 3 yds. 2 ft. 10 in., (i) by 44, (ii) by 132.
15. 5 A. 2 R. 15 P. by 350.
16. 8 qrs. 5 bus. 2 pk. by 420.
17. 12 d. 13 hrs. 14 min. 12 secs. by 65.
18. 5 tons 7 cwt. 1 qr. 15 lbs. by 94.
19. 3 lbs. 4 oz. 12 dwts. 12 grs. by 124.
20. 8 A. 3 R. 15 P. by 327.

21. 15 sq. yds. 7 sq. ft. 82 sq. in. by 1212.
22. 6 tons 15 cwt. 1 qr. 7 lbs. 3 oz. by 2341.
23. 2 lbs. 4 oz. 16 dwt. 18 grs. by 3124.
24. 1 m. 2 fur. 15 p. 4 yds., (i) by 5, (ii) by 9.
25. 3 A. 2 R. 25 P. 16 Sq. Yds., (i) by 8, (ii) by 12.
26. 3 m. 5 fur. 23 p. 2 yds. 1 ft. (i) by 25, (ii) by 55.
27. 3 A. 2 R. 15 P. 20 Sq. Yds. 4 Ft. by 30.
28. 5 m. 3 fur. 14 p. 2 yds. by 1314.

EXAMPLES III. O.

Divide

1. 22 d. 1 hr. 12 min. by 6.
2. 37 cwt. 3 qrs. 7 lbs. by 7.
3. 44 lbs. 2 oz. 8 drs. by 8.
4. 52 lbs. 10 oz. 13 dwts. by 9.
5. 153 yds. 2 ft. 1 in. by 11.
6. 95 A. 1 R. 24 P. by 12.
7. 185 lbs. 8 oz. 17 dwts. by 54.
8. 123 d. 10 hrs. 45 min. by 50.
9. 1052 yds. 1 ft. by 132.
10. 251 A. 3 R. 13 P. by 121.
11. 19 tons 14 cwt. 26 lbs. 4 oz. 12 drs. by 500.
12. 214 tons 11 cwt. 3 qrs. 14 lbs. by 196.
13. 12 tons 3 cwt. 1 qr. 25 lbs. by 37.
14. 309 tons 12 cwt. 2 qrs. 19 lbs. by 47.
15. 10 tons 6 cwt. 3 qrs. 10 lbs. 15 oz. by 57.
16. 37 yds. 2 ft. 3 in. by 151.
17. 35 tons 2 cwt. 20 lbs. 2 oz. by 289.
18. 279 qrs. 5 bus. 1 pk. 1 gal. 3 qts. by 253.
19. 61 tons 1 cwt. 3 qrs. by 2896.
20. 24 m. 58 yds. 2 ft. 4 in. by 1234.
21. 36 m. 4 fur. 23 po. 3 yds. 1 ft. 6 in. by 10.
22. 208 A. 3 R. 11 Sq. Yds. by 12.
23. 55 m. 7 fur. 26 po. 1 yd. 1 ft. by 43.
24. 298 A. 2 R. 31 P. 18 Sq. Yds. 2 Ft. 108 In. by 73.

25. On the different days of a certain week a tradesman took £5. 6*s.* 11*d.*, £4. 19*s.* 6*d.*, £4. 17*s.* 9*d.*, £5. 1*s.* 11*d.*, £5. 7*s.* 8*d.*, and £13. 17*s.* 9*d.*, respectively. What was the total amount for the week, and what was the average daily amount?

[The **average** (or the **mean**) of any number of quantities is that quantity which when taken as many times as there are quantities will give the same total amount. The average is therefore found by adding all the quantities and dividing the sum by the number of the quantities.]

26. The heights of 5 boys are 4 ft. 9 in., 4 ft. 11 in., 5 ft., 5 ft. 3 in., and 5 ft. 6 in., respectively. What is their average height?

27. The weights of the men in a boat are 9 st. 12 lbs., 10 st. 3 lbs., 10 st. 10 lbs., 11 st. 1 lb., 11 st. 13 lbs., 12 st. 2 lbs., 12 st. 10 lbs., and 11 st. 9 lbs. What is the average weight of the men?

EXAMPLES III. P.

1. Divide 2 tons 5 cwt. by 9 cwt.

2. Divide 6 oz. 10 dwts. by 13 dwts.

3. Divide 3 A. 2 R. 10 P. by 19 P.

4. Divide 2 qrs. 4 bus. 1 pk. by 2 bus. 1 pk.

5. How many pieces each 3 yds. 1 ft. long can be cut from a rope whose length is 180 yards?

6. A wheel revolves once every 2 m. 15 secs.; how many times does it revolve in 1 hr. 48 m.?

7. The circumference of a tricycle wheel is 12 feet, how many times does the wheel turn round in a journey of 10 miles?

8. A field of 12 A. 2 R. 35 P. is divided into allotments each containing 1 R. 15 P. How many allotments are there?

9. A man's average step is 2 ft. 11 in. How many steps does he take in walking $3\frac{1}{2}$ miles?

10. How many jars, each containing 2 gals. 3 qts. 1 pt. can be filled out of a cask containing 46 gallons?

11. How many rails, each weighing 4 cwt. 1 qr. 14 lbs., can be made out of 61 tons 2 qrs. 14 lbs. of iron? What will each rail cost at $1\frac{1}{2}d.$ a lb.?

12. How many times does 2 miles 76 yds. contain 14 yds. 1 ft. 6 in.?

13. Each of a certain number of articles weighs 1 qr. 14 lbs. 1 oz., and the total weight is 2 tons 10 cwt. 2 qrs. 22 lbs. 7 oz. How many are there?

14. How many times is 36 lbs. 3 oz. 3 dwts. contained in 543 lbs. 11 oz. 5 dwts.?

15. How many bullets each weighing $2\frac{1}{2}$ oz. can be made from a quantity of lead weighing 7 cwt. 1 qr. 13 lbs.?

16. A sovereign weighs 123 grains; how many can be made out of 3 lbs. 5 oz. of standard gold?

MISCELLANEOUS EXAMPLES II.

1. A certain chapter of a book begins at the top of the 357th page and ends at the bottom of the 435th page. How many pages are there in the chapter?

2. Find the number nearest to 1000 which is exactly divisible by 39.

3. Shew that there are the same number of farthings in £59. 12*s.* $1\frac{3}{4}d.$ as there are lbs. in 25 tons 10 cwt. 3 qrs. 19 lbs.

4. A man bought a silver tea set weighing 5 lbs. 8 oz. for 5*s.* 6*d.* an oz., and sold it for £30. How much did he gain?

5. The circuit of a racing path is 137 yards 1 ft. 6 in.; how many times must a bicyclist go round in order to complete 5 miles?

---

6. What number subtracted eighty-eight times from eighty thousand and eighty will leave eighty-eight as a remainder?

7. Multiply, in three lines, 61795 by 2880729.

8. Exactly 20 years ago a man was four times as old as his son, whose present age is 28. What is the present age of the father?

9. How many pounds Troy are there in 144 pounds Avoir.?

10. How many payments of 14*s.* $11\frac{3}{4}d.$ can be made out of £89. 2*s.* $6\frac{1}{4}d.$?

---

11. Multiply three thousand and fifty-seven by two thousand four hundred and sixty-eight, and divide the product by three thousand seven hundred and two.

12. A certain number was divided by 35 by 'short' divisions; the quotient was 72, the first remainder was 2 and the second remainder was 6. What was the dividend?

13. A grocer buys 572 pounds of tea at 1*s.* $9\frac{1}{2}d.$ a lb., and sells at 2*s.* 1*d.* a lb. What does he gain by the transaction?

14. Shew that, if the price in shillings of 1 cwt. of goods be multiplied by 3 and divided by 7, the result will be the price in farthings of 1 lb. of the goods.

15. A person's income for 1888 was £38. 10*s.* a calendar month, and he spent £1. 4*s.* $7\frac{1}{2}d.$ a day; how much had he left at the end of the year?

---

16. A certain number divided by 243 gives quotient 1306 and remainder 92. Find the number.

17. What is the greatest and what is the least number of three digits which is exactly divisible by 31?

18. What day of the year was July 20th, 1891?

19. How many half-pint glasses of ale can be drawn from a barrel containing 36 gallons (allowing nothing for waste); and what will be the whole profit got by retailing the ale at three half-pence a glass, the original cost being fourteen pence a gallon?

20. How many plots each containing 2 r. 16 p. 8 sq. yds. are there in 10 a. 36 p. 15 sq. yds.?

---

21. The successful candidate at an election had a majority of 572 over the unsuccessful candidate; and 6920 votes were given altogether for the two candidates. How many did each get?

22. How many days would it take to count three million sovereigns, counting 100 a minute and going on for 10 hours every day?

23. Reduce 14058 inches to poles.

24. A grocer buys 6 tons 15 cwt. of sugar at £1. 0*s.* 6*d.* a cwt., and sells the sugar at $2\frac{1}{2}d.$ a lb. How much does he gain by the transaction?

25. How many rings each weighing 4 dwts. 18 grs. can be made from a mixture of 1 lb. 10 oz. 1 dwt. 18 grs. of pure gold with 7 oz. 7 dwts. 6 grs. of alloy?

---

26. Express 99, 564, 1749 and 6259 by means of Roman numerals.

27. What number multiplied by 51 will give the same product as 2431 multiplied by 171?

28. Find the greatest and least numbers of six digits which are exactly divisible by 789.

29. How many francs each worth $9\frac{1}{2}d.$ are equal to £15. 17*s.* $4\frac{1}{2}d.$?

30. A bill of £9. 8*s.* $10\frac{1}{4}d.$ has to be paid by several persons in equal shares; and it is found that 14 of them have to pay altogether £3. 11*s.* $5\frac{1}{2}d.$ How many are there to share the whole cost?

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31. The sum of two numbers is 9269 and their difference is 2581 ; what are the numbers ?

32. Find the final remainder after subtracting 3175 as many times as possible from ten millions.

33. 22 yards in 5 seconds is how many miles in an hour ?

34. Divide £1468. 16*s.* 3*d.* by 426.

35. Two pieces of cloth of the same length cost £11. 3*s.* 6*d.* and £14. 8*s.* respectively ; and the price of the first was 6*s.* 2½*d.* a yard. How many yards were there in each piece, and what was the price per yard of the second piece ?

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36. Add together fourteen hundred and seventeen, four thousand eleven hundred and nine, six million fifteen thousand, and eighteen million twelve hundred and nineteen.

37. What is the least number which must be added to ten million in order that the sum may be exactly divisible by 365 ?

38. A train is going at the rate of 30 miles an hour ; how many feet would it go in a second ?

39. A silver tankard weighs 350 oz. *Troy* ; what is its weight *Avoir.* ?

40. A sum of £50. 7*s.* 5*d.* is to be equally divided among the boys of a certain school, and it is found that each boy will get a half-sovereign, a half-crown, a sixpence and a penny. How many boys are there in the school ?

---

41. From the difference between 16391 and 4787 take the sum of 3509 and 1976.

42. A man bought 374 eggs at 2 a penny, and some others at 3 for twopence, and he paid altogether £1. 9*s.* 11*d.* How many did he buy at 3 for twopence ?

43. If telegraph posts be placed 14 p. 3 yds. apart, how many will there be in a mile ?

44. An estate is planted with 472 young trees at a cost of £1. 2*s.* 9*d.* a dozen. What is the whole cost ?

45. Into how many lots can 57 tons 5 cwt. 1 qr. 7 lbs. be divided so that each lot is 1 ton 12 cwt. 2 qrs. 25 lbs. ?

CHAPTER IV.

MEASURES. MULTIPLES. PRIME FACTORS. G. C. M. AND L. C. M.

76. A **Factor**, or a **Measure**, of a number is any number which divides it exactly.

Thus 2, 3, 4, 6, 8, 12, 24 are *factors* of 24.

A **Multiple** of a number is the product of that number and any number whatever. Or, a **multiple** of a number is any number which is exactly divisible by it.

Thus 6×1, 6×2, 6×6, 6×300, &c., that is 6, 12, 36, 1800, &c., are all multiples of 6.

It will be seen at once that a number has a limited number of factors, but an unlimited number of multiples.

NOTE. Throughout the rest of this chapter *divide* will be used to mean *divide without remainder.*

77. A **Prime number** is a number which is not divisible by any number except itself and 1. A prime number is often called a **prime.**

Thus 2, 3, 5, 7, &c. are *primes.*

Every number which has other factors beside itself and unity is called a **composite** number.

Thus 4, 6, 8, 9, &c. are *composite* numbers.

Two numbers which cannot both be divided by any number (except unity) are said to be **prime to one another,** and each is said to be *prime to the other.*

Thus 4 and 9 are prime to one another; they are, however, both composite numbers.

78. The following simple conditions of divisibility will be found to be useful.

(i) A number is or is not divisible by **2** or by **5** according as the number expressed by its last digit is or is not divisible by 2 or by 5 respectively.

Thus all numbers which end in 0, 2, 4, 6, or 8 are divisible by 2, and those which end in 1, 3, 5, 7, or 9 are not divisible by 2.

Also all numbers which end in 0 or 5 are divisible by 5, and no other numbers.

Numbers are called **Even** or **Odd** according as they are or are not divisible by 2.

(ii) A number is or is not divisible by **4** or by **25** according as the number expressed by its last two digits is or is not divisible by 4 or by 25 respectively.

Thus, we can see without trial that 67215736 is divisible by 4, by using 36 as a test.

(iii) A number is or is not divisible by **8** or by **125** according as the number expressed by the last three digits is or is not divisible by 8 or by 125 respectively.

(iv) A number is or is not divisible by **3** (or by **9**) according as the sum of its digits is or is not divisible by 3 (or by 9).

The sum of the digits of the number 56174154, namely

$$5+6+1+7+4+1+5+4 \text{ is } 33;$$

and 33 is divisible by 3, but is not divisible by 9. Thus the number 56174154 is divisible by 3 but not by 9.

(v) A number is divisible by **11** when the difference between the sum of the first, third, fifth, &c. digits and the sum of the second, fourth, sixth, &c. digits is zero or a multiple of 11, and not otherwise.

Thus 3572129 is seen to be divisible by 11 since $9+1+7+3$ differs from $2+2+5$ by 11.

EXAMPLES IV. A.

Which of the numbers 2, 4, 8, 3, 9, 5, 25, 125, 11 can be seen by inspection to be factors of

1.	964.	**2.**	225.	**3.**	1925.	**4.**	7326.
5.	6975.	**6.**	4125.	**7.**	94680.	**8.**	29304.
9.	76164.	**10.**	49125.	**11.**	307890.	**12.**	264792.

79. The following are important general theorems:—

I. *Every divisor or factor of each of several numbers is a divisor of their sum.*

If, for example, each of several numbers is divisible by 12, then each can be arranged in groups of *twelve*, and therefore their sum consists of a certain number of *twelves*. Similarly for any other divisor.

II. *Every divisor of a number is a divisor of any multiple of that number.*

If, for example, any number is divisible by 12, it can be arranged in groups of *twelves*, and so also can any number of repetitions of the number.

III. *Every divisor of two numbers is a divisor of the sum or of the difference of any multiples of the numbers.*

If, for example, two numbers are both divisible by 12, they can both be arranged in groups of *twelves*, and so also can any multiples of either. These multiples can then be added together, or one can be taken from the other, *without taking to pieces any of the groups.*

To make the above theorems quite clear to a beginner it would be well to have actual counters to deal with, which could be tied up by *twelves* in bags or bundles. The pupil would then see that the different additions and subtractions could be performed *without undoing any of the bags or bundles,* and therefore the final result must be a certain number of *twelves.*

***80.** The different tests of divisibility given in Art. 78 can be proved as follows:

To prove (i). Excluding the unit's digit, the remaining digits of any number represent so many *tens*. Now 10 is divisible by 2 (or by 5), and therefore any multiple of *ten* is divisible by 2 (or by 5).

Hence, if the unit's digit of a number is divisible by 2 (or by 5) the number itself is divisible by 2 (or by 5).

For example 7534 is 753 *tens* + 4. Now 753 *tens* is a multiple of 2 (or of 5), and 4 is a multiple of 2 (but not of 5). Therefore 7534 is a multiple of 2 (but not of 5).

(ii) Excluding the two last digits, the remaining digits of any number represent so many *hundreds*. Now 100 is divisible by 4 (or by 25), and therefore any number of *hundreds* is divisible by 4 (or by 25).

Hence, if the number represented by the last two digits of any number be divisible by 4 (or by 25), the whole number must be divisible by 4 (or by 25).

Similarly, any number is divisible by 8 (or by 125) if the number formed by the last *three* digits is divisible by 8 (or by 125).

(iv) Consider, for example, the number 3576. Since $10=9+1$, $100=99+1$, $1000=999+1$, &c., we have

$$\begin{aligned} 3000 &= 999\times 3+1\times 3 = \text{a multiple of } 9+3,\\ 500 &= 99\times 5+1\times 5 = \text{a multiple of } 9+5,\\ 70 &= 9\times 7+1\times 7 = \text{a multiple of } 9+7,\\ 6 &= 6. \end{aligned}$$

Hence, by addition,

3576 = a multiple of 9 + the sum of the digits 3, 5, 7, 6.

Thus *the difference between the number* 3576 *and the sum of its digits is a multiple of* 9 (and therefore also of 3); and hence the number is divisible by 9 (or by 3) when the sum of its digits is divisible by 9 (or by 3).

The above proof is clearly applicable to any number whatever.

(v) Consider, for example, the number 742543.

Since $10=11-1$, $100=99+1$, $1000=990+11-1$,

$10000=9999+1$, $100000=99990+11-1$, &c.,

and since 99, 9999, 999999, and all other numbers which are made up of an *even* number of nines are clearly divisible by 11, it follows that

10 = a multiple of 11 − 1,		therefore 40 = a multiple of 11 − 4.	
100 = ...	+1,	500 = ...	+5.
1000 = ...	−1,	2000 = ...	−2.
10000 = ...	+1,	40000 = ...	+4.
100000 = ...	−1,	700000 = ...	−7.
	Also	3 =	3.

Hence, by addition,

742543 = a multiple of 11 plus $3-4+5-2+4-7$.

Hence in order that 742543 may be a multiple of 11 the difference between $3+5+4$ and $4+2+7$ must be zero or a multiple of 11, which is not the case.

81. The Sieve of Eratosthenes. The different prime numbers can be found in order by the following method, called the Sieve of Eratosthenes.

Write down in their natural order the numbers from 1 to any extent that may be required: thus

1,	2,	3,	4̇,	5,	6̈,	7,	8̇,	9̇,	10̇,
11,	12̇,	13,	14̇,	15̇,	16,	17,	18̇,	19,	20̇,
21̇,	22,	23,	24̇,	25.	26,	27,	28̇,	29,	30̇,
31,	32,	33,	34,	35̇,	36̇,	37,	38,	39,	40̇, &c.

Now take the first prime number, 2, and over every second number from 2 place a dot: we thus mark all the multiples of 2. Then, leaving 3 unmarked, place a dot over every third number from 3: we thus mark all multiples of 3. The number next to 3 left unmarked is 5; and, leaving 5 unmarked, place a dot over every fifth number from 5: we thus mark all multiples of 5. And so for multiples of 7, &c.

By proceeding in this way all multiples of the prime numbers 2, 3, 5, 7, &c. are struck out; also multiples of all composite numbers are necessarily struck out at the same time, for example all multiples of 6 are struck out as being multiples of either of its prime factors 2 or 3. Hence all the numbers which are left unmarked are primes, for no one of them is divisible by any number (except unity) which is smaller than itself.

We can thus find in order as many prime numbers as we please.

The primes less than 100 will be found to be

1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

82. To find whether a given number is or is not a prime, we have only to see whether it is divisible by any one of the *prime* numbers, 2, 3, 5, 7, &c.

Ex. 1. Is 233 a prime number?

By trial it will be found that 233 is *not* divisible by 2, or by 3, or by 5, or by 7, or by 11, or by 13, or by 17. Now it is not necessary to try any other primes, for $233 \div 17$ gives a quotient less than 17; if therefore 233 were divisible by a prime greater than 17, the quotient would be less than 17, and 233 would be divisible by this quotient, that is by a number less than 17, which we know is not the case. Hence 233 is a prime number.

83. Resolution into prime factors.

The following examples will suffice to shew how to express any number whatever as the product of factors each of which is a prime.

The method is applicable to all numbers however large, provided we find as many prime numbers as may be necessary by means of the 'sieve'; the method would however be extremely tedious in the case of a very large number.

Ex. 1. Express 28028 as the product of prime factors.

The number is divisible by 2, and $28028 = 2 \times 14014$; then again 14014 is divisible by 2, and we have $28028 = 2 \times 2 \times 7007$. Since 7007 is not divisible by 2, we pass on to the next prime. By inspection 7007 is *not* divisible by 3 or by 5; we therefore pass on to the next prime, namely 7. Then $28028 = 2 \times 2 \times 7 \times 1001$. By trial 1001 is divisible by 7; and we have

$$28028 = 2 \times 2 \times 7 \times 7 \times 143.$$

Finally we find that $143 = 11 \times 13$. The work would be written

$$28028 = 2 \times 14014 = 2 \times 2 \times 7007 = 2 \times 2 \times 7 \times 1001$$
$$= 2 \times 2 \times 7 \times 7 \times 143 = 2 \times 2 \times 7 \times 7 \times 11 \times 13.$$

Ex. 2. Find the prime factors of 3978.

The number is divisible by 2, and we have $3978 = 2 \times 1989$. Then 1989 is divisible by 3, and we have $3978 = 2 \times 3 \times 663$. Again 663 is divisible by 3, and we have $3978 = 2 \times 3 \times 3 \times 221$. It will be found that $221 = 13 \times 17$. Hence $3978 = 2 \times 3 \times 3 \times 13 \times 17$.

EXAMPLES IV. B.

Express the following numbers as the product of prime factors:

1. 6, 9, 10, 15, 24, 30, 36, 39, 45, 48.
2. 49, 50, 54, 60, 68, 70, 75, 77, 81, 86.
3. 100, 105, 120, 130, 136, 148, 154, 156.
4. 184, 196, 273, 275, 364, 391, 525, 585.
5. 350, 459, 715, 728, 750, 792, 825, 999.
6. 1092, 2618, 6611, 3885, 3731, 51051, 74613, 462462.

Greatest Common Measure.

84. A **common measure,** or **common factor** of two or more numbers is a number which exactly divides each of them.

For example, 2, 3 and 6 are *common measures* of 18 and 24.

The **greatest common measure** [written G.C.M.] of two or more numbers is the greatest number which exactly divides each of them.

Thus, 6 is the greatest common measure of 18 and 24.

85. When numbers are expressed as the product of prime factors, their G.C.M. can be seen by inspection.

Consider, for example, the numbers

$$2\times2\times2\times3\times3\times11 \text{ and } 2\times2\times2\times2\times3\times7\times13.$$

Here we see that 2 and 3 are the only prime factors which are common to the two numbers; we see also that $2\times2\times2$ is the highest power of 2, and that 3 is the highest power of 3 which is common.

Hence the G. C. M. is $2\times2\times2\times3$.

Again, to find the G. C. M. of 720 and 1080.

Since $720=2\times2\times2\times2\times3\times3\times5$, and $1080=2\times2\times2\times3\times3\times3\times5$; we see that the G. C. M. is $2\times2\times2\times3\times3\times5=2^3\times3^2\times5$.

EXAMPLES IV. C.

Find the G.C.M. of

1.	12 and 18.	**2.**	20 and 25.	**3.**	30 and 42.
4.	18 and 30.	**5.**	60 and 84.	**6.**	54 and 90.
7.	45 and 105.	**8.**	72 and 90.	**9.**	126 and 315.
10.	189 and 273.	**11.**	132 and 252.	**12.**	315 and 357.
13.	693 and 819.	**14.**	792 and 924.	**15.**	891 and 1221.

16. 48, 60 and 72.

17. 72, 108 and 180.

18. 264, 360 and 600.

19. 630, 756 and 1155.

86. We must now shew how to find the G.C.M. of two numbers without first going through the troublesome process of expressing the numbers as the product of prime factors.

The method depends on the following theorem, proved in Art. 79:

Any common factor of two numbers is also a factor of the sum or of the difference of any multiples of the numbers.

Suppose that we have two numbers whose G.C.M. is required.

If we divide the greater number by the smaller; then, by the nature of division,

(i) the remainder is equal to the difference between the greater number and some multiple of the smaller;

also (ii) the greater number is equal to the sum of the remainder and some multiple of the smaller.

From (i) it follows that **any common factor of the original numbers** is a factor of the remainder, and therefore **is a common factor of the remainder and the smaller number.**

And, from (ii) it follows that **any common factor of the remainder and the smaller number** is a factor of the greater number also, and therefore **is a common factor of the two original numbers.**

The G.C.M. of the two original numbers must therefore be the same as the G.C.M. of the smaller number and the remainder.

Thus the problem of finding the G.C.M. of the two original numbers is reduced to that of finding the G.C.M. of the smaller number and the remainder.

Ex. 1. Find the G. C. M. of 3663 and 5439.

First divide the greater by the less

$$\begin{array}{r} 3663 \overline{)\,5439\,(\,1} \\ 3663 \\ \hline 1776 \end{array}$$

Hence the G. C. M. required is the same as the G. C. M. of 1776 and 3663. Divide the greater of these by the less.

$$\begin{array}{r} 1776 \overline{)\,3663\,(\,2} \\ 3552 \\ \hline 111 \end{array}$$

The problem is now reduced to finding the G. C. M. of 111 and 1776. Again divide

$$\begin{array}{r} 111 \overline{)\,1776\,(\,16} \\ 111 \\ \hline 666 \\ 666 \\ \hline \end{array}$$

Thus 111 is a factor of 1776, and therefore 111 is the G. C. M. of 111 and 1776.

But the G. C. M. of 111 and 1776 is the G. C. M. required.

The successive divisions are usually written down in the more compact form

$$
\begin{array}{l}
3663\,)\,5439\,(\,1 \\
\quad 3663 \\
\quad \overline{1776}\,)\,3663\,(\,2 \\
\quad\quad 3552 \\
\quad\quad \overline{111}\,)\,1776\,(\,16 \\
\quad\quad\quad 111 \\
\quad\quad\quad \overline{666} \\
\quad\quad\quad 666 \\
\quad\quad\quad \overline{}
\end{array}
$$

Ex. 2. Find the G. C. M. of 311 and 331.

$$
\begin{array}{l}
311\,)\,331\,(\,1 \\
\quad 311 \\
\quad \overline{20}\,)\,311\,(\,15 \\
\quad\quad 20 \\
\quad\quad \overline{111} \\
\quad\quad 100 \\
\quad\quad \overline{11}\,)\,20\,(\,1 \\
\quad\quad\quad 11 \\
\quad\quad\quad \overline{9}\,)\,11\,(\,1 \\
\quad\quad\quad\quad 9 \\
\quad\quad\quad\quad \overline{2}\,)\,9\,(\,4 \\
\quad\quad\quad\quad\quad 8 \\
\quad\quad\quad\quad\quad \overline{1}\,)\,2\,(\,2 \\
\quad\quad\quad\quad\quad\quad 2 \\
\quad\quad\quad\quad\quad\quad \overline{}
\end{array}
$$

Here the G. C. M. of 311 and 331 is the same as the G. C. M. of 1 and 2, so that the numbers are prime to one another.

In this example it would be a great waste of time to proceed to the end; for the G. C. M. required is the G. C. M. of *any* divisor and the corresponding dividend, and as soon as it is obvious that one such pair have no common factors it is not necessary to proceed further. Now the only prime factors of 20 are 2 and 5, and by inspection neither of these is a factor of 311.

NOTE. The Greatest Common Measure is sometimes called the Highest Common Factor [H. C. F.].

EXAMPLES IV. D.

Find the G.C.M. (or H.C.F.) of

1. 221 and 247.	2. 357 and 391.	3. 899 and 1073.
4. 663 and 923.	5. 1517 and 1927.	6. 1785 and 2485.
7. 3499 and 3953.	8. 4899 and 5893.	9. 9709 and 22849.
10. 11663 and 12091.	11. 17947 and 29737.	
12. 11453 and 12961.	13. 3834038 and 4169594.	
14. 132038 and 369792.	15. 5411728 and 10902416.	

87. To find the G.C.M. of three or more numbers, we have only to find the G.C.M. of the first two numbers; then the G.C.M. of this result and the third number; and so on.

Ex. Find the G.C.M. of 286, 338 and 585.

The G.C.M. of 286 and 338 will be found to be 26. Then the G.C.M. of 26 and 585 will be found to be 13.

EXAMPLES IV. E.

Find the G.C.M. of

1. 165, 198, 242.	2. 312, 429, 572.	3. 222, 370, 550.
4. 2387, 2821, 4433.	5. 3157, 3321, 4059.	6. 4732, 5824, 6643.

LEAST COMMON MULTIPLE.

88. A **common multiple** of two or more numbers is a number which is exactly divisible by each of them.

For example, 200 is a common multiple of 20 and 25.

The **least common multiple** [written L.C.M.] of two or more numbers is the least number which is exactly divisible by each of them.

For example, 100 is the least common multiple of 20 and 25.

89. When numbers are expressed as the product of prime factors, their L.C.M. can be seen by inspection.

Consider, for example, the numbers

$$2\times2\times2\times3\times5,\ 2\times2\times3\times3\times7,\ 3\times5\times5\times5\times11.$$

Here we see that a common multiple must contain the prime factors 2, 3, 5, 7 and 11. Also a multiple of the first number must contain the *third* power at *least* of the factor 2; a multiple of the second must contain the *second* power at *least* of the factor 3; a multiple of the third number must contain the *third* power at *least* of the factor 5; and we must also have the first power at least of the factors 7 and 11.

The least common multiple must therefore be

$$2\times2\times2\times3\times3\times5\times5\times5\times7\times11.$$

From the above example it will be seen that the L. C. M. *of two or more numbers is the continued product of all the different prime factors which occur in the different numbers, each prime factor being taken to the highest power in which it occurs.*

Ex. Find the L. C. M. of 105, 126 and 196.

The numbers are $3\times5\times7$, $2\times3\times3\times7$, $2\times2\times7\times7$.

Hence the L. C. M. is $2\times2\times3\times3\times5\times7\times7=8820$.

90. The usual method by which the L.C.M. of numbers is found is as follows:

Having written the numbers in a row, first strike out any numbers which are factors of any of the others (for every multiple of a number is a multiple of any factor of that number). Then divide by any **prime number** *which divides at least* two *of the numbers, and put the quotients below those numbers which can be divided, and bring down all those numbers which are not divisible by that prime. Operate on the second row in the same manner as on the first, and go on until a row is arrived at in which no two numbers have any common factor. Then the* L. C. M. *is the continued product of all the divisors and all the numbers left in the last row.*

Thus, to find the L. C. M. of 2, 5, 15, 24, 25, 30, 36.

The process is written as under

$$\begin{array}{rl} 2) & \cancel{2},\ \cancel{5},\ \cancel{15},\ 24,\ 25,\ 30,\ 36 \\ 2) & 12,\ 25,\ 15,\ 18 \\ 3) & 6,\ 25,\ 15,\ 9 \\ & 2,\ 25,\ \cancel{5},\ 3 \end{array}$$

Hence the L. C. M. is $2\times2\times3\times2\times25\times3=1800$.

First 2, 5 and 15 are struck out since each is contained in one or other of the remaining numbers. We then divide by 2, the lowest prime which divides more than one of the numbers; we then again divide by 2, and then by 3. In the fourth row 5 is struck out because it is a factor of 25, and no prime will divide more than one of the remaining numbers in that row.

The reason the L. C. M. is given by the above process is that the L. C. M. of the numbers in the first row = 2 × L. C. M. of the numbers in the second row; and so on to the end.

EXAMPLES IV. F.

Find the L.C.M. of

1. 12, 16. **2.** 20, 25. **3.** 25, 30. **4.** 8, 12, 20.
5. 6, 8, 12. **6.** 15, 20, 30. **7.** 30, 45, 54.
8. 10, 14, 35. **9.** 6, 12, 18, 63. **10.** 9, 36, 45, 81.
11. 3, 11, 18, 33, 36. **12.** 25, 27, 33, 55.
13. 16, 20, 22, 33, 36. **14.** 3, 4, 10, 12, 14, 16, 18.
15. 3, 14, 12, 56 and 28. **16.** 16, 18, 20, 24, 30, 36.
17. 12, 15, 18, 21, 25, 35, 210. **18.** 12, 42, 49, 54.
19. 30, 35, 42, 60, 72. **20.** 18, 54, 90, 102, 120, 144.

91. To find the L.C.M. of numbers which cannot be readily resolved into factors, we must first find the G.C.M.

For example, to find the L. C. M. of 4592 and 5371.

The G. C. M. will be found to be 41. And by division

$$4592 = 41 \times 112 \text{ and } 5371 = 41 \times 131.$$

Hence, as 112 and 131 are prime to one another, the L. C. M. is

$$41 \times 112 \times 131.$$

It should be noticed that the L. C. M. $\times$ G. C. M. $= 41 \times 112 \times 41 \times 131$ = product of the numbers.

To find the L.C.M. of more than two numbers we can find the L.C.M. of two of the numbers, and then the L.C.M. of this result and of the third number, and so on to the end.

EXAMPLES IV. G.

Find the L. C. M. of

1. 357, 391. **2.** 851, 943. **3.** 3497, 4035.
4. 4899, 5893. **5.** 165, 198, 242. **6.** 312, 429, 572.
7. 360, 1350, 1500. **8.** 195, 546, 286.
9. 420, 630, 1050 and 1470. **10.** 1365, 2288, 2640.

92. To find the G. C. M. and L. C. M. of concrete quantities.

Ex. 1. Find the G. C. M. of 4 yds. 10 in. and 5 yds. 2 in.

First express both quantities in terms of the common unit inches; then we have 154 inches and 182 inches. Now the G. C. M. of the numbers 154 and 182 will be found to be 14. Hence 14 *inches* is the greatest length which is exactly contained in both the given lengths, that is 14 inches is the G. C. M. required.

Ex. 2. To find the L. C. M. of 4 yds. 10 in. and 5 yds. 2 in.

Here we have to find the smallest length which is an exact multiple of both the given lengths.

Reduce the given lengths to a common unit, namely inches; then we have 154 inches and 182 inches.

It will be found that $14 \times 11 \times 13 = 2002$ is the L. C. M. of 154 and 182.

Hence the L. C. M. required is 2002 inches = 55 yds. 1 ft. 10 in.

EXAMPLES IV. H.

Find the G. C. M. and the L. C. M. of

1. 2*s.* 1*d.* and 2*s.* 11*d.*
2. 3*s.* 4*d.* and 5*s.* 4*d.*
3. 8*s.* 2*d.* and 12*s.* 10*d.*
4. 16*s.* 3*d.* and £1. 3*s.* 8*d.*
5. 7 ft. and 12 ft. 10 in.
6. 2 yds. 2 ft. 3 in. and 3 yds. 1 ft. 1 in.
7. 1 cwt. 2 qrs. 12 lbs. and 2 cwt. 1 qr. 3 lbs.
8. 1 cwt. 2 qrs. 21 lbs. and 1 cwt. 1 qr. 4 lbs.
9. 5 gallons 2 qts. 1 pt. and 8 gallons 1 pt.
10. 6*s.* 5*d.*, 10*s.* 1*d.* and 11*s.* 11*d.*

MISCELLANEOUS EXAMPLES III.

1. Divide the product of 8978 and 55112 by 5561.

2. How many minutes are there in 5 years beginning with 1890?

3. How much would it cost to make a road a mile long at £32. 12*s.* a chain?

4. Divide £210 between *A* and *B*, so that *A* may have five times as much as *B*.

5. Find the cost of a steel hammer weighing 225 tons at the rate of 3*d.* for 2 lbs.

6. Find the G. C. M. of 3465 and 3696.

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7. What number is the same multiple of 7 that 21560 is of 55?

8. Reduce 95536 inches to miles, furlongs, poles, &c.

9. What is the price of a silver bowl weighing 5 lbs. 8 oz. at 5*s.* 10*d.* an ounce?

10. The 'Standard' newspaper of Thursday January 1, 1891, was numbered 20741; what was the number of the paper issued on July 1, 1890? [The Standard is published every day throughout the year except on Sundays.]

11. Two equal sums were respectively divided among 12 men and a certain number of boys. Each man received 5*s.* and each boy 1*s.* 6*d.* How much was divided altogether, and how many boys were there?

12. Find the G. C. M. of 5610, 11781 and 1309.

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13. After multiplying 375 by 29 and 131 by some other number, the results when added together amounted to 13888. What was the other number?

14. Divide £124. 9*s.* 1*d.* by 17.

15. How many half-crowns are there in 189550 half-guineas?

16. How many Fridays will there be in 7 years beginning Thursday January 1, 1891?

17. Find the least number by which 222 must be multiplied in order that the product may be a multiple of 1295.

18. Four bells toll at intervals of 3, 4, 5 and 6 seconds respectively. If they all begin to toll at the same instant, how long will it be before they again all toll together?

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19. The radius of the moon being 1886720 yards, and her distance from the earth 221 times her radius; find her distance in miles.

20. Multiply £31. 3*s.* 5*d.* by 117 and divide the result by 39.

21. Find the number of feet in 43 miles 5 furlongs 6 yards.

22. Find the number of seconds from the beginning of the year 1890 up to 10 o'clock in the morning of March 25th.

23. An ounce of standard gold is worth £3. 17*s.* 10½*d.* How many sovereigns are there in 100 lbs. of standard gold?

24. What numbers less than 200 leave 2 as remainder when divided by either of the numbers 3, 4, 5 or 6?

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25. What is the least number of times that 315 must be added to 1594 that the sum may exceed a million?

26. Multiply 67412 by 9997 as shortly as you can.

27. How many tons, cwt., &c. are there in a million drams?

28. A farmer lost one lamb out of every four born in a certain season, and 351 survived; how many were born?

29. Find the prime factors of 1176 and 19404, and hence write down their G. C. M. and L. C. M.

30. Find the length of the longest chain in terms of which 9 po. 4 yds. 6 in. and 44 po. 3 yds. 1 ft. can both be expressed as integers.

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31. When 829653 is divided by a certain number the quotient is 811 and the remainder is 811; find the number.

32. Reduce 1 cwt. 6 lbs. 6 oz. to Troy weight.

33. A person's income being £556. 12*s.* 6*d.* a year, how much less per day can he afford to spend in a leap year than in an ordinary year?

34. Find the cost of 286 quarters of wheat at 4*s.* 3*d.* a bushel.

35. Find the L. C. M. of 12, 64, 80, 96, 120, 160.

36. The length and breadth of a room are respectively 20 feet and 14 ft. 8 in.: find their G. C. M.

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37. In a division sum the quotient is twice the divisor, and the remainder (which is 50) is one-fifth part of the quotient. Find the dividend.

38. Multiply 6178 by 125, and 123456 by 172814412, as shortly as you can.

39. Reduce 630000 farthings to guineas, and 12 cwt. 1 qr. 2 lbs. 15 oz. to ounces.

40. Multiply £26. 7*s.* 8¾*d.* by 287, and prove your result by division.

41. Find the L. C. M. of 21, 22, 24, 28, 32, 33, also of 16, 18, 20, 24, 30, 36.

42. Find the greatest sum of money of which both £2. 18*s.* 7*d.* and £5. 1*s.* 9*d.* are multiples.

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43. What number is it which after being subtracted 19 times from 1000 leaves a remainder 12?

44. How many half-pence are there in £182. 7*s.* 8*d.* and how many inches in 8 miles 60 yards?

45. How many oranges can be bought for 6*s.* 3*d.* at the rate of 2 for $1\frac{1}{2}d.$?

46. Divide £1560 among *A*, *B* and *C*, so that *B* may have £100 more than *A*, and *C* may have £100 more than *B*.

47. Find the greatest number of which both 387476 and 509642 are multiples.

48. Find the least number which can be divided by 7, 20, 28, 35 and leave 3 as remainder in each case.

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49. If a number when divided by 35 give a remainder 27, what remainder will it give when divided by 7?

50. What is the greatest and what is the least number of four digits which is exactly divisible by 73?

51. Find the sum of one million pence, one thousand shillings and one hundred half-crowns.

52. How many more farthings are there in £25. 0*s.* $4\frac{1}{2}d.$ than in £23. 17*s.* $2\frac{3}{4}d.$?

53. A farmer bought 83 lambs at £1. 2*s.* 6*d.* each, 54 sheep at £2. 2*s.* 9*d.* each, 13 cows at £15. 10*s.* 6*d.* each, and a horse for £27. 10*s.* How much will the total cost be less than £500?

54. Find the L. C. M. of 54, 105, 120, 90, 18, and the G. C. M. of 1215 and 9375.

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55. Multiply three thousand and eighty-seven by seventy-two thousand nine hundred and thirty. What numbers less than 12 will exactly divide the product?

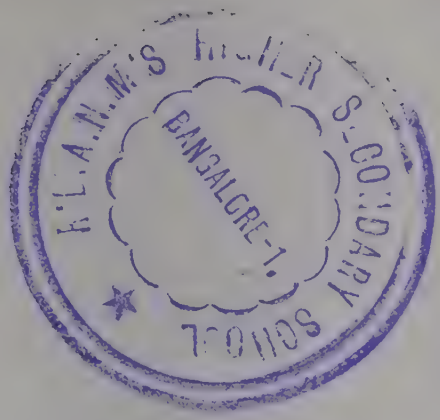
56. A person bought books at 2*s.* $1\frac{1}{2}d.$ each and sold them at 3*s.* each; how much did he gain by selling $13\frac{1}{2}$ dozen of them?

57. Reduce 17 wks. 5 d. 16 hrs. 17 min. 25 secs. to seconds.

58. Knowing that the first of January 1891 was on Thursday, find the day of the week of January 1st, 1900, and also of May 1st, 1900.

59. Find the G. C. M. of 10481 and 17617; also of 156695 and 692307.

60. Four men can walk 30, 35, 40 and 45 miles a day respectively; what is the least distance they can all walk in an exact number of days?



CHAPTER V.

FRACTIONS.

93. If the unit quantity be divided into a number of equal parts, one or more of these parts is called a **fraction** of the unit.

If the unit be divided into 2, 3, 4, 5, &c. equal parts, these parts are called halves, third-parts, fourth-parts, fifth-parts, &c., or more shortly and more generally *halves, thirds, fourths, fifths,* &c.

Thus if a unit be divided into 7 equal parts, the portion made up of 3 of these parts is called 3 *sevenths*.

We have hitherto only considered numbers which denote collections of whole units. These are distinguished from fractions by being called **whole numbers** or **integers.**

94. A **Vulgar Fraction** is a fraction represented by two numbers placed one above the other with a line between them. The lower number is called the **denominator** and shews the number of equal parts into which the unit is divided; the upper number is called the **numerator** and shews how many of those parts are to be taken to form the fraction.

Thus $\frac{3}{5}$ is read *three-fifths*, and denotes that the unit is divided into 5 equal parts and that 3 of these parts are taken to form the fraction.

95. If we have 3 units and divide each of them into 5 equal parts, and then take one of the parts from each divided unit, we shall take one part out of every five, that is, one-fifth of the whole three units; but each of the parts is one-fifth of a single unit and we take 3 of them, we therefore take 3 *fifths* of one unit.

Thus 3 *fifths of* 1 *unit is the same as* 1 *fifth of* 3 *units.*

Hence $\frac{3}{5}$, which by definition denotes 3 *fifths* of 1 unit, may also be considered to stand for 1 *fifth* of 3 units.

The same holds good for all other fractions; for example

$\frac{3}{5}$ of £1 = $\frac{1}{5}$ of £3;

and $\frac{7}{9}$ of 1 yard = $\frac{1}{9}$ of 7 yards.

EXAMPLES V. A.

1. Write in figures the following fractions :—five-ninths, six-elevenths, eleven twenty-thirds, sixteen twenty-sevenths, seventeen ninety-oneths, ninety-five one hundred and fourths.

2. Write in words :—$\frac{3}{7}$, $\frac{5}{8}$, $\frac{2}{11}$, $\frac{9}{23}$, $\frac{5}{31}$, $\frac{17}{56}$, $\frac{7}{81}$, $\frac{20}{111}$, $\frac{15}{219}$, $\frac{119}{2000}$.

Find the value of

3.	$\frac{2}{5}$ of £1.	**4.**	$\frac{3}{4}$ of 1*s.*	**5.**	$\frac{2}{9}$ of £1. 10*s.*
6.	$\frac{7}{9}$ of 6*s.*	**7.**	$\frac{4}{7}$ of 2*s.* 11*d.*	**8.**	$\frac{3}{11}$ of 3*s.* 8*d.*
9.	$\frac{5}{7}$ of a guinea.	**10.**	$\frac{3}{10}$ of 5 guineas.	**11.**	$\frac{7}{15}$ of a crown.
12.	$\frac{1}{8}$ of 1 cwt.	**13.**	$\frac{3}{7}$ of a stone.	**14.**	$\frac{3}{14}$ of a cwt.
15.	$\frac{3}{22}$ of a mile.	**16.**	$\frac{3}{10}$ of an acre.	**17.**	$\frac{5}{9}$ of 2 gals. 1 qt.
18.	$\frac{2}{9}$ of 3 bus. 1 pk. 1 gal.	**19.**	$\frac{3}{35}$ of 3 oz. 12 dwts. 22 grs.		
20.	$\frac{7}{120}$ of a day.	**21.**	$\frac{3}{16}$ of 1 hour.	**22.**	$\frac{11}{140}$ of 1 d. 4 hrs.

96. The numerator and denominator of a fraction are called its **terms**.

When the numerator is less than the denominator the fraction is called a **proper** fraction; and when the numerator is greater than the denominator the fraction is called an **improper** fraction.

Thus a proper fraction is less and an improper fraction is greater than 1.

A **mixed number** is a number made up of a fraction and an integer.

Thus $2\frac{1}{4}$, which means $2 + \frac{1}{4}$, is a mixed number.

97. A mixed number can be expressed as an improper fraction.

Consider, for example, $3\frac{2}{7}$.

Each unit contains 7 *sevenths*, therefore 3 units contain 3×7 *sevenths*.

Hence $3\frac{2}{7} = 3 \times 7$ *sevenths* $+ 2$ *sevenths* $= \frac{21+2}{7} = \frac{23}{7}$.

Again $7\frac{2}{9} = \frac{7 \times 9 + 2}{9} = \frac{65}{9}$. Also $6 = 6 \times 4$ *fourths* $= \frac{24}{4}$.

From the above it will be seen that a *mixed number is equivalent to an improper fraction* whose denominator is the denominator of the fractional part of the mixed number, and whose numerator is obtained by multiplying the whole number by the denominator of the fraction and adding in its numerator.

It should be noticed that a whole number can be expressed as a fraction with *any* given denominator. For example,

$6 = 6 \times 7$ *sevenths* $= \frac{42}{7}$; also $6 = 6 \times 13$ *thirteenths* $= \frac{78}{13}$.

98. Conversely, an improper fraction can be expressed as a mixed number, or a whole number.

Consider, for example, $\frac{23}{7}$.

Since 7 *sevenths* make 1 unit,

$\frac{23}{7} = 3 \times 7$ *sevenths* $+ 2$ *sevenths* $= 3 + 2$ *sevenths* $= 3\frac{2}{7}$,

Again $\frac{24}{4} = 6 \times 4$ *fourths* $= 6$, since 4 *fourths* $= 1$.

From the above it will be seen that an *improper fraction is reduced to a mixed number* by dividing its numerator by its denominator, the quotient giving the whole number and the remainder, if any, the numerator of the proper fraction whose denominator is the same as that of the improper fraction.

EXAMPLES V. B.

Express as whole or mixed numbers:

1. $\frac{12}{4}$, $\frac{20}{5}$, $\frac{35}{7}$, $\frac{90}{15}$.
2. $\frac{10}{3}$, $\frac{13}{4}$, $\frac{17}{5}$, $\frac{25}{6}$.
3. $\frac{17}{11}$, $\frac{35}{9}$, $\frac{83}{10}$, $\frac{79}{12}$.
4. $\frac{35}{17}$, $\frac{42}{19}$, $\frac{60}{21}$, $\frac{70}{29}$.
5. $\frac{115}{27}$, $\frac{135}{29}$, $\frac{144}{31}$, $\frac{151}{34}$.
6. $\frac{422}{111}$, $\frac{516}{17}$, $\frac{357}{41}$, $\frac{819}{101}$.
7. Express 3, 5, 9 and 13 as fractions with a denominator 7.

Express as improper fractions

8. $1\frac{1}{2}$, $1\frac{3}{4}$, $2\frac{1}{3}$, $3\frac{1}{5}$.
9. $5\frac{2}{7}$, $1\frac{3}{5}$, $3\frac{4}{9}$, $4\frac{3}{11}$.
10. $7\frac{3}{16}$, $6\frac{5}{12}$, $7\frac{2}{13}$, $1\frac{17}{31}$.
11. $4\frac{3}{19}$, $5\frac{6}{17}$, $2\frac{112}{113}$, $7\frac{11}{212}$.

Find the value of

12. $2\frac{5}{6}$ of £1. 12*s*.
13. $7\frac{3}{11}$ of £6. 12*s*. $5\frac{1}{2}d$.
14. $6\frac{4}{9}$ of £1. 3*s*. $5\frac{1}{4}d$.
15. $3\frac{4}{9}$ of 2 yds.
16. $1\frac{3}{7}$ of 2 cwt.
17. $15\frac{4}{15}$ of 5 oz. 6 dwts. 21 grs.

99. The following is a very important truth :—

The value of a fraction is not altered by multiplying the numerator and denominator by the same number.

Consider, for example, the fraction $\frac{5}{7}$.

To obtain this fraction the unit must be divided into 7 equal parts and 5 of those parts must be taken.

Now suppose that each of the parts is sub-divided into any number, say 8, equal portions. There will be in the whole unit 7×8 of these portions; and the 5 parts we are to take will contain 5×8 of these portions.

Hence the fraction will consist of 5×8 portions out of the 7×8 equal portions into which the unit is divided; that is the same fraction as before will now be expressed by $\frac{5 \times 8}{7 \times 8}$.

Hence $$\frac{5}{7} = \frac{5 \times 8}{7 \times 8} = \frac{40}{56}.$$

Precisely the same reasoning will apply to any other fractions besides $\frac{5}{7}$, and will hold good however many sub-divisions may be made.

Since a fraction is unaltered by *multiplying* its numerator and denominator by the same number, it is clear that we may reverse the process; thus a **fraction is not altered in value by dividing its numerator and denominator by the same number.**

100. A fraction is said to be in its **lowest terms** when the numerator and denominator have no common factor.

Thus the fractions $\frac{2}{3}$, $\frac{5}{6}$, $\frac{12}{25}$ are in their lowest terms; but the fractions $\frac{4}{6}$, $\frac{10}{12}$, $\frac{24}{50}$ are not in their lowest terms, for in each case the numerator and denominator have 2 as common factor.

Reduction of fractions to their lowest terms.

Example. To reduce $\frac{825}{1540}$ to its lowest terms.

We have proved that we may, without altering the value of a fraction, divide its numerator and denominator by the same number.

To reduce to the *lowest* terms we must divide by the H. C. F. of the numerator and denominator; for we thus obtain an equivalent fraction whose numerator and denominator have no common factors.

In the present case the H. C. F. will be found to be 55.

Thus $$\frac{825}{1540} = \frac{825 \div 55}{1540 \div 55} = \frac{15}{28}.$$

Instead of reducing a fraction to its lowest terms by dividing the numerator and denominator by their H. C. F., we may divide by *any* common factor, and repeat the process until the fraction is reduced to its lowest terms. Thus

$$\frac{825}{1540} = \frac{165}{308} = \frac{15}{28}.$$

We see at once that 5 is a common factor; we therefore divide the numerator and denominator by 5, and obtain the equivalent fraction $\frac{165}{308}$. We now see that 11 is a common factor, and having divided the numerator and denominator by 11 we have the equivalent fraction $\frac{15}{28}$, which is at once seen to be in its lowest terms.

EXAMPLES V. C.

Reduce the following fractions to their lowest terms:

1. $\frac{2}{4}, \frac{4}{6}, \frac{9}{12}, \frac{12}{16}, \frac{5}{15}, \frac{15}{20}, \frac{12}{18}, \frac{8}{20}, \frac{15}{24}, \frac{24}{30}$.
2. $\frac{14}{21}, \frac{42}{54}, \frac{56}{77}, \frac{18}{30}, \frac{42}{48}, \frac{24}{60}, \frac{25}{75}, \frac{56}{70}, \frac{70}{84}, \frac{64}{96}$.

3.	$\frac{144}{156}$.	4.	$\frac{120}{135}$.	5.	$\frac{120}{280}$.	6.	$\frac{231}{273}$.
7.	$\frac{792}{1683}$.	8.	$\frac{840}{3080}$.	9.	$\frac{1001}{1155}$.	10.	$\frac{1110}{1221}$.
11.	$\frac{735}{1323}$.	12.	$\frac{2028}{2340}$.	13.	$\frac{3663}{4477}$.	14.	$\frac{728}{2128}$.
15.	$\frac{219877}{289870}$.	16.	$\frac{285714}{428571}$.	17.	$\frac{174528}{191808}$.	18.	$\frac{13828}{449008}$.

101. When we have two or more fractions with different denominators, it is often necessary to reduce them to equivalent fractions which all have the same denominator; and it is generally best to use the smallest possible common denominator. This is called *reducing the fractions to their lowest common denominator.*

Reduction to lowest common denominator.

Consider the fractions $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{8}{9}$.

The L. C. M. of the denominators 4, 6 and 9 is easily seen to be 36. Since 36 is a multiple of each denominator, all the fractions can be reduced to equivalent fractions which have 36 for denominator provided the numerator and denominator of each of the fractions be multiplied by a suitable number, namely by the numbers $36 \div 4$, $36 \div 6$ and $36 \div 9$ respectively, that is by 9, 6 and 4 respectively.

Thus
$$\frac{3}{4}=\frac{3\times 9}{4\times 9}=\frac{27}{36},$$
$$\frac{5}{6}=\frac{5\times 6}{6\times 6}=\frac{30}{36},$$
and
$$\frac{8}{9}=\frac{8\times 4}{9\times 4}=\frac{32}{36}.$$

Again, to reduce $\frac{7}{12}$, $\frac{17}{35}$ to their lowest common denominator.

The L. C. M. of 12 and 35 is 12×35.

Then $\frac{7}{12}=\frac{7\times 35}{12\times 35}=\frac{245}{420}$, and $\frac{17}{35}=\frac{17\times 12}{35\times 12}=\frac{204}{420}$.

102. Comparison of fractions.

Of two fractions which have the same denominator the greater is that which has the greater numerator, for the parts being the same the greater fraction is that which has the most of them.

Again, of two fractions which have the same numerator the greater is that which has the smaller denominator, for the number of parts being the same in both, the greater is that in which the parts are the greater, that is in which the unit has been divided into the smaller number of equal parts.

We can therefore see at once which of a number of fractions is the greatest and which is the least, provided the fractions are first of all reduced to equivalent fractions with the same denominator. For this particular purpose it would do equally well to reduce the fractions to equivalent fractions with the same *numerator*, but it is for other purposes much less convenient to reduce fractions to equivalent fractions with the same numerator.

Ex. Which is the greatest and which is the least of the fractions $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$.

As in the preceding article, the fractions are equivalent to $\frac{27}{36}$, $\frac{30}{36}$ and $\frac{32}{36}$ respectively; they are therefore in ascending order of magnitude.

EXAMPLES V. D.

Reduce to equivalent fractions with the lowest common denominator, and arrange in ascending order of magnitude.

1. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$.
2. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{8}{27}$.
3. $\frac{1}{5}$, $\frac{3}{10}$, $\frac{7}{20}$.
4. $\frac{5}{6}$, $\frac{7}{12}$, $\frac{11}{18}$.
5. $\frac{4}{9}$, $\frac{7}{15}$, $\frac{9}{40}$.
6. $\frac{5}{14}$, $\frac{11}{21}$, $\frac{21}{35}$.
7. $\frac{7}{8}$, $\frac{6}{7}$, $\frac{4}{5}$.
8. $\frac{7}{12}$, $\frac{11}{16}$, $\frac{13}{20}$.
9. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{7}$.
10. $\frac{5}{6}$, $\frac{8}{9}$, $\frac{9}{11}$.
11. $\frac{11}{16}$, $\frac{13}{18}$, $\frac{19}{24}$.
12. $\frac{22}{27}$, $\frac{19}{24}$, $\frac{29}{36}$.

13. $\frac{8}{15}, \frac{22}{45}, \frac{31}{60}$.
14. $\frac{5}{7}, \frac{2}{9}, \frac{1}{2}, \frac{9}{14}, \frac{8}{15}$.
15. $\frac{5}{24}, \frac{11}{30}, \frac{21}{40}, \frac{29}{48}$.
16. $\frac{4}{7}, \frac{7}{10}, \frac{7}{12}, \frac{19}{35}$.
17. $\frac{3}{8}, \frac{5}{12}, \frac{7}{18}, \frac{11}{30}$.
18. $\frac{6}{7}, \frac{7}{12}, \frac{17}{18}, \frac{19}{21}, \frac{25}{28}$.

Reduce to equivalent fractions which have the lowest common numerator.

19. $\frac{5}{8}, \frac{10}{17}, \frac{15}{23}$.
20. $\frac{12}{17}, \frac{16}{19}, \frac{18}{23}$.
21. $\frac{15}{37}, \frac{18}{43}, \frac{20}{49}$.

103. Addition of fractions.

The sum of two or more fractions which have *the same denominator* is found at once by adding the numerators.

For, just as 5 *tons* + 4 *tons* + 2 *tons* = 11 *tons*,

so also 5 *sevenths* + 4 *sevenths* + 2 *sevenths* = 11 *sevenths*,

that is, $\frac{5}{7}+\frac{4}{7}+\frac{2}{7}=\frac{11}{7}$.

When fractions have not the same denominator they must first be reduced to *equivalent fractions which have the same denominator:* their sum can then be immediately found by adding the numerators.

Ex. 1. Find the sum of $\frac{5}{6}$, $\frac{7}{12}$ and $\frac{4}{9}$.

The L. C. M. of 6, 12 and 9 is 36.

Reducing to equivalent fractions with 36 as denominator, we have

$$\frac{5}{6}+\frac{7}{12}+\frac{4}{9}=\frac{5\times 6}{6\times 6}+\frac{7\times 3}{12\times 3}+\frac{4\times 4}{9\times 4}$$

$$=\frac{30}{36}+\frac{21}{36}+\frac{16}{36}$$

$$=\frac{30+21+16}{36}=\frac{67}{36}=1\frac{31}{36}.$$

After a little practice some of the steps might be omitted.

Ex. 2. Add together $2\frac{5}{6}+3\frac{7}{12}$.

Adding the whole numbers we have $2+3=5$.

Then $5+\frac{5}{6}+\frac{7}{12}=5+\frac{10}{12}+\frac{7}{12}=5+\frac{17}{12}=5+1\frac{5}{12}=6\frac{5}{12}$.

The result should in all cases be reduced to its lowest terms, and an improper fraction should be expressed as a mixed number.

EXAMPLES V. E.

Find the sum of the following fractions :

1. $\frac{1}{4}$ and $\frac{3}{4}$.
2. $\frac{3}{8}$ and $\frac{1}{8}$.
3. $\frac{5}{12}$ and $\frac{3}{12}$.
4. $\frac{7}{11}$ and $\frac{6}{11}$.
5. $\frac{5}{9}$ and $\frac{8}{9}$.
6. $\frac{11}{16}$ and $\frac{13}{16}$.
7. $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$ and $\frac{13}{16}$.
8. $\frac{5}{24}$, $\frac{7}{24}$, $\frac{11}{24}$, $\frac{13}{24}$, $\frac{17}{24}$ and $\frac{23}{24}$.
9. $\frac{1}{2}$ and $\frac{3}{4}$.
10. $\frac{3}{4}$ and $\frac{1}{8}$.
11. $\frac{2}{3}$ and $\frac{4}{9}$.
12. $\frac{3}{4}$ and $\frac{5}{6}$.
13. $\frac{5}{6}$ and $\frac{7}{8}$.
14. $\frac{3}{14}$ and $\frac{8}{21}$.
15. $\frac{5}{16}$ and $\frac{7}{20}$.
16. $\frac{4}{15}$ and $\frac{7}{18}$.
17. $\frac{5}{21}$ and $\frac{9}{28}$.
18. $2\frac{3}{5}$ and $1\frac{7}{10}$.
19. $5\frac{1}{8}$ and $2\frac{7}{12}$.
20. $7\frac{5}{6}$ and $\frac{4}{9}$.
21. $2\frac{5}{6}$ and $3\frac{8}{9}$.
22. $3\frac{2}{3}$ and $1\frac{1}{4}$.
23. $1\frac{2}{27}$ and $7\frac{19}{24}$.
24. $\frac{3}{5}$, $\frac{4}{9}$, $\frac{7}{15}$ and $\frac{13}{45}$.
25. $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{7}$ and $\frac{2}{9}$.
26. $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{6}$ and $\frac{11}{15}$.
27. $\frac{5}{6}$, $\frac{3}{8}$, $\frac{14}{15}$ and $\frac{5}{18}$.
28. Find $\frac{2}{7}+\frac{3}{14}+\frac{4}{21}+\frac{5}{28}+\frac{6}{35}$.
29. Find $\frac{4}{9}+\frac{5}{18}+\frac{7}{27}+\frac{8}{45}+\frac{11}{60}$.
30. Find $3\frac{3}{5}+\frac{7}{12}+5\frac{3}{16}+7\frac{4}{15}$.
31. Find $3\frac{1}{4}+\frac{3}{10}+5\frac{1}{35}+3\frac{5}{28}+7\frac{20}{63}$.
32. Find $3\frac{1}{6}+5\frac{3}{8}+\frac{11}{24}+3\frac{7}{30}$.
33. Find $\frac{11}{30}+2\frac{7}{45}+11\frac{3}{40}+5\frac{13}{60}$.
34. Find $3\frac{3}{7}+7\frac{7}{10}+\frac{5}{12}+2\frac{19}{35}$.
35. Find $2\frac{13}{357}+5\frac{27}{391}$.
36. Find $3\frac{9}{889}+11\frac{9}{1073}$.
37. Find $3\frac{56}{165}+5\frac{49}{198}+11\frac{105}{242}$.
38. Find $10\frac{121}{312}+11\frac{98}{429}+7\frac{255}{572}$.

104. Subtraction of fractions.

The difference of two fractions which have *the same denominator* is found at once by taking the difference of their numerators.

For, just as *5 pounds − 3 pounds = 2 pounds*,
so also *5 sevenths − 3 sevenths = 2 sevenths*,
that is $\frac{5}{7}-\frac{3}{7}=\frac{2}{7}$.

When the fractions have not the same denominator they must first be reduced to equivalent fractions which have the same denominator: their difference can then be immediately found by taking the difference of their numerators.

Ex. 1. Subtract $\frac{5}{8}$ from $\frac{11}{12}$.

The L. C. M. of 8 and 12 is 24. And $\frac{11}{12}=\frac{22}{24}$, $\frac{5}{8}=\frac{15}{24}$;

$$\therefore \frac{11}{12}-\frac{5}{8}=\frac{22}{24}-\frac{15}{24}=\frac{7}{24}.$$

Ex. 2. Subtract $3\frac{5}{8}$ from $5\frac{11}{12}$.

$$5\tfrac{11}{12}-3\tfrac{5}{8}=5\tfrac{22}{24}-3\tfrac{15}{24}=2\tfrac{7}{24},$$

by subtracting the whole numbers and the fractions separately.

Ex. 3. Subtract $3\frac{11}{12}$ from $5\frac{5}{8}$.

$5\frac{5}{8}-3\frac{11}{12}=5\frac{15}{24}-3\frac{22}{24}=2\frac{15}{24}-\frac{22}{24}$, by taking 3 from 5.

Now we cannot take $\frac{22}{24}$ from $\frac{15}{24}$; we must therefore re-arrange $2\frac{15}{24}$ by changing one of its units into *twenty-fourths*. Thus we proceed

$$1+\tfrac{24}{24}+\tfrac{15}{24}-\tfrac{22}{24}=1+\tfrac{39}{24}-\tfrac{22}{24}=1\tfrac{17}{24}.$$

Ex. 4. Simplify $3\frac{1}{8}-2\frac{5}{6}+8\frac{3}{4}-5\frac{7}{12}-2\frac{7}{24}+\frac{17}{48}$.

Taking the whole numbers first we have $3-2+8-5-2=2$.

The L. C. M. of the denominators is 48. Hence we have

$$2+\frac{6-40+36-28-14+17}{48}=2+\frac{59-82}{48}=1+\frac{48+59-82}{48}=1\tfrac{25}{48}.$$

EXAMPLES V. F.

Simplify:

1. $\frac{7}{12}-\frac{5}{12}$.	**2.** $\frac{5}{8}-\frac{3}{8}$.	**3.** $\frac{9}{16}-\frac{5}{16}$.	**4.** $\frac{17}{32}-\frac{9}{32}$.
5. $\frac{7}{12}-\frac{3}{8}$.	**6.** $\frac{13}{16}-\frac{7}{10}$.	**7.** $\frac{3}{14}-\frac{2}{21}$.	**8.** $\frac{7}{18}-\frac{13}{54}$.
9. $\frac{5}{16}-\frac{7}{24}$.	**10.** $\frac{3}{5}-\frac{4}{7}$.	**11.** $\frac{7}{9}-\frac{3}{16}$.	**12.** $\frac{5}{12}-\frac{2}{7}$.
13. $\frac{8}{15}-\frac{7}{20}$.	**14.** $\frac{8}{65}-\frac{3}{26}$.	**15.** $\frac{9}{68}-\frac{11}{85}$.	**16.** $\frac{6}{19}-\frac{21}{76}$.
17. $3\frac{3}{4}-2\frac{1}{2}$.	**18.** $7\frac{19}{21}-5\frac{11}{42}$.		**19.** $2\frac{35}{36}-2\frac{19}{30}$.
20. $3\frac{5}{6}-2\frac{7}{18}$.	**21.** $7\frac{8}{15}-5\frac{3}{10}$.	**22.** $19\frac{11}{21}-12\frac{5}{14}$.	
23. $7\frac{3}{14}-6\frac{9}{77}$.	**24.** $3\frac{7}{45}-2\frac{3}{25}$.	**25.** $6\frac{7}{8}-2\frac{15}{16}$.	
26. $5\frac{4}{15}-3\frac{7}{20}$.	**27.** $16\frac{7}{30}-5\frac{11}{36}$.	**28.** $9\frac{7}{63}-4\frac{98}{105}$.	

Find the difference between

29. $3\frac{9}{11}$ and $5\frac{6}{17}$.	**30.** $7\frac{8}{21}$ and $8\frac{7}{24}$.	**31.** $6\frac{20}{77}$ and $15\frac{25}{99}$.
32. $7\frac{11}{42}$ and $5\frac{25}{63}$.	**33.** $8\frac{21}{40}$ and $12\frac{17}{48}$.	**34.** $6\frac{21}{52}$ and $12\frac{17}{39}$.
35. $143\frac{9}{65}$ and $127\frac{11}{117}$.	**36.** $85\frac{60}{697}$ and $72\frac{81}{615}$.	

Simplify

37. $2\frac{1}{2}+3\frac{1}{4}-4\frac{1}{8}$.	**38.** $6\frac{11}{12}-2\frac{7}{8}+1\frac{5}{16}$.	**39.** $5\frac{3}{4}-3\frac{7}{16}+\frac{17}{20}-2\frac{7}{10}$.
40. $15\frac{7}{27}-13\frac{5}{12}+16\frac{7}{18}-9\frac{25}{54}$.	**41.** $12\frac{5}{12}-10+7\frac{10}{21}-\frac{4}{9}-5\frac{27}{35}$.	
42. $4\frac{3}{8}-2\frac{7}{18}+2\frac{13}{28}-3\frac{5}{36}$.	**43.** $6\frac{4}{25}-\frac{3}{8}-2\frac{7}{40}+5\frac{2}{125}+\frac{3}{1000}$.	

105. Multiplication by a whole number.

Just as $5 \text{ tons} \times 3 = 15 \text{ tons}$,

so also $5 \text{ sevenths} \times 3 = 15 \text{ sevenths}$,

that is $\frac{5}{7} \times 3 = \frac{15}{7}$.

Again, $\frac{5}{18} \times 3 = \frac{5 \times 3}{18} = \frac{5}{18 \div 3}$ [Art. 99] $= \frac{5}{6}$.

Hence **to multiply a fraction by a whole number, we must multiply the numerator, or** (when possible) **divide the denominator, by that whole number.**

The product should always be reduced to its lowest terms, and an improper fraction should be expressed as a mixed number.

Ex. Multiply $\frac{5}{18}$ by 15.

$$\frac{5}{18} \times 15 = \frac{5 \times 15}{18} = \frac{75}{18} = \frac{25}{6} \text{ [Art. 99]} = 4\tfrac{1}{6}.$$

106. Division by a whole number.

Just as $15 \text{ tons} \div 3 = 5 \text{ tons}$,

so also $15 \text{ sevenths} \div 3 = 5 \text{ sevenths}$,

that is $\frac{15}{7} \div 3 = \frac{5}{7}$.

Again, to divide $\frac{5}{6}$ by 3.

Here 5 is not a multiple of 3. But, since $\frac{5}{6} = \frac{5 \times 3}{6 \times 3}$ [Art. 99],

$$\frac{5}{6} \div 3 = \frac{5 \times 3}{6 \times 3} \div 3 = \frac{5 \times 3 \div 3}{6 \times 3} = \frac{5}{6 \times 3} = \frac{5}{18}.$$

[We can see at once, without reference to Art. 99, that a fraction is divided by a whole number by multiplying its denominator by the whole number. For example, in $\frac{5}{6 \times 3}$ there are the same number of parts as in $\frac{5}{6}$, namely five, but the unit in the former case is divided into 3 times as many parts as in the latter, and therefore each of the parts in $\frac{5}{6 \times 3}$ is one-third of each of the parts in $\frac{5}{6}$.]

Hence **to divide a fraction by a whole number we must multiply the denominator, or** (when possible) **divide the numerator, by that whole number.**

Ex. 1. Divide $3\frac{1}{5}$ by 7.

$$3\tfrac{1}{5} \div 7 = \frac{16}{5} \div 7 = \frac{16}{5 \times 7} = \frac{16}{35}.$$

Ex. 2. Divide $215\frac{3}{7}$ by 9.

We first divide 215 by 9, the quotient being 23 and the remainder 8; we have then to divide $8\frac{3}{7}$ by 9. Thus

$$215\tfrac{3}{7}\div 9 = 23 + 8\tfrac{3}{7}\div 9 = 23 + \tfrac{59}{7}\div 9 = 23 + \tfrac{56}{63} = 23\tfrac{59}{63}.$$

107. We have seen in Art. 95 that the fraction $\frac{3}{5}$, for example, expresses the result of dividing 3 units into 5 equal parts. We are thus able to express the **complete quotient** in any case of the division of one whole number by another.

For example, $23\div 5 = \frac{23}{5} = 4\frac{3}{5}$.

It should however be noticed that this will only apply in those cases when the units under consideration are capable of being divided into any number of equal parts. We could not, for example, say that 35 boys could be divided into 4 equal classes each containing $8\frac{3}{4}$ boys.

From the above it will be seen that *a fraction may be considered as the quotient obtained by dividing the numerator by the denominator.*

EXAMPLES V. G.

Multiply, and reduce to their simplest forms,

1.	$\frac{2}{7}$ by 2.	**2.**	$\frac{5}{19}$ by 3.	**3.**	$\frac{4}{21}$ by 5.	**4.**	$\frac{3}{17}$ by 4.
5.	$\frac{3}{7}$ by 4.	**6.**	$\frac{7}{12}$ by 5.	**7.**	$\frac{5}{9}$ by 4.	**8.**	$\frac{7}{11}$ by 6.
9.	$\frac{7}{9}$ by 6.	**10.**	$\frac{4}{15}$ by 10.	**11.**	$\frac{5}{16}$ by 6.	**12.**	$\frac{7}{12}$ by 8.
13.	$2\frac{3}{4}$ by 6.	**14.**	$5\frac{3}{8}$ by 10.	**15.**	$7\frac{8}{15}$ by 25.	**16.**	$9\frac{41}{50}$ by 15.

Simplify

17.	$\frac{4}{5}\div 2$.	**18.**	$\frac{8}{15}\div 4$.	**19.**	$\frac{6}{25}\div 3$.	**20.**	$\frac{24}{25}\div 8$.
21.	$\frac{4}{5}\div 3$.	**22.**	$\frac{8}{15}\div 5$.	**23.**	$\frac{6}{25}\div 5$.	**24.**	$\frac{24}{25}\div 7$.
25.	$\frac{15}{16}\div 6$.	**26.**	$\frac{24}{25}\div 16$.	**27.**	$\frac{36}{49}\div 30$.	**28.**	$\frac{25}{27}\div 15$.
29.	$2\frac{1}{5}\div 11$.	**30.**	$7\frac{1}{5}\div 6$.	**31.**	$9\frac{1}{7}\div 8$.	**32.**	$12\frac{2}{9}\div 11$.
33.	$5\frac{3}{8}\div 5$.	**34.**	$7\frac{4}{9}\div 6$.	**35.**	$8\frac{3}{14}\div 15$.	**36.**	$7\frac{11}{16}\div 15$.
37.	$85\frac{2}{7}\div 9$.	**38.**	$214\frac{1}{12}\div 7$.	**39.**	$174\frac{15}{16}\div 18$.	**40.**	$254\frac{7}{15}\div 25$.

108. Multiplication by a fraction. We have already defined multiplication to be the taking one number as many times as there are units in another. Thus, to multiply 5 by 4, we take as many fives as there are units in four. Now 4 is $1+1+1+1$, and 5×4 is $5+5+5+5$.

Thus *to multiply one number by a second is to do to the first what is done to the unit to obtain the second.*

This modified form of the definition of multiplication applies equally well to multiplication by a fraction.

For example, to multiply $\frac{5}{7}$ by $\frac{3}{4}$, we must do to $\frac{5}{7}$ what is done to the unit to obtain $\frac{3}{4}$, that is we must divide $\frac{5}{7}$ into 4 equal parts and take 3 of those parts. Each of the 4 parts into which $\frac{5}{7}$ is divided will be $\frac{5}{7\times4}$, and by taking 3 of these parts we get $\frac{5\times3}{7\times4}$.

Thus $$\frac{5}{7}\times\frac{3}{4}=\frac{5\times3}{7\times4}.$$

Hence **the product of any two fractions is another fraction whose numerator is the product of their numerators and whose denominator is the product of their denominators.**

The continued product of any number of fractions is obtained by continued application of the above rule.

Thus, to find the continued product of $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{8}{9}$.

$$\frac{2}{3}\times\frac{4}{5}\times\frac{8}{9}=\frac{2\times4}{3\times5}\times\frac{8}{9}=\frac{2\times4\times8}{3\times5\times9}=\frac{64}{135}.$$

Hence, **the product of any number of fractions is another fraction whose numerator is the product of their numerators and whose denominator is the product of their denominators.**

It should be noticed that *the product of one fraction by a second is equal to the product of the second by the first.*

For, $$\frac{5}{7}\times\frac{3}{4}=\frac{5\times3}{7\times4}=\frac{3\times5}{4\times7}\ [\text{Art. 43}]=\frac{3}{4}\times\frac{5}{7}.$$

Ex. 1. Multiply $2\frac{1}{4}$ by $3\frac{5}{8}$.

The mixed numbers must first be reduced to improper fractions.

Thus $$2\tfrac{1}{4}\times3\tfrac{5}{8}=\frac{9}{4}\times\frac{29}{8}=\frac{9\times29}{4\times8}=\frac{261}{32}=8\tfrac{5}{32}.$$

Ex. 2. Multiply $\frac{6}{35}$ by $\frac{7}{27}$.

$$\frac{6}{35}\times\frac{7}{27}=\frac{\overset{2}{\cancel{6}}\times\overset{1}{\cancel{7}}}{\underset{5}{\cancel{35}}\times\underset{9}{\cancel{27}}}=\frac{2}{45}.$$

Instead of actually performing the multiplications indicated, and then reducing the resulting fraction to its lowest terms, it is best first to divide both numerator and denominator by any common factor which may be obvious. In the above, 3 is a common factor of 6 and 27. We therefore divide 6 and 27 by 3, crossing out these numbers and putting the quotients, namely 2 and 9 respectively in their places. Again 7 is a common factor of 7 and 37, we therefore divide these numbers by 7, and put the quotients 1 and 5 in their places.

The above process is called **cancelling** common factors.

Ex. 3. Simplify $\frac{2}{3} \times \frac{3}{4} \times \frac{2}{5}$.

$$\frac{2}{3} \times \frac{3}{4} \times \frac{2}{5} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}} \times \underset{2}{\cancel{4}} \times 5} = \frac{1}{5}.$$

EXAMPLES V. H.

Simplify

1. $\frac{1}{3} \times \frac{1}{2}$.
2. $\frac{2}{3} \times \frac{5}{7}$.
3. $\frac{3}{5} \times \frac{3}{8}$.
4. $\frac{5}{7} \times \frac{4}{9}$.
5. $1\frac{1}{2} \times 2\frac{1}{4}$.
6. $2\frac{1}{4} \times 3\frac{2}{5}$.
7. $4\frac{3}{8} \times 2\frac{3}{4}$.
8. $2\frac{5}{12} \times 6\frac{5}{9}$.
9. $\frac{2}{3} \times \frac{3}{4}$.
10. $\frac{3}{5} \times \frac{20}{33}$.
11. $\frac{5}{6} \times \frac{12}{25}$.
12. $\frac{7}{11} \times \frac{22}{35}$.
13. $\frac{5}{9} \times \frac{27}{40}$.
14. $\frac{15}{16} \times \frac{8}{45}$.
15. $\frac{23}{40} \times \frac{40}{69}$.
16. $\frac{3}{7} \times \frac{21}{60}$.
17. $1\frac{5}{8} \times 2\frac{6}{13}$.
18. $3\frac{2}{5} \times 1\frac{3}{17}$.
19. $2\frac{3}{8} \times 1\frac{1}{7}$.
20. $5\frac{2}{11} \times 3\frac{9}{19}$.
21. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.
22. $\frac{5}{8} \times \frac{2}{5} \times \frac{4}{7}$.
23. $\frac{3}{7} \times \frac{8}{11} \times \frac{7}{24}$.
24. $\frac{6}{7} \times 1\frac{3}{4} \times 5\frac{5}{6}$.
25. $2\frac{1}{4} \times 3\frac{1}{5} \times 4\frac{1}{6}$.
26. $5\frac{1}{5} \times 7\frac{1}{7} \times \frac{7}{10}$.
27. $\frac{3}{17} \times 5\frac{3}{13} \times 1\frac{3}{12}$.
28. $2\frac{1}{10} \times 3\frac{3}{7} \times 6\frac{1}{9}$.
29. $1\frac{7}{15} \times 2\frac{2}{9} \times 1\frac{7}{11}$.
30. $6\frac{9}{17} \times \frac{34}{65} \times 1\frac{28}{37}$.
31. $\frac{65}{114} \times 2\frac{11}{26} \times 5\frac{9}{11} \times \frac{19}{160}$.
32. $1\frac{1}{33} \times 1\frac{23}{40} \times \frac{20}{57} \times 1\frac{90}{119}$.
33. $\frac{39}{44} \times 1\frac{2}{5} \times 5\frac{7}{13} \times 2\frac{13}{21}$.

109. A fraction of a fraction is called a **compound fraction.**

Thus $\frac{2}{3}$ of $\frac{5}{7}$ is a compound fraction.

To take $\frac{2}{3}$ of $\frac{5}{7}$ we must divide $\frac{5}{7}$ into 3 equal parts and take 2 of those parts.

Hence $\frac{2}{3}$ of $\frac{5}{7}$ is the same as $\frac{5}{7} \times \frac{2}{3}$.

Ex. 1. Multiply $\frac{3}{7}$ of $2\frac{1}{5}$ by $\frac{5}{9}$ of $1\frac{3}{4}$.

$\frac{3}{7}$ of $2\frac{1}{5} = \frac{3}{7} \times \frac{11}{5}$, and $\frac{5}{9}$ of $1\frac{3}{4} = \frac{5}{9} \times \frac{7}{4}$;

hence the required product $= \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{7}}} \times \frac{11}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{7}}}{4} = \frac{11}{12}$.

EXAMPLES V. I.

Simplify

1. $\frac{2}{3}$ of $\frac{3}{4}$.
2. $\frac{4}{5}$ of $\frac{5}{6}$.
3. $\frac{7}{8}$ of $\frac{8}{9}$.
4. $\frac{5}{6}$ of $\frac{6}{25}$.
5. $1\frac{3}{4}$ of $2\frac{2}{7}$.
6. $3\frac{1}{3}$ of $6\frac{1}{4}$.
7. $7\frac{7}{8}$ of $2\frac{2}{9}$.
8. $3\frac{1}{5}$ of $3\frac{1}{8}$.
9. $\frac{5}{6}$ of $\frac{2}{7}$ of $\frac{7}{10}$.
10. $2\frac{1}{2}$ of $3\frac{1}{4}$ of $\frac{4}{5}$.
11. $6\frac{2}{7}$ of $2\frac{3}{11}$ of $1\frac{2}{5}$.
12. $\frac{2}{3}$ of $\frac{5}{9} \times \frac{9}{25}$ of $2\frac{1}{4}$.
13. $\frac{7}{9}$ of $2\frac{2}{5} \times 1\frac{3}{13}$ of $2\frac{1}{8}$.
14. $1\frac{1}{7}$ of $3\frac{1}{5} \times \frac{15}{16} \times 5\frac{1}{4}$.
15. $1\frac{4}{5}$ of $3\frac{3}{4} \times 5\frac{1}{3}$ of $7\frac{1}{9}$.

110. Two fractions which are such that the numerator of each is the denominator of the other are called **reciprocal** fractions.

Thus the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$; also the reciprocal of $\frac{1}{5}$ is $\frac{5}{1}$, that is 5.

It follows from Art. 108 that the product of two reciprocal fractions is 1. For example $\frac{2}{3}\times\frac{3}{2}=1$.

111. Division by a fraction. We have seen [Art. 51] that in division the dividend is equal to the product of the divisor and the quotient. To divide one fraction by another we therefore proceed as follows.

For example, to divide $\frac{2}{3}$ by $\frac{5}{7}$.

The required quotient $\times\frac{5}{7}=\frac{2}{3}$.

$\therefore$ the required quotient $\times\frac{5}{7}\times\frac{7}{5}=\frac{2}{3}\times\frac{7}{5}$.

But $\frac{5}{7}\times\frac{7}{5}=1$. [Art. 108.]

$\therefore$ the required quotient $=\frac{2}{3}\times\frac{7}{5}$,

thus to divide by $\frac{5}{7}$ we multiply by the reciprocal fraction, namely $\frac{7}{5}$. The same reasoning will apply to all cases.

Hence, **to divide by a fraction we must multiply by the reciprocal fraction.**

Ex. 1. Divide $\frac{5}{6}$ by $\frac{15}{32}$.

$$\frac{5}{6}\div\frac{15}{32}=\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}}\times\frac{\overset{16}{\cancel{32}}}{\underset{3}{\cancel{15}}}=\frac{16}{9}=1\tfrac{7}{9}.$$

Ex. 2. Divide $2\frac{1}{7}$ by $1\frac{13}{14}$.

The mixed numbers must first be expressed as improper fractions. Then

$$2\tfrac{1}{7}\div1\tfrac{13}{14}=\frac{15}{7}\div\frac{27}{14}=\frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{7}}}\times\frac{\overset{2}{\cancel{14}}}{\underset{9}{\cancel{27}}}=\frac{10}{9}=1\tfrac{1}{9}.$$

EXAMPLES V. J.

Simplify

1. $\frac{3}{4}\div\frac{2}{3}$.	2. $\frac{5}{6}\div\frac{2}{3}$.	3. $\frac{2}{9}\div\frac{3}{4}$.	4. $\frac{1}{5}\div\frac{3}{7}$.
5. $\frac{1}{4}\div\frac{3}{4}$.	6. $\frac{7}{12}\div\frac{11}{12}$.	7. $\frac{3}{8}\div\frac{5}{8}$.	8. $\frac{7}{16}\div\frac{9}{16}$.
9. $\frac{1}{2}\div\frac{1}{4}$.	10. $\frac{1}{4}\div\frac{1}{2}$.	11. $\frac{2}{3}\div\frac{1}{6}$.	12. $\frac{1}{6}\div\frac{2}{3}$.
13. $2\frac{3}{4}\div2\frac{1}{4}$.	14. $5\frac{1}{10}\div1\frac{7}{10}$.	15. $6\frac{2}{9}\div1\frac{5}{9}$.	16. $4\frac{11}{16}\div1\frac{9}{16}$.
17. $6\frac{3}{5}\div11$.	18. $6\frac{3}{7}\div9$.	19. $6\frac{9}{11}\div25$.	20. $2\frac{11}{12}\div7$.
21. $\frac{25}{81}\div\frac{35}{36}$.	22. $\frac{13}{64}\div\frac{39}{68}$.	23. $\frac{64}{125}\div\frac{48}{55}$.	24. $\frac{121}{144}\div\frac{143}{180}$.
25. $2\frac{6}{25}\div1\frac{1}{35}$.	26. $4\frac{4}{15}\div1\frac{29}{35}$.	27. $1\frac{61}{64}\div1\frac{7}{48}$.	28. $1\frac{23}{121}\div1\frac{37}{143}$.
29. $1\frac{143}{145}\div2\frac{59}{68}$.	30. $5\frac{7}{8}\div2\frac{13}{32}$.	31. $11\frac{22}{117}\div12\frac{89}{351}$.	32. $21\frac{35}{187}\div5\frac{11}{221}$.

112. A **complex fraction** is a fraction whose numerator, or denominator, or both, are fractional.

Thus $\frac{\frac{3}{5}}{7}$, $\frac{\frac{2}{3}}{\frac{5}{7}}$ and $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}+\frac{1}{2}}$ are complex fractions.

Complex fractions are simplified by making use of the modified definition of a fraction given in Art. 107.

Ex. 1. Simplify $\frac{\frac{2}{3}}{\frac{5}{7}}$.

$$\frac{\frac{2}{3}}{\frac{5}{7}}=\frac{2}{3}\div\frac{5}{7}=\frac{2}{3}\times\frac{7}{5}\quad\text{[Art. 111]}\quad=\frac{2\times7}{3\times5}=\frac{14}{15}.$$

Ex. 2. Simplify $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}+\frac{1}{2}}$.

$$\text{The fraction}=\frac{\frac{3+2}{6}}{\frac{3+2}{4}}=\frac{\frac{5}{6}}{\frac{5}{4}}$$

$$=\frac{5}{6}\div\frac{5}{4}=\frac{\cancel{5}}{\cancel{6}_{3}}\times\frac{\overset{2}{\cancel{4}}}{\cancel{5}}=\frac{2}{3}.$$

A complex fraction is unaltered in value by multiplying its numerator and denominator by the same number.

For example $\frac{\frac{5}{7}}{\frac{4}{9}}=\frac{\frac{5}{7}\times11}{\frac{4}{9}\times11}$. For $\frac{\frac{5}{7}}{\frac{4}{9}}=\frac{5}{7}\div\frac{4}{9}=\frac{5}{7}\times\frac{9}{4}$.

Also $$\frac{\frac{5}{7}\times11}{\frac{4}{9}\times11}=\frac{5}{7}\times11\div\frac{4}{9}\div11=\frac{5}{7}\times11\times\frac{9}{4}\times\frac{1}{11}$$

$$=\frac{5}{7}\times\frac{9}{4}\times11\times\frac{1}{11}\text{ [Art. 108]}=\frac{5}{7}\times\frac{9}{4}.$$

Ex. 1. Simplify, $\frac{\frac{3}{4}-\frac{2}{3}}{\frac{7}{8}-\frac{5}{6}}$.

Multiply the numerator and denominator by 24, the L. C. M. of 3, 4, 6, 8. Then we have

$$\frac{\frac{3}{4}-\frac{2}{3}}{\frac{7}{8}-\frac{5}{6}}=\frac{18-16}{21-20}=\frac{2}{1}=2.$$

Ex. 2. Simplify $\dfrac{3}{5+\dfrac{2}{7-\dfrac{3}{4+\frac{1}{2}}}}$.

$$\frac{3}{5+\dfrac{2}{7-\dfrac{3}{4+\frac{1}{2}}}} = \frac{3}{5+\dfrac{2}{7-\frac{6}{9}}} = \frac{3}{5+\frac{18}{57}} = \frac{171}{303} = \frac{57}{101}.$$

First multiply the numerator and denominator of the lowest complex fraction, namely $\dfrac{3}{4+\frac{1}{2}}$ by 2, and we get $\frac{6}{9}$. Next multiply the numerator and denominator of the fraction $\dfrac{2}{7-\frac{6}{9}}$ by 9, and we get $\frac{18}{57}$. Then multiply the numerator and denominator of $\dfrac{3}{5+\frac{18}{57}}$ by 57, and we get $\frac{171}{303}$, which is then reduced to its lowest terms.

A fraction of this type is called a **continued fraction.**

EXAMPLES V. K.

Simplify

1. $\dfrac{5}{\frac{5}{6}}$.
2. $\dfrac{7}{\frac{3}{4}}$.
3. $\dfrac{\frac{3}{4}}{6}$.
4. $\dfrac{1\frac{2}{3}}{10}$.
5. $\dfrac{\frac{2}{3}}{\frac{4}{9}}$.
6. $\dfrac{\frac{4}{9}}{\frac{2}{3}}$.
7. $\dfrac{1\frac{1}{12}}{\frac{5}{6}}$.
8. $\dfrac{\frac{3}{7}}{\frac{5}{14}}$.
9. $\dfrac{2\frac{1}{5}}{4\frac{2}{5}}$.
10. $\dfrac{5\frac{1}{4}}{4\frac{1}{5}}$.
11. $\dfrac{6\frac{2}{9}}{1\frac{5}{9}}$.
12. $\dfrac{7\frac{3}{7}}{11\frac{5}{9}}$.
13. $\dfrac{\frac{2}{3}+\frac{1}{6}}{\frac{3}{4}+\frac{1}{8}}$.
14. $\dfrac{\frac{11}{12}-\frac{3}{8}}{\frac{11}{12}+\frac{3}{8}}$.
15. $\dfrac{\frac{8}{9}+\frac{3}{11}}{\frac{2}{9}+\frac{7}{11}}$.
16. $\dfrac{\frac{7}{12}-\frac{5}{13}}{\frac{8}{13}-\frac{5}{12}}$.
17. $\dfrac{3\frac{5}{8}-1\frac{7}{8}}{2\frac{3}{4}-\frac{5}{8}}$.
18. $\dfrac{7\frac{3}{11}-4\frac{6}{11}}{5\frac{7}{12}-2\frac{8}{9}}$.
19. $\dfrac{15\frac{3}{14}-10\frac{10}{21}}{18\frac{5}{7}-15\frac{9}{56}}$.
20. $\dfrac{2\frac{3}{4}-1\frac{5}{8}}{\frac{3}{4}\text{ of }\frac{8}{9}}$.
21. $\dfrac{3\frac{1}{2}\text{ of }4\frac{1}{3}}{2\frac{1}{3}\text{ of }6\frac{1}{2}}$.
22. $\dfrac{\frac{3}{5}\div 2\frac{1}{2}}{1\frac{1}{15}\div 2\frac{2}{5}}$.
23. $\dfrac{3\frac{1}{4}+4\frac{1}{3}}{6\frac{1}{2}+1\frac{1}{12}}$.
24. $\dfrac{\frac{1}{2}+\frac{1}{4}+\frac{1}{6}}{\frac{8}{25}+\frac{9}{100}+\frac{7}{50}}$.
25. $\dfrac{\frac{14}{15}+\frac{13}{14}-\frac{9}{10}-\frac{8}{9}}{\frac{1}{10}+\frac{1}{9}-\frac{1}{15}-\frac{1}{14}}$.
26. $\dfrac{22}{1+\dfrac{5}{8+\frac{3}{4}}}$.
27. $\dfrac{35}{5-\dfrac{4}{7-\frac{3}{5}}}$.

28. $\dfrac{5\frac{1}{2}-3\frac{3}{4}}{\frac{2}{5}+\frac{19}{3\frac{1}{2}+2\frac{1}{3}}}$

29. $\dfrac{1}{2-\dfrac{1}{3+\dfrac{1}{2-\frac{1}{3}}}}$.

30. $\dfrac{2}{1+\dfrac{3}{2+\dfrac{4}{3+\frac{5}{4}}}}$.

31. $\dfrac{7}{5+\dfrac{3}{1-\dfrac{1}{3-\frac{5}{7}}}}$.

32. $\dfrac{7\frac{1}{2}}{3+\dfrac{2}{5-\dfrac{2}{1-\frac{1}{7}}}}$.

33. $\dfrac{3\frac{4}{11}}{3\frac{1}{2}-\dfrac{2}{3-\dfrac{3}{4+\frac{1}{2}}}}$.

BRACKETS.

113. It is now necessary to make a digression to explain the nature and use of brackets.

Several numbers are included in a bracket when they are to be *treated as a whole,* some sign being put outside the bracket to shew *how* this whole is to be treated.

Thus $17+(4+8)$ denotes that 8 is to be added to 4 and that this result is then to be added to 17.

Again $117-6\times(4+8)$ denotes that 6 is to be multiplied by the sum of 4 and 8, and that the product is to be subtracted from 117. Also $36\div(24\div4)$ denotes that 24 is to be divided by 4, and that 36 is to be divided by this result.

When two brackets come together without any sign between them the sign of multiplication must be understood.

Thus $(5+7)(9-3)$ is put for $(5+7)\times(9-3)$.

Also $6(5+7)$ is put for $6\times(5+7)$.

Instead of enclosing numbers in brackets a line is sometimes drawn over them.

Thus $17+\overline{4+8}$ may be used instead of $17+(4+8)$.

Sometimes brackets are put within brackets: the different brackets must then be of different shapes to avoid confusion. The different shapes generally used are ( ), { } and [ ].

For example $150-3\{13-(9-3)\}$.

In order to simplify when there is more than one bracket, the *innermost* bracket must be cleared away first.

Thus $150-3\{13-(9-3)\}=150-3\{13-6\}=150-3\times7$
$=150-21=129.$

Numbers enclosed in brackets must be reduced to their simplest form before performing any of the operations indicated by signs outside the bracket.

EXAMPLES V. L.

Simplify

1. $8+(7+3)$.
2. $15-(9+3)$.
3. $27-(11-4)$.
4. $15-2(8-5)$.
5. $3(7-2)-4(8-6)$.
6. $7(3+9)-5(12-4)$.
7. $(9+7)(4+11)$.
8. $(13-5)(15-7)$.
9. $(23-12)(18-15)$.
10. $3(13-4)(15-7)$.
11. $8(17-11)(13-6)$.
12. $5(11+5)(11-5)$.
13. $25-(17-2)+(18-12)$.
14. $27-3(20-11)+4(8-3)$.
15. $18+\{17-2-(15-4)\}$.
16. $23-\{41-(27-6)\}$.
17. $63-2\{83-15-4(17-5)\}$.
18. $75-5\{13+19-4(15-10)\}$.
19. $2\frac{1}{2}-(1\frac{3}{4}-\frac{5}{6})$.
20. $5\frac{3}{4}-2\frac{7}{8}-2(4\frac{5}{6}-3\frac{7}{8})$.
21. $\frac{2}{3}$ of $(\frac{1}{3}+\frac{5}{6})$.
22. $\frac{4}{21}$ of $(5\frac{1}{4}-2\frac{1}{7})$.

114. We now proceed to give examples of a more complicated nature; it will be well, however, for the student to consider carefully the following cases in which mistakes are frequently made in the meaning of the signs employed.

I. Operations of multiplication and division are to be performed in order from left to right, and each sign is a direction to multiply or divide what precedes the sign by **the number** that follows **next after it.**

For example, $36\times6\div3=216\div3=72$,

$36\div6\div3=6\div3=2$,

and $36\div6\times3=6\times3=18$.

So also, $\frac{2}{3}\times\frac{3}{5}\div\frac{8}{15}=\frac{2\times3}{3\times5}\div\frac{8}{15}=\frac{2\times3}{3\times5}\times\frac{15}{8}=\frac{3}{4}$,

$\frac{2}{3}\div\frac{5}{6}\div\frac{4}{5}=\frac{2}{3}\times\frac{6}{5}\div\frac{4}{5}=\frac{2}{3}\times\frac{6}{5}\times\frac{5}{4}=1$,

and $\frac{2}{3}\div\frac{5}{6}\times\frac{4}{5}=\frac{2}{3}\times\frac{6}{5}\times\frac{4}{5}=\frac{16}{25}$.

II. Quantities connected by the sign '**of**' must be considered as a **single quantity,** just as if they were enclosed in brackets.

Thus $\frac{14}{15} \div \frac{2}{5}$ of $\frac{7}{8} = \frac{14}{15} \div \frac{2 \times 7}{5 \times 8} = \frac{14}{15} \times \frac{5 \times 8}{2 \times 7} = \frac{8}{3}$.

Again $\frac{3}{4}$ of $\frac{5}{8} \div \frac{3}{8}$ of $\frac{5}{6} = \frac{3 \times 5}{4 \times 8} \div \frac{3 \times 5}{8 \times 6} = \frac{3 \times 5}{4 \times 8} \times \frac{8 \times 6}{3 \times 5} = \frac{3}{2}$.

III. All multiplications and divisions must be performed, also complex and compound fractions must be reduced to simple fractions, before performing any operations of addition or subtraction.

Thus $\frac{2}{3} + \frac{3}{4}$ of $\frac{5}{6} + \frac{3}{4} = \frac{2}{3} + \frac{3 \times 5}{4 \times 6} + \frac{3}{4}$

$= \frac{2}{3} + \frac{5}{8} + \frac{3}{4} = \frac{16 + 15 + 18}{24} = \frac{49}{24} = 2\frac{1}{24}$.

It is a very common mistake to work a question of this kind as if it meant $\left(\frac{2}{3} + \frac{3}{4}\right)$ of $\left(\frac{5}{6} + \frac{3}{4}\right)$.

EXAMPLES V. M.

Simplify

1. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.
2. $\frac{2}{3} \div \frac{3}{4} \times \frac{4}{5}$.
3. $\frac{2}{3} \div \frac{3}{4} \div \frac{4}{5}$.
4. $\frac{5}{8} \div \frac{7}{12} \div \frac{5}{7}$.
5. $1\frac{1}{8} \div \frac{1}{2} \div 2\frac{1}{4}$.
6. $1\frac{1}{8} \div \frac{1}{2} \times 2\frac{1}{4}$.
7. $\frac{2}{3} \div \frac{5}{6} \times \frac{6}{35} \div \frac{2}{7}$.
8. $\frac{5}{12} \div \frac{3}{7} \times \frac{9}{14} \div 3\frac{1}{8}$.
9. $6\frac{2}{3} \times 4\frac{3}{8} \div 5\frac{1}{4} \div 2\frac{2}{3}$.
10. $\frac{2}{3} \div \frac{5}{6}$ of $\frac{3}{10}$.
11. $\frac{5}{7}$ of $\frac{3}{4} \div \frac{9}{14}$.
12. $\frac{3}{7}$ of $3\frac{1}{3} \div \frac{5}{7}$.
13. $\frac{5}{12}$ of $3\frac{3}{7} \div 2\frac{6}{7}$.
14. $2\frac{1}{5} \div 1\frac{3}{8}$ of $3\frac{1}{5}$.
15. $284\frac{3}{4}$ of $\frac{16}{19} \div 17\frac{12}{19}$.
16. $\frac{4}{15} \div 1\frac{1}{7}$ of $4\frac{1}{5}$ of $1\frac{7}{9}$.
17. $\frac{2}{5}$ of $\frac{2}{8} \div 13\frac{1}{2}$ of $\frac{4}{9}$.
18. $4\frac{3}{4}$ of $1\frac{3}{8} \div 4\frac{1}{8}$ of $2\frac{1}{9}$.
19. $2\frac{1}{5} \div \frac{5}{6}$ of $\frac{2}{2}\frac{4}{5} \div 5\frac{1}{2}$.
20. $1\frac{1}{4} \div 3\frac{1}{4}$ of $\frac{3}{5} \times 6\frac{1}{2}$.
21. $2\frac{1}{3} \div 1\frac{5}{7}$ of $1\frac{1}{15} \div 1\frac{3}{32}$.
22. $\frac{1}{2} + \frac{1}{3}$ of $\frac{3}{4} - \frac{3}{8}$.
23. $\frac{1}{2}$ of $\frac{1}{3} + \frac{3}{4} - \frac{3}{8}$.
24. $\frac{1}{3} - \frac{1}{4}$ of $\frac{1}{5} - \frac{1}{6}$.
25. $\frac{1}{3}$ of $\frac{1}{4} - \frac{1}{5}$ of $\frac{1}{6}$.
26. $2\frac{1}{2} + 1\frac{1}{4}$ of $2\frac{1}{5} - 3\frac{7}{8}$.
27. $2\frac{1}{2}$ of $1\frac{1}{4} + 2\frac{1}{5}$ of $3\frac{7}{8}$.
28. $3\frac{1}{5} - \frac{3}{10}$ of $2\frac{1}{4} - 1\frac{1}{8}$.
29. $\frac{5}{9}$ of $3\frac{3}{5} - 2\frac{1}{5}$ of $\frac{5}{16}$ of $2\frac{3}{11}$.

30. $1\frac{1}{2}$ of $\frac{7}{9} \div 3\frac{1}{9} - \frac{5}{16}$.

31. $\frac{5}{4}$ of $2\frac{2}{3} - 4\frac{1}{3}$ of $5\frac{5}{6} \div 2\frac{1}{3}$ of $3\frac{1}{4}$.

32. $\frac{5}{8} + 1\frac{1}{6}$ of $2\frac{1}{7} - \frac{1}{2} \div \frac{4}{25}$.

33. $3\frac{1}{2}$ of $1\frac{1}{14} + 7\frac{1}{2} - 1\frac{1}{4} \div \frac{2}{3}$ of $\frac{5}{21}$.

34. $\left(\frac{5}{7} + \frac{11}{2\frac{1}{5}} \text{ of } 2\frac{3}{5} - \frac{27}{35}\right) \div 3\frac{1}{24}\frac{7}{15}$.

35. $\dfrac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{13}{36}}$.

36. $\dfrac{\frac{1}{2} - \frac{3}{10}}{\frac{2}{3} + 1\frac{13}{21}} \div \dfrac{\frac{3}{2} \text{ of } \frac{4}{15}}{2 \div \frac{7}{8}}$.

37. $\dfrac{\frac{1}{3} + \frac{1}{7} + \frac{5}{63}}{\frac{1}{2} - \frac{1}{18}} \div \dfrac{\frac{5}{14} \text{ of } 2\frac{1}{4}}{\frac{2}{7} \text{ of } 4\frac{1}{2}}$.

38. $\dfrac{\frac{1}{27} + \frac{1}{5} \text{ of } \frac{2}{5} \text{ of } \frac{4}{5}}{\frac{4}{25} - \frac{1}{3} \text{ of } \frac{2}{5} + \frac{1}{9}} \div \left(\frac{7}{9} - \frac{8}{15}\right)$.

39. $\dfrac{2\frac{1}{2} - \frac{1}{3} \text{ of } 1\frac{1}{5} + \frac{4}{5}}{\frac{3}{5} + \frac{1}{3} \text{ of } \frac{5}{8} \div \frac{4}{5} \div 2\frac{1}{12}}$.

115. **The value of a given fraction of a given concrete quantity** is found at once by definition.

For example, to find $\frac{5}{18}$ of 3 lbs. 4 oz. Troy, we must divide 3 lbs. 4 oz. into 18 equal parts and then take 5 of those parts, that is we must divide by 18 and then multiply by 5. We may however [Art. 95] first multiply by 5 and then divide by 18. Thus

	lbs.	oz.
	3 .	4
		5
18 | 2 |	16 .	8
9 |	8 .	4

11 oz. 2 dwts. $5\frac{1}{3}$ grs.

EXAMPLES V. N.

1. Find $\frac{4}{5}$, $\frac{8}{9}$, $\frac{20}{21}$ and $6\frac{3}{7}$ of £3. 18*s.* 9*d.*

2. Find $\frac{3}{11}$, $\frac{5}{13}$, $4\frac{8}{17}$ and $6\frac{9}{13}$ of £2. 10*s.* $7\frac{3}{4}d.$

3. Add together $\frac{2}{3}$ of 2*s.* 6*d.*, $2\frac{4}{5}$ of 1*s.* 8*d.* and $1\frac{1}{11}$ of 6*s.* 5*d.*

4. Add together $1\frac{1}{2}$ of 4*s.* 9*d.*, $3\frac{5}{8}$ of £1. 6*s.* 4*d.*, and $3\frac{7}{11}$ of £5. 16*s.* 5*d.*

5. Find $11\frac{2}{3}$ of $4\frac{1}{2}d. + 1\frac{1}{11}$ of £1. 10*s.* $0\frac{1}{4}d. - 2\frac{1}{2}$ of $\frac{1}{3\frac{1}{4}}$ of £1. 6*s.*

6. Find $\frac{3}{4}$ of £5. 7*s.* $11d. + 1\frac{1}{3}$ of £20. 5*s.* $9\frac{3}{4}d. - 1\frac{7}{9}$ of £11. 12*s.* 6*d.*

7. Find $\frac{1}{16}$ of £44. 12*s.* $4d. - \frac{3}{7}$ of £1. 12*s.* $8d. + 9s.$ $9\frac{1}{4}d. \div 3\frac{1}{2}$.

8. Find $3\frac{3}{8}$ of £1. 2*s.* $6d. - \frac{2}{5}$ of 17*s.* $6d. - \frac{1}{6}$ of 3 guineas.

9. Find $3\frac{7}{9}$ of a guinea $- 8\frac{3}{8}$ of a crown $+ 1\frac{2}{3}$ of $3\frac{1}{7}$ of £5. 19*s.*

10. Find the difference between $\frac{3}{8}$ of 2*s.* 6*d.* and $\frac{4}{9}$ of 1*s.* 6*d.*; also between $1\frac{3}{5}$ of £2. 7*s.* 11*d.* and $2\frac{5}{6}$ of £1. 13*s.* $4\frac{1}{2}$*d.*

11. Find, to the nearest farthing, the sum of $\frac{1}{4}$ of a guinea, $\frac{1}{9}$ of a crown, and $\frac{1}{7}$ of a shilling.

12. Find, to the nearest penny, the sum of $3\frac{7}{11}$ of 5*s.* 8*d.*, $5\frac{3}{8}$ of 4*s.* 7*d.* and $2\frac{3}{7}$ of 4*s.* 6*d.*

13. Add together $1\frac{1}{2}$ of 1 qr. 4 lbs., $3\frac{1}{3}$ of 1 cwt. 8 lbs., and $\frac{5}{11}$ of 2 cwt. 3 qrs. 22 lbs.

14. Multiply 3 cwt. 2 qrs. 27 lbs. 9 oz. by $20\frac{2}{7}$.

15. Add $4\frac{1}{4}$ of 4 ft. 5 in., and $3\frac{5}{8}$ of 2 ft. 10 in.

16. Add $7\frac{1}{5}$ of 1 yd. 2 ft. 3 in. and $3\frac{7}{10}$ of 2 ft. 8 in.

17. Add $3\frac{1}{3}$ of 1 mile 1 fur. 21 poles and $2\frac{5}{16}$ of 3 miles 2 fur. 16 po.

18. Find the difference between $\frac{2}{3}$ of $3\frac{1}{4}$ of 3 acres 10 poles and $\frac{1}{2}$ of $4\frac{1}{3}$ of 2 acres 2 roods 12 poles.

19. Find the difference between $\frac{2}{3}$ of a mile and $\frac{5}{6}$ of 3 fur. 8 p. 2 yds.

20. What weight must be added to $\frac{3}{8}$ of $\frac{5}{4}$ of half a cwt. to make it up to $\frac{2}{5}$ of $3\frac{3}{4}$ of a stone?

116. To express one quantity as a fraction of another we proceed as follows.

Ex. 1. Express 14*s.* 6*d.* as a fraction of 15*s.* 8*d.*

$$14s.\ 6d. = 174d. \text{ and } 15s.\ 8d. = 188d.$$

Now $$1d. = \tfrac{1}{188} \text{ of } 188d.;$$

$$\therefore\ 174d. = \tfrac{174}{188} \text{ of } 188d.$$

Hence the fraction required is $\frac{174}{188} = \frac{87}{94}$.

Ex. 2. Express $2\frac{1}{5}$ of 1*s.* $7\frac{1}{2}$*d.* as a fraction of £1.

$$2\tfrac{1}{5} \text{ of } 1s.\ 7\tfrac{1}{2}d. = \frac{11}{5} \text{ of } \frac{39}{2}\,d. = \frac{11 \times 39}{5 \times 2}\,d. = \frac{429}{10}\,d.$$

Also $$£1 = 240d.$$

Now $$1d. = \tfrac{1}{240} \text{ of } £1.$$

$$\therefore\ \tfrac{429}{10}d. = \tfrac{429}{10} \times \tfrac{1}{240} \text{ of } £1 = \tfrac{429}{2400}£.$$

Hence the fraction required $= \frac{429}{2400} = \frac{143}{800}$.

EXAMPLES V. O.

1. Express 6*d.*, 8*d.*, 9*d.*, $4\frac{1}{2}d.$, $7\frac{1}{2}d.$, $10\frac{1}{2}d.$, $2\frac{1}{2}d.$, $2\frac{1}{4}d.$ as fractions of 1 shilling.

2. Express 15*s.*, 13*s.* 4*d.*, 12*s.* 6*d.*, 7*s.* 6*d.*, 2*s.* 6*d.*, 3*s.* 4*d.*, 2*s.* 8*d.*, 5*s.* 4*d.*, 1*s.* 8*d.* and 1*s.* 3*d.*, as fractions of £1.

3. What fraction of 4*s.* $7\frac{1}{2}d.$ is $9\frac{1}{4}d.$?

4. What fraction of £18. 2*s.* 1*d.* is £4. 5*s.* 7*d.*?

5. What fraction of £5. 13*s.* 9*d.* is $\frac{2}{3}$ of £3. 15*s.* 10*d.*?

6. Express $1\frac{1}{4}$ of $3\frac{4}{5}$ of 1*s.* 2*d.* as a fraction of $7\frac{1}{8}$ of $2\frac{1}{3}$ of 3*s.* 4*d.*

7. How many times does 4 yds. 2 ft. 8 in. contain 1 yd. 1 ft. 7 in.?

8. What fractional part of 9 A. 3 R. 35 P. is 4 A. 2 R. 5 P.?

9. What would be the measure of 9 tons 13 cwt. 2 lbs., if 2 tons 13 cwt. 2 qrs. 13 lbs. were taken as the unit?

10. Express 5 lbs. 9 oz. 12 dwts. 3 grs. in terms of 13 lbs. 6 oz. 8 dwts. 7 grs. as unit.

11. Express 11 dwts. 16 grs., 1 oz. 8 dwts., and 2 oz. 17 dwts. 18 grs. (1) as fractions of 1 lb. Troy, and (2) as fractions of 1 lb. Avoir.

12. What fraction of 7 m. 4 fur. 30 p. is 9 m. 1 fur. 25 p. 2 yds. 1 ft. 6 in.?

13. What fraction of £2. 8*s.* $1\frac{1}{2}d.$ must be added to $\frac{2}{3}$ of $\frac{4}{5}$ of 17*s.* 6*d.* that the sum may be 19*s.* $7\frac{3}{4}d.$?

14. What fraction of £19. 2*s.* $11\frac{1}{4}d.$ must be added to $\frac{3}{20}$ of £1. 14*s.* 7*d.* in order that the sum may be £2?

15. Express $\frac{3}{20}$ of 4 guineas as a fraction of £7.

16. Reduce 3 lbs. 10 oz. to the fraction of 14 cwt. 2 qrs.

17. Express $1\frac{11}{16}$ fathoms $+\frac{7}{8}$ yds. $+3\frac{3}{16}$ ft. $+9\frac{3}{4}$ in. as a fraction of 16 yds. 2 ft. 3 in. [1 fathom = 6 feet.]

18. Reduce $2\frac{1}{8}$ of £5. 11*s.* to the fraction of $2\frac{5}{16}$ of £4. 5*s.*

19. Reduce $\frac{2}{5}$ of £4. 12*s.* 6*d.* to the fraction of £11. 2*s.*

20. Express $\frac{2}{3}$ of a ton $-\frac{1}{3}$ of a cwt. $-33\frac{1}{4}$ lbs. as a fraction of 1 cwt. 1 qr.

MISCELLANEOUS EXAMPLES IN FRACTIONS.

1. Reduce $5\frac{1}{7}$, $8\frac{3}{11}$ and $25\frac{3}{25}$ to improper fractions.

2. Simplify $\frac{3}{4}-\frac{3}{8}+\frac{5}{12}-\frac{5}{16}+\frac{7}{24}-\frac{7}{48}$.

What must be added to $5\frac{7}{8}$ that the sum may be $12\frac{5}{6}$?

3. Multiply $2\frac{2}{3}$ of $5\frac{3}{8}$ by $3\frac{1}{7}\div 6\frac{1}{7}$.

4. Simplify $\dfrac{3\frac{1}{4}-\frac{5}{6}\times\frac{6}{7}+\frac{3}{14}}{\frac{3}{14}+\frac{1}{3}\text{ of }\frac{5}{4}-\frac{5}{21}}$.

5. Add together $\frac{2}{5}$ of £1. 1*s.* $4\frac{1}{4}$*d.* and $\frac{5}{112}$ of a guinea.

---

6. Arrange, in ascending order of magnitude, the fractions

$$\frac{7}{12},\ \frac{5}{9},\ \frac{8}{15},\ \frac{13}{20}.$$

7. From the sum of $\frac{1}{2}$ and $\frac{1}{3}$ take the difference between $\frac{1}{4}$ and $\frac{1}{6}$.

8. Simplify $2\frac{2}{5}$ of $4\frac{3}{8}$ of $5\frac{1}{7}$.

9. Simplify $\dfrac{3\frac{1}{2}+2\frac{1}{3}-4\frac{5}{6}}{\frac{7}{12}\div\frac{2}{3}}\times\frac{5}{16}$.

10. Find the value of $\frac{7}{8}$ of £1 $+\frac{3}{5}$ of a guinea $-\frac{2}{5}$ of 6*s.* 8*d.* $+\frac{3}{100}$ of a crown $-\frac{3}{16}$ of a shilling.

---

11. Reduce $\frac{410200}{750008}$ and $\frac{2518194}{881879}$ to their lowest terms.

12. Reduce to a common denominator

$$\frac{1}{504},\ \frac{2}{819},\ \frac{1}{468},\text{ and }7\frac{3}{728}.$$

13. Simplify $3\frac{3}{4}+2\frac{2}{5}$ of $1\frac{1}{9}-4\frac{1}{5}$.

14. Simplify $\dfrac{\frac{1}{5}-\frac{1}{6}+\frac{1}{9}-\frac{1}{18}}{\frac{1}{4}-\frac{1}{5}+\frac{1}{9}-\frac{2}{45}}$.

15. Find the value of $\frac{3}{4}$ of 1*s.* 7*d.* $+2\frac{1}{2}$ of $1\frac{3}{5}$ of 5*s.* $2\frac{1}{4}$*d.* $+\frac{3}{70}$ of $2\frac{1}{2}$ guineas.

---

16. Add $\frac{1}{12}$, $\frac{5}{64}$, $\frac{7}{80}$, $\frac{13}{160}$, $\frac{27}{320}$ and $\frac{41}{480}$.

17. Subtract $5\frac{13}{15}$ from $7\frac{9}{20}$; also $5\frac{3}{8}+2\frac{5}{6}$ from $12\frac{1}{4}$.

18. Divide $2\frac{2}{15}+2\frac{27}{35}-3\frac{4}{21}$ by $2\frac{5}{21}+3\frac{1}{3}-4\frac{6}{7}$.

19. Simplify $\dfrac{1\frac{1}{2}+2\frac{1}{4} \text{ of } 5\frac{1}{3}-12\frac{3}{4}}{\frac{3}{4}+\frac{4}{5}\times 8\frac{3}{5}-6\frac{1}{2}}$.

20. What fraction of 35 shillings is equal to the sum of $2\frac{3}{4}$ of 7*s.* 6*d.* and $\frac{3}{2}\times\frac{5}{4}$ of 3*s.*?

---

21. Reduce $\frac{34726}{65541}$, $\frac{100010}{205902}$ and $\frac{1795224}{2524525}$ to their lowest terms.

22. Shew that $\dfrac{1}{2}+\dfrac{1}{2\times3\times4}+\dfrac{1}{2\times3\times4\times5\times6}$ is less than $\dfrac{6}{11}$ but greater than $\dfrac{8}{15}$.

23. Simplify $\frac{23}{23}\frac{5}{1}\times\frac{66}{65}\times\frac{91}{94}$.

24. Simplify $\dfrac{2\frac{1}{4}+\frac{2}{3} \text{ of } 1\frac{1}{2}-3\frac{1}{8}}{\frac{3}{4}+\frac{1}{4}\div\frac{2}{3}-\frac{1}{8}}$ and $\cfrac{1}{4-\cfrac{1}{4+\cfrac{1}{4-\frac{1}{4}}}}$.

25. What fraction is 13*s.* $9\frac{3}{4}$*d.* of £2. 0*s.* $4\frac{1}{2}$*d.*?

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26. Simplify $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}-1\frac{3}{6}-\frac{1}{20}$.

27. Subtract $2\frac{3}{34}\frac{3}{}$ from $5\frac{4}{51}\frac{7}{}$, and $8\frac{1}{4}\frac{6}{5}$ from $12\frac{7}{60}$.

28. Multiply $3\frac{1}{8}$ of $5\frac{1}{7}$ by $4\frac{1}{5}$ of $\frac{3}{2}$, and divide the result by $4\frac{1}{2}$ of $1\frac{7}{8}$.

29. Simplify $\dfrac{\frac{2}{3}+\frac{3}{7}}{\frac{3}{4}+\frac{2}{5}}\div\dfrac{2\frac{2}{3}}{\frac{3}{4}-\frac{2}{5}}$ and $\dfrac{8\frac{2}{17} \text{ of } (\frac{7}{6}-\frac{6}{7})}{(8-\frac{1}{3})\div 6\frac{1}{5} \text{ of } (1-\frac{4}{39})}$.

30. Express $\frac{1}{19}$ of £1. 2*s.* $6\frac{3}{4}$*d.* $+\frac{1}{8}$ of 11*s.* 8*d.* as a fraction of 3*s.* 6*d.*

---

31. By how much does the sum of $1\frac{1}{6}$, $\frac{5}{8}$ and $\frac{1}{10}$ fall short of the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{11}{12}$?

32. Simplify $2\frac{1}{2}-\frac{1}{3} \text{ of } 4\frac{1}{5}+2\frac{1}{5}\div 3\frac{1}{5}\times 2\frac{1}{4}-\frac{47}{320}$.

33. How many pieces each $\frac{3}{5}$ of 1 inch can be cut from a wire whose length is $5\frac{1}{2}$ inches; and what will be the length of the piece left over?

34. Simplify $\dfrac{4\frac{1}{2}-3\frac{1}{3}+5\frac{1}{12}}{7\frac{1}{2}-4\frac{1}{3}+11\frac{1}{12}}-\dfrac{11\frac{3}{4}-5\frac{7}{15}}{11\frac{1}{2}+5\frac{3}{5}}$.

35. Find the difference between $\frac{5}{9}$ of 31 acres 2 roods 27 poles and $\frac{7}{8}$ of 18 acres 1 rood.

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36. Arrange the fractions $\frac{8}{35}$, $\frac{11}{45}$, $\frac{13}{63}$ in ascending order of magnitude, and find the difference between the greatest and the least of them.

37. Find the difference between

$$1+\tfrac{1}{3}+\tfrac{1}{5}+\tfrac{1}{7} \text{ and } \tfrac{1}{2}-\tfrac{1}{4}+\tfrac{1}{6}-\tfrac{1}{8}.$$

38. Divide $3\frac{1}{4}$ of $4\frac{1}{3}$ by $(2\frac{1}{2}-\frac{1}{3})$ of $(3\frac{1}{2}-\frac{1}{4})$.

39. Simplify $\frac{1}{17}\frac{8}{1}(1-\frac{64}{81})+\frac{8}{11}\times\frac{1}{6}(\frac{1}{2}+\frac{5}{12})$.

40. What fraction of a mile is 4 fur. 32 poles? Express 37 p. 1 yd. 2 ft. 6 in. as a fraction of 1 fur. 10 p. 5 yds.

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41. Take the sum of $\frac{3}{4}$ and $\frac{1}{2}$ from the sum of $\frac{5}{6}$ and $\frac{2}{3}$.

42. After taking away $\frac{1}{2}$ and $\frac{2}{5}$ of a certain quantity, what fraction of the whole will be left?

43. Multiply $1\frac{1}{3}+3\frac{4}{5}$ by $3\frac{1}{2}+2\frac{4}{7}$, and divide the result by $5\frac{1}{2}$ of $5\frac{2}{3}$.

44. Simplify $\dfrac{2\frac{6}{8}\frac{9}{8}-\frac{5}{13} \text{ of } 4\frac{8}{11}+3\frac{11}{12}\div 7\frac{5}{6}}{3\frac{3}{11} \text{ of } 10\frac{1}{12}-3\frac{1}{3}\div 4\frac{4}{9}}$.

45. Find the value of $3\frac{3}{11}$ of $4\frac{5}{7}$ of $1\frac{5}{9}$ of 1*s.* $1\frac{1}{2}$*d.*

---

46. Add together $\frac{2}{5}$, $\frac{35}{80}$, $\frac{14}{100}$, $\frac{3}{140}$ and $\frac{3}{2800}$.

47. Simplify $\frac{110}{47}\div 7\frac{6}{7}\div\frac{182}{611}$.

48. Simplify $\dfrac{2\frac{1}{4}-\frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3}+\frac{7}{8}}$ and $\cfrac{1}{1-\cfrac{1}{2+\cfrac{1}{3-\cfrac{1}{4}}}}$.

49. There are three partners in a certain business, one of whom provided $\frac{2}{5}$ of the whole capital, and another provided $\frac{3}{8}$. What fraction of the whole was supplied by the third partner?

50. Find the difference between $\frac{6}{7}$ of 15 acres 3 roods 35 poles and $\frac{7}{9}$ of 31 acres 2 roods 27 poles.

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51. Reduce $\frac{23}{67}\frac{12}{76}$ and $\frac{47}{61}\frac{81}{47}$ to their lowest terms.

52. Simplify $12\frac{2}{3}-2\frac{1}{3} \text{ of } \frac{6}{7}-5\frac{1}{6}$.

53. If $\frac{4}{7}$ of an estate be worth £660, how much will the whole be worth, and how much will $\frac{8}{33}$ be worth?

54. Simplify $\dfrac{9\frac{1}{9}+8\frac{1}{8}-7\frac{1}{7}}{9\frac{1}{9}-6\frac{9}{56}+7\frac{1}{7}}$ and $\dfrac{1-\dfrac{1}{2-\frac{1}{3}}}{1+\dfrac{1}{2+\frac{1}{3}}}$.

55. If $\frac{3}{10}$ of $\frac{5}{7}$ of 12*s.* 3*d.* be taken from $\frac{7}{15}$ of $2\frac{1}{2}$ guineas, what fraction of £1. 5*s.* will remain?

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56. Simplify $\frac{11}{12}-\frac{10}{11}+\frac{9}{10}-\frac{8}{9}$.

57. Simplify $8\,(1\frac{1}{4}-\frac{1}{3})-3\{2\frac{1}{2}-3\,(1\frac{1}{4}-\frac{7}{8})\}$.

58. Divide $1\frac{1}{2}$ of $5\frac{3}{8}$ by $2\frac{3}{4}$ of $7\frac{9}{11}$.

59. Simplify $\dfrac{4\frac{1}{2}-3\frac{1}{3}+5\frac{1}{12}}{7\frac{1}{2}-4\frac{1}{3}+11\frac{1}{12}}-\dfrac{11\frac{7}{10}-5\frac{7}{15}}{11\frac{1}{2}+5\frac{3}{5}}$.

60. Find the value of $\frac{2}{5}$ of $\frac{3}{8}$ of a sovereign $-\frac{1}{7}$ of $\frac{5}{6}$ of a guinea; and express the difference as a fraction of £1. 4*s.* 9*d.*

---

61. Arrange in ascending order of magnitude, and find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$ and $\frac{25}{36}$.

62. How much must be taken from $\frac{2}{3}$ of $5\frac{1}{4}$ that the remainder may equal the sum of $1\frac{2}{5}$ and $\frac{3}{7}$ of $1\frac{1}{5}$?

63. Simplify $2\frac{1}{2}$ of $(\frac{4}{5}+\frac{3}{8})\div 5\frac{7}{8}-1\frac{1}{4}$ of $\frac{2}{7}+3\frac{3}{4}$ of $1\frac{3}{7}$.

64. Simplify $\dfrac{1\frac{2}{3}+3\frac{4}{5}\text{ of }3\frac{1}{3}}{1\frac{4}{5}-\frac{3}{7}\text{ of }1\frac{2}{9}}$ of $9\frac{4}{7}-2\frac{6}{17}$ of $\dfrac{1+\frac{1}{3}-\frac{1}{5}}{1+\dfrac{1}{4\frac{1}{3}}}$.

65. Find the value of $\frac{11}{16}$ of $\frac{2}{5}$ of £1 + $\frac{2}{3}$ of $\frac{3}{4}$ of 13*s.* 6*d.*, and reduce the result to the fraction of £1. 17*s.* 6*d.*

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66. Reduce $\frac{729}{1917}$ and $\frac{1887}{2109}$ to their lowest terms.

67. Add together $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{7}$, $\frac{1}{14}$ and $\frac{9}{56}$.

68. After spending $\frac{3}{5}$ of his money, a man found that $\frac{3}{7}$ of the remainder was 2 guineas. What had he at first?

69. Reduce to its simplest form

$$\frac{(3\frac{1}{5}+5\frac{1}{9}-\frac{1}{45})(4\frac{1}{5}-3\frac{1}{4})}{1\frac{5}{11}+2\frac{1}{8}-(2\frac{9}{16}-\frac{1}{8}-\frac{1}{22})}.$$

70. Express 1 lb. Troy as a fraction of 1 lb. Avoir.; also express 3 oz. 3 dwts. 9 grs. as a fraction of 4 oz. 2 dwts. 21 grs.

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71. Find the value of $7\frac{17}{20} - 2\frac{3}{8} + 1\frac{1}{6} - 1\frac{13}{14}$.

72. A man travelled $\frac{3}{4}$ of a certain distance by railway, $\frac{4}{21}$ of the whole distance by coach, and walked the rest of the way which was 15 miles. What was the length of the whole journey?

73. By what must $7\frac{2}{3}$ of $3\frac{3}{4}$ be multiplied that the product may be equal to $4\frac{3}{5}$ of $2\frac{5}{8}$?

74. Simplify $\dfrac{6\frac{3}{4} - \frac{3}{7} \text{ of } 15\frac{3}{4} + 2\frac{2}{35} \div 1\frac{11}{25}}{\frac{3}{4} \text{ of } 7\frac{3}{7} - 5\frac{3}{5} \div 3\frac{4}{15}}$.

75. Express $\frac{3}{50}$ of £1. 2*s.* 6*d.* $+ \frac{7}{8}$ of $\frac{11}{5}$ of 4*d.* $- \frac{4}{5}$ of $\frac{1}{8}$ of 2*s.* 5*d.* as a fraction of 8*s.* 9*d.*

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76. Subtract $10\frac{11}{15}$ from $23\frac{3}{5}$, and $15\frac{23}{48}$ from $20\frac{39}{84}$.

77. A man, after spending $\frac{2}{5}$ and $\frac{1}{9}$ of his money, finds he has £1. 2*s.* left. How much had he at first?

78. Simplify $(3\frac{1}{7} + 2\frac{1}{4} + 4\frac{2}{5}) \div (\frac{1}{2} + \frac{2}{5})$ of $(\frac{1}{2} - \frac{3}{7})$.

79. Simplify

(i) $\dfrac{\frac{3}{4} - \frac{2}{3} \times \frac{1}{2} - \frac{1}{6}}{(4\frac{1}{2} - 2\frac{1}{3}) \div (3\frac{1}{3} - 1\frac{1}{6})}$, (ii) $\dfrac{3\frac{2}{13}}{2\frac{1}{2} - \dfrac{3}{4 - \frac{3}{4}}}$.

80. Find the value of $2\frac{1}{7}$ of 7*s.* $3\frac{1}{2}$*d.* $+ \frac{2}{7}$ of $\frac{2}{9}$ of a guinea $+ 2\frac{3}{8}$ of £1.

---

81. The sum of two numbers is $5\frac{2}{3}$ and the smaller is $2\frac{7}{8}$; what is the greater?

82. What fraction added to $\frac{2}{3}$ of $1\frac{1}{2} - \frac{1}{12}$ makes the sum equal to $8\frac{1}{4} \div 2\frac{1}{12}$?

83. Subtract $\frac{2}{3}$ of $\frac{5}{6}$ from $1\frac{1}{2}$ of $\frac{4}{9}$, and divide the result by $(\frac{2}{3} - \frac{4}{7})(\frac{4}{5} - \frac{5}{8})$.

84. A man pays away $\frac{3}{5}$ of his money and afterwards $\frac{7}{8}$ of the remainder. What fraction of the whole had he then left?

85. If $\frac{5}{14}$ of an estate be worth £1282. 3*s.* 4*d.*, what is the value of $\frac{3}{8}$ of it?

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86. Reduce to a common denominator, and arrange in order of magnitude the fractions

$$\tfrac{5}{12},\ \tfrac{7}{15},\ \tfrac{9}{20},\ \tfrac{13}{32},\ \tfrac{19}{48}.$$

87. Multiply the difference between $3\frac{1}{2}$ of $1\frac{1}{14}+7\frac{1}{2}$ and $2\frac{1}{7}\div\frac{2}{3}$ of $\frac{5}{21}$ by the sum of $\dfrac{1}{5\frac{1}{7}}$ and $\dfrac{1}{7\frac{1}{5}}$.

88. Simplify $\frac{1}{7}$ of $\dfrac{6\frac{7}{9}-2\frac{2}{9}}{\frac{1}{9}\text{ of }18\frac{1}{2}}\div\dfrac{\frac{2}{7}\left(6\frac{4}{3}+19\frac{2}{3}\right)}{\frac{8}{11}+\frac{12}{13}\text{ of }6\frac{1}{2}}$.

89. After spending $\frac{2}{5}$ of his money, a boy found that $\frac{4}{7}$ of the remainder was 1*s.* 9*d.* What had he at first?

90. Add together $\frac{1}{14}$ of a furlong, $\frac{9}{14}$ of a pole and $1\frac{3}{4}$ of a yard.

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91. Reduce to their lowest terms $\frac{2173}{4033}$, $\frac{14960}{15330}$ and $\frac{289071}{1542241}$.

92. Add together $8\frac{7}{11}$ of $\frac{3}{5}$, $9\frac{1}{2}$ of $\frac{13}{76}$, $3\frac{3}{4}$ of $\frac{8}{15}$ and $6\frac{8}{9}$ of $\frac{15}{22}$.

93. Divide $3\frac{5}{12}$ by $4\frac{11}{28}$, and $25\frac{25}{26}$ by $3\frac{6}{13}$.

94. Simplify $\dfrac{4\frac{1}{6}}{\frac{7}{16}}$ of $3\frac{1}{5}-\frac{2}{7}$ of $\dfrac{2\frac{2}{3}+1\frac{1}{6}}{2-\frac{4}{25}}$.

95. *A* gave $\frac{1}{3}$ of his marbles to *B*, $\frac{1}{6}$ to *C*, $\frac{1}{8}$ to *D*, $\frac{1}{16}$ to *E*, and then had 105 left. How many did each receive?

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96. What must be added to $3\frac{3}{4}+2\frac{5}{8}$ that the sum may be $7\frac{1}{4}$?

97. Simplify $1\frac{3}{4}$ of $\frac{2}{3}+5\frac{1}{2}\div(5-1\frac{7}{8})$ of $\frac{1}{25}$.

98. Multiply $3\frac{9}{11}$ of $\frac{3}{20}$ of $6\frac{1}{2}$ by $2\frac{2}{9}$ of $\frac{2}{39}$.

99. Divide the sum of $2\frac{2}{3}$, $3\frac{3}{4}$ and $5\frac{3}{8}$ by the sum of $2\frac{2}{5}$ and $3\frac{8}{9}$, and to the quotient add the difference of $10\frac{9}{10}$ and $5\frac{31}{40}$.

100. A man spent $\frac{2}{5}$ of his money, then $\frac{1}{3}$ of the remainder, and then $\frac{1}{8}$ of what he still had left. After this he had £7 left. How much had he at first?

CHAPTER VI.

DECIMALS.

117. We have already seen in Chapter III. how the measurement of continuous magnitudes leads to subdivisions of the unit of measurement. In our ordinary system of weights and measures the subdivisions and multiples of the standard unit are exceedingly inconvenient and troublesome.

In the **decimal** system of subdivision the unit is divided into ten equal parts, called tenth-parts or **tenths**; these tenths again into ten equal parts called **hundredths**; and so on for **thousandths, ten-thousandths,** &c.

The sum of any number of tenths, hundredths, &c. is called a **decimal fraction** or a **decimal.**

It will be at once seen how any magnitude may be measured in terms of any chosen unit and its decimal multiples and sub-multiples, namely *tens*, *hundreds*, &c. and *tenths*, *hundredths*, &c.; and it will often be found to be advantageous to use the decimal system in calculations, although unfortunately our own different weights and measures are not connected together in so simple a manner.

118. We have now to shew how these *tenths*, *hundredths*, *thousandths*, &c. can be denoted by means of figures.

In the ordinary system of notation for whole numbers the place-value of any digit diminishes, for every move from left to right, to one-tenth of what it was before.

Thus, moving from the fourth place to the third changes *thousands* to *hundreds*, from the third to the second changes *hundreds* to *tens*, and from the second to the first changes *tens* to *ones*.

If then we put some mark to shew the position of the units figure, and *suppose the same law to be continued,* we have at once a method by which *tenths, hundredths,* &c. can be represented.

The mark adopted for shewing the units-place is a **point,** called the **decimal point,** which is placed next after the units digit.

Thus 2·5 stands for 2 *units* and 5 *tenths*; also 45·627 stands for 4 *tens* 5 *units* 6 *tenths* 2 *hundredths* 7 *thousandths.*

Again ·35 stands for 3 *tenths* and 5 *hundredths*; also ·006 stands for 6 *thousandths.*

119. If we count from the units-place, and call right to left *up,* and left to right *down*; then

tens are 1 place up, and *tenths* are 1 place down,
hundreds ... 2 ....., ... *hundredths* ... 2 .....,
thousands ... 3 ....., ... *thousandths*... 3 .....,

and so on.

120. By definition, ·465 stands for 4 *tenths,* 6 *hundredths* and 5 *thousandths.* But, since 1 *tenth* = 10 *hundredths* = 100 *thousandths,* it follows that ·465 = 465 *thousandths.*

So also ·0372 = 372 *ten-thousandths,* and similarly in other cases.

Thus a decimal may be '**read off**' as an integral number of the units corresponding to the last place on the right.

For example, ·506 may be read five hundred and six *thousandths*; and ·3070 may be read three thousand and seventy *ten-thousandths.*

To '**read off**' decimals it is, however, usual only to give the digits in order.

Thus ·615 is read 'decimal six, one, five'; 15·0524 is read 'fifteen, decimal nought, five, two, four'; and 1567·0082 is read 'one thousand five hundred and sixty seven, decimal nought, nought, eight, two.'

121. It has been already noticed that noughts at the *extreme left of an integer* are without any value.

Thus 00516 means just the same as 516.

So also noughts at the *extreme right of a decimal* are without any value.

For ·20 is 2 *tenths* 0 *hundredths* which is just the same as ·2; and similarly in other cases.

EXAMPLES. VI. A.

Write out in words the separate value of every figure in

1. 3·5. **2.** 15·7. **3.** 125·34. **4.** 12·53. **5.** 800·17.

6. 1200·63. **7.** ·875. **8.** 3·427. **9.** 50·037. **10.** 250·0034.

11. 8000·1009. **12.** 5·00107. **13.** 560002·19007.

14. 27·050603.

Write in figures

15. Twelve and three-tenths. **16.** Eighteen and seven-tenths.

17. Three hundred and four and nine-tenths.

18. Twenty-five, three-tenths and four-hundredths.

19. Four, six-tenths and seven-thousandths.

20. One million, four-tenths and three-millionths.

21. Read off, as a number of units of the lowest order represented, ·15, ·0127, 31·5, 5·016 and 2·0084.

122. The **reduction of a decimal to a vulgar fraction** follows at once from the definition.

For example, ·25 is by definition $\frac{2}{10}+\frac{5}{100}$, which is equal to $\frac{25}{100}$. Again, $\cdot304=\frac{3}{10}+\frac{4}{1000}=\frac{304}{1000}$, and $2\cdot06=2\frac{6}{100}$.

Thus a decimal can be at once written as a vulgar fraction whose denominator is a power of 10; the numerator being the number represented by the figures of the decimal, and the denominator being 1 *followed by as many noughts as there are figures after the decimal point.*

Thus $\cdot2153=\frac{2153}{10000}$, $\cdot056=\frac{056}{1000}=\frac{56}{1000}=\frac{7}{125}$, and

$$3\cdot005=3\frac{005}{1000}=3\frac{1}{200}.$$

EXAMPLES. VI. B.

Express the following decimals as vulgar fractions with powers of 10 for their denominators, and then reduce the fractions to their lowest terms.

1. ·2, ·3, ·4, ·5, ·8, ·25, ·35, ·45, ·75, ·88.
2. ·125, ·375, ·625, 1·325, 5·424, 30·056, 15·068.
3. 15·3125, 17·0625, 8·00125, 103·0375.
4. ·040625, 3·003125, 4·09375, 1·50128.
5. Write as decimals :—$\frac{5}{10}$, $\frac{17}{100}$, $\frac{193}{1000}$ and $\frac{214}{100}$.
6. Write as decimals :—$\frac{5}{100}$, $\frac{17}{1000}$, $\frac{9}{1000}$, $\frac{97}{10000}$ and $\frac{123}{1000000}$.

123. Multiplication by 10, 100, &c. A decimal is multiplied by 10 by simply *moving the decimal point one place to the right,* for by so doing each figure is moved one place up, so that the units are turned into tens, the tens into hundreds, &c.; also the tenths are turned into units, the hundredths into tenths, &c. Thus each of the parts of the decimal, and therefore the sum of them all, is multiplied by 10.

For example, $2{\cdot}35 \times 10 = 23{\cdot}5$; for, by moving the decimal point, 2 *units* is changed into 2 *tens*, 3 *tenths* into 3 *units* and 5 *hundredths* into 5 *tenths*. Thus each of the parts of the decimal, and therefore the sum of them all, has been multiplied by 10.

By again moving the decimal point one place to the right the number is again multiplied by 10, so that a decimal is multiplied by 10×10, that is by 100, by moving the decimal point *two* places to the right. So also a decimal is multiplied by 1000 by moving the point three places to the right, and so on.

Thus $\cdot 3214 \times 100 = 32{\cdot}14$, $\cdot 0056 \times 1000 = 005{\cdot}6$, that is 5·6.

Again $\cdot 25 \times 100 = 25\cdot$ [The decimal point is however omitted when no figures follow it.]

Also $\cdot 25 \times 1000 = 250\cdot$, that is 250, noughts being put on the right, when, as in this case, there are not enough figures.

124. Division by 10, 100, &c. A decimal is divided by 10 by simply *moving the decimal point one place to the left.*

For example $23{\cdot}5 \div 10 = 2{\cdot}35$; for moving the decimal point changes 2 *tens* into 2 *units*, 3 *units* into 3 *tenths*, and 5 *tenths* into 5 *hundredths*. Thus each of the parts of the decimal, and therefore the whole decimal, has been divided by 10.

So also a decimal is divided by 100 by moving the decimal point *two* places to the left; it is divided by 1000 by moving the decimal point *three* places to the left; and so on.

Thus $32{\cdot}14 \div 100 = {\cdot}3214$, $250 \div 100 = 2{\cdot}50 = 2{\cdot}5$.

Also $5{\cdot}6 \div 1000 = {\cdot}0056$, noughts being prefixed on the left, when, as in this case, there are not enough figures.

EXAMPLES. VI. C.

Multiply each of the following decimals by 10.

1. 2·34, 51·67, 230·7, 21·08, 512·6, 314·5.
2. 7·123, 123·4, ·562, ·0034, 501·7, ·0006.

Write down the following

3.	$2{\cdot}157 \times 100$.	4.	${\cdot}0067 \times 100$.	5.	$210{\cdot}35 \times 100$.
6.	$1{\cdot}2308 \times 1000$.	7.	$2{\cdot}1345 \times 1000$.	8.	$3{\cdot}501 \times 1000$.
9.	$1{\cdot}27 \times 100$.	10.	$12{\cdot}7 \times 100$.	11.	$31{\cdot}5 \times 100$.
12.	${\cdot}003 \times 1000$.	13.	${\cdot}07 \times 1000$.	14.	$3{\cdot}92 \times 1000$.
15.	$35{\cdot}7 \div 10$.	16.	$27{\cdot}3 \div 10$.	17.	$5{\cdot}16 \div 10$.
18.	$21{\cdot}74 \div 10$.	19.	${\cdot}315 \div 10$.	20.	${\cdot}637 \div 10$.
21.	${\cdot}2143 \div 10$.	22.	${\cdot}058 \div 10$.	23.	${\cdot}0067 \div 10$.
24.	$56{\cdot}23 \div 100$.	25.	$38{\cdot}27 \div 100$.	26.	$71{\cdot}08 \div 100$.
27.	$5{\cdot}813 \div 100$.	28.	$7{\cdot}092 \div 100$.	29.	$3{\cdot}721 \div 100$.
30.	${\cdot}4106 \div 100$.	31.	${\cdot}7102 \div 100$.	32.	${\cdot}51 \div 100$.
33.	${\cdot}019 \div 100$.	34.	${\cdot}0084 \div 100$.	35.	${\cdot}0718 \div 100$.
36.	$6234{\cdot}1 \div 1000$.	37.	$623{\cdot}41 \div 1000$.	38.	$6{\cdot}2341 \div 1000$.
39.	${\cdot}57 \div 10000$.	40.	${\cdot}174 \div 10000$.	41.	${\cdot}098 \div 10000$.

ADDITION AND SUBTRACTION OF DECIMALS.

125. In order to add any number of decimals they must first be arranged so that *their decimal points are in a vertical column.* This will ensure that all the tenths shall be in the same vertical column, and so for the hundredths, &c.; and so also for the units, tens, hundreds, &c.

We have now only to add the figures in each vertical column in order, beginning on the right, carrying the number of tens in any column as units to the next column. The decimal point in the result being under the other decimal points.

Ex. 1. Add 2·415, 6·307, 8·1738 and ·6947.

2·415	Mental work,	7, **15**;
6·307		1, 5, 8, 15, **20**;
8·1738		2, 11, 18, **19**;
·6947		1, 7, 8, 11, **15**;
17·5905		1, 9, 15, **17**.

Ex. 2. Subtract 3·615 from 7·43.

7·43	5 and **5** make 10;
3·615	1 (carried), 2 and **1** make 3;
3·815	6 and **8** make 14;
	1 (carried), 4 and **3** make 7.

EXAMPLES. VI. D.

Find the sum of

1. 21·63, 5·24, 170·63, 27·59, 17.
2. 301·7, 30·17, 3·017, ·3017, ·03017.
3. 170·2, 3·605, 17·35, 15·609, ·0086.
4. ·0037, 21·85, 169·4, 17·9375, ·90087.
5. 4·1372, 41·372, 4137·2, ·41372, 41372.

Find the difference between

6. 3·726 and 5·949.
7. 3·008 and 3·08.
8. 14·753 and 6·876.
9. ·217 and ·271.
10. 1 and ·888.
11. 20 and ·675.
12. ·00013 and ·00175.
13. ·8017 and ·00693.
14. Find $3·17 + 4·216 - 5·8004 + 2·0097 - ·99873$.
15. Find $21·09 - 3·985 - 7·0095 + ·09372 - 4·38009 + 2·60009$.
16. Subtract from 11·214 the sum of 2·301, 1·7293, 2·0507 and 3·62743.
17. Subtract from 20 the sum of 3·416, 2·6008, 5·73124 and 1·5063.
18. By how much does the sum of 3·5612 and 4·71305 exceed the sum of 1·70862 and 5·91927?

Multiplication of Decimals.

126. In order to see how to multiply one decimal by another, consider the following examples.

Ex. 1. Multiply 15·978 by 6.

We proceed exactly as in ordinary multiplication, 8 *thousandths* multiplied by 8 are 48 *thousandths*, that is 4 *hundredths* and 8 *thousandths*. We put down 8 in the column for *thousandths* and carry the 4 *hundredths*; and proceed in this way to the end. It is not, however, necessary to specify the kind of units which are being dealt with at any stage. [See Note p. 14.] The whole product will be found to be 95·868, which is placed under the multiplicand so that units of the same order are in a vertical column.

Thus

$$\begin{array}{r} 15{\cdot}978 \\ \times\quad 6 \\ \hline 95{\cdot}868 \end{array}$$

Ex. 2. Multiply 31·524 by 736.

We may multiply by 6, by 30 and by 700 separately, and then add these partial products. To multiply by 6 we proceed as in Ex. 1, and thus obtain 189·144.

In order to multiply by 30 we may multiply by 3 and 10 in succession. Now $31{\cdot}524 \times 3$ will be found as above to be 94·572; and the product $94{\cdot}572 \times 10$ is obtained at once by moving the decimal point one place to the right [Art. 123], or by *keeping the decimal point fixed and shifting all the figures one place to the left.* Similarly to multiply by 700 we multiply by 7, and then shift all the figures *two* places to the left, keeping the decimal point fixed. Now this shifting of the figures one place to the left when we multiply by 3 *tens*, and two places to the left when we multiply by 7 *hundreds*, *brings the first figure of each line of the multiplication under the figure from which that line arises;* units of the same order in the multiplicand and in the partial products being in vertical columns. Thus

$$\begin{array}{r} 31{\cdot}524 \\ \times\quad 736 \\ \hline 189{\cdot}144 \\ 945{\cdot}72 \\ 22066{\cdot}8 \\ \hline 23201{\cdot}664 \end{array}$$

Ex. 3. Multiply 81·726 by 2·34.

We may multiply separately by 2, by 3 *tenths* and by 4 *hundredths*, and then add these partial products.

To multiply by 2 we proceed as in Ex. 1, and thus obtain 163·452. In order to multiply by 3 *tenths* we may multiply by 3 and then divide by 10. Now $81{\cdot}726 \times 3 = 245{\cdot}178$; and $245{\cdot}178 \div 10$ is obtained by moving the decimal point one place to the left [Art. 124], or by *keeping the decimal point fixed and shifting all the figures one place to the right.* Similarly, to multiply by 4 *hundredths*, we multiply by 4, and then shift all the figures *two* places back, keeping the decimal point fixed. Thus, if we place the multiplier so that its *units figure is under the last figure of the multiplicand;* then the first figure in each line of the multiplication, after being shifted back as we have seen to be necessary, will be under the figure from which that line arises, and *units of the same order in the multiplicand and in the partial products will be in vertical columns.* Thus

$$\begin{array}{r} 81{\cdot}726 \\ \quad 2{\cdot}34 \\ \hline 163{\cdot}452 \\ 24{\cdot}517\,8 \\ 3{\cdot}269\,04 \\ \hline 191{\cdot}238\,84 \end{array}$$

From the above examples it will be seen that the product of one decimal by another is obtained as follows:

First place the multiplier so that its **units figure is under the last figure of the multiplicand,** then multiply by the figures of the multiplier in order from left to right, placing the **first figure of each line of the multiplication under the figure from which that line arises.** Then **the decimal points** in the partial products **will all be under the decimal points in the multiplicand.**

Thus to multiply 321·4 by 68·04, ·31 by ·17, and ·316 by ·0207, we proceed as under:

321·4	·31	·316
6 8·04	0·17	0·0207
19284·	·03 1	·006 32
2571·2	·02 17	·000 2212
12·8 56	·05 27	·006 5412
21868·0 56		

127. It will be readily seen from the last article that the number of decimal places in the product is equal to the *sum* of the number of decimal places in

the multiplicand and multiplier together. We may therefore find the product by *multiplying without any regard to the decimal points, and marking off from the right of the product so obtained as many decimal places as there are in the multiplicand and multiplier together; adding noughts to the left, if necessary.*

For example, the products $21{\cdot}4 \times 8{\cdot}04$ and $\cdot 316 \times \cdot 0217$ may be found thus:—

21·4	·316
8·04	·0217
856	2212
1712	632
172·056	·0065412

The reason for this rule can also be seen thus:

Taking the last example, to find $21{\cdot}4 \times 8{\cdot}04$.

$21{\cdot}4 \times 10 = 214$, and $8{\cdot}04 \times 100 = 804$. [Art. 123.]

Hence $21{\cdot}4 \times 8{\cdot}04 \times 10 \times 100 = 214 \times 804 = 172056$.

Thus $21{\cdot}4 \times 8{\cdot}04 = 172056 \div 10 \div 100$
$= 172056 \div 1000 = 172{\cdot}056$, by Art. 124.

EXAMPLES. VI. E.

Perform the multiplications indicated below:

1. $1{\cdot}6 \times 4$. **2.** $5{\cdot}7 \times 8$. **3.** $1{\cdot}25 \times 7$. **4.** $3{\cdot}72 \times 9$.

5. $12{\cdot}56 \times 17$. **6.** $31{\cdot}09 \times 23$. **7.** $92{\cdot}74 \times 37$. **8.** $7{\cdot}314 \times 84$.

9. $12{\cdot}34 \times 2{\cdot}4$. **10.** $38{\cdot}24 \times 3{\cdot}9$. **11.** $\cdot 1729 \times \cdot 24$.

12. $\cdot 3462 \times \cdot 75$. **13.** $3{\cdot}4165 \times 3{\cdot}57$. **14.** $\cdot 2675 \times 3{\cdot}85$.

15. $3{\cdot}9017 \times \cdot 215$. **16.** $\cdot 0827 \times \cdot 2413$. **17.** $\cdot 0237 \times \cdot 5214$.

18. $\cdot 00167 \times \cdot 0589$. **19.** $\cdot 13579 \times \cdot 0246$. **20.** $\cdot 009235 \times \cdot 00713$.

21. $2{\cdot}1046 \times 4{\cdot}0035$. **22.** $2{\cdot}31575 \times 4{\cdot}0824$.

23. $\cdot 21089 \times \cdot 003904$. **24.** $\cdot 021628 \times \cdot 002828$.

25. $\cdot 1 \times \cdot 1 \times \cdot 1$. **26.** $\cdot 01 \times \cdot 01 \times \cdot 01$. **27.** $6 \times \cdot 6 \times \cdot 06$.

28. $\cdot 31 \times \cdot 41 \times \cdot 51$. **29.** $\cdot 5 \times \cdot 05 \times \cdot 005$. **30.** $2{\cdot}5 \times \cdot 25 \times \cdot 025$.

31. Simplify $(1{\cdot}416 + \cdot 495 + 3{\cdot}219) \times (6{\cdot}714 - 2{\cdot}834)$.

32. Simplify $(3{\cdot}8016 - 2{\cdot}794) \times (1{\cdot}8093 - \cdot 078)$.

Division of Decimals.

128. Case I. *To divide a decimal by a whole number.*

The method will be seen from the following examples:

Ex. 1. Divide 6975·85 by 133, and 6·97585 by 133.

We proceed in the same manner as in the division of one whole number by another. First separate off from the left of the dividend the least number of figures such that the number represented by these figures alone, *disregarding the decimal point*, is not less than the dividend: in the present cases this will be 697, and 697 divided by 133 gives 5 for quotient. But, since 697 actually represents 697 *tens* in the first case and 697 *hundredths* in the second, the quotient will be 5 *tens* in the first case and 5 *hundredths* in the second. Having thus determined the first figure of the quotient, the figures of the dividend on the right of those in the group first considered can be brought down one at a time and the corresponding figures of the quotient will follow the first in order, just as in the division of whole numbers. Thus

133) 6975·85 (52·45	133) 6·97585 (·05245
665	665
325	325
266	266
598	598
532	532
665	665
665	665

From the above it will be seen that **the first significant figure of the quotient is of the same order as the last on the right of those figures of the dividend which are used in obtaining it.**

Ex. 2. Divide ·019 by 125.

In this case we must add a nought on the right of the dividend in order to obtain a group which is not less than the divisor [see Art. 121]. Thus to obtain the first figure of the quotient we divide 190 *ten-thousandths* by 125, so that the first figure of the quotient is 1 *ten-thousandth*. We then continue, as in Example 1, adding as many noughts on the right of the dividend as may be necessary.

$$
\begin{array}{r|l}
125 & \cdot 019000\,(\cdot 000152 \\
 & \underline{125} \\
 & \quad 650 \\
 & \underline{625} \\
 & \quad 250 \\
 & \quad 250
\end{array}
$$

Ex. 3. Divide 31·57 by 8.

As the divisor is not greater than 12, we proceed by Short Division, as under:

$$
\begin{array}{r|l}
8 & 31\cdot 57000 \\
\hline
 & 3\cdot 94625
\end{array}
$$

After finding the quotient to the same order as the last significant figure of the dividend, there will be as remainder 5 *hundredths*. We then reduce this remainder to the next lower order, namely *thousandths*, and proceed with the division until there is no remainder, or until the quotient is found to the requisite degree of accuracy.

EXAMPLES. VI. F.

Divide

1. 16·4 by 2.	**2.** 32·7 by 3.	**3.** 17·2 by 4.
4. ·156 by 6.	**5.** ·288 by 9.	**6.** ·135 by 9.
7. 125·6 by 20.	**8.** 31·83 by 30.	**9.** 11·7215 by 50.
10. 215·4 by 80.	**11.** ·0321 by 60.	**12.** ·174 by 120.
13. 5·22 by 48.	**14.** ·171 by 72.	**15.** ·012 by 1600.
16. ·027 by 45.	**17.** 2·355 by 75.	**18.** 2·715 by 48.
19. 52·7 by 17.	**20.** 43·7 by 23.	**21.** 166·6 by 119.

22. 3751·5 by 123.
23. 3·7515 by 1230.
24. 375·15 by 125.
25. 37·515 by 1250.
26. 67·77 by 135.
27. ·006777 by 1350.
28. 1·036656 by 207.
29. ·1036656 by 5008.
30. ·001036656 by 2070.
31. ·651714 by 3156.
32. ·0651714 by 315600.
33. 2·94596405 by 13579.
34. ·0722646078 by 56294000.
35. ·028207026232 by 3109720.

Find, to 4 places of decimals

36. 12·15 ÷ 148.	**37.** 2·374 ÷ 156.	**38.** 41·75 ÷ 89.
39. 135 ÷ 17.	**40.** 17 ÷ 135.	**41.** 121 ÷ 170.

42. Simplify ·026 × ·0493 ÷ 221.

43. Simplify 35·4775 ÷ 115 × 2·3.

129. CASE II. *To divide any number by a decimal.*

This case is reduced to the preceding by making use of the following principle :—

The quotient obtained by dividing one number by any other is unaltered by multiplying both dividend and divisor by the same number.

The principle follows at once from the fact that the factors of a product can be taken in any order. [Art. 43 and 108.] An example will shew the method of proof.

Since $24 = 6 \times 4$; $24 \times 8 = 6 \times 4 \times 8 = 6 \times 8 \times 4$;

$$\therefore (24 \times 8) \div (6 \times 8) = 4 = 24 \div 6.$$

The way in which the principle is applied will be seen from the following example:

Ex. 1. Divide 69·7585 by 1·33.

The quotient is the same as the quotient obtained when both dividend and divisor are multiplied by 100; and [Art. 123] we should then have to divide 6975·85 by 133, which comes under Case I.

Hence to divide any number by a decimal we first **multiply both the dividend and divisor by a power of 10 which will make the divisor a whole number, and then proceed as in the case of division by a whole number.**

Ex. 2. Divide 87·69 by ·237.

To make ·237 a whole number its decimal point must be moved *three* places to the right; and in order that the quotient may be unaltered, the dividend must be multiplied by the same power of 10, and therefore the decimal point in the dividend must also be moved *three* places to the right, and it will then become 87690. Thus we shall find the quotient required by dividing 87690 by 237.

Ex. 3. Divide ·21 by ·0125.

First move the decimal points in the divisor and dividend 4 places to the right; they will then become 125 and 2100. Then, as in Case I,

$$\begin{array}{r} 125\,)\,2100{\cdot}0\,(\,16{\cdot}8 \\ 125 \\ \hline 850 \\ 750 \\ \hline 1000 \\ 1000 \end{array}$$

130. Note. It should be noticed that although the *quotient* is unaltered by multiplying both dividend and divisor by the same number, the *remainder*, if any, is not unaltered, but is equal to the original remainder multiplied by the number by which the original divisor and dividend were multiplied.

For example $27 \div 6$ gives quotient 4 and remainder 3; but

$(27 \times 8) \div (6 \times 8)$ gives quotient 4 and remainder 3×8.

Example. How many pieces each 1·02 inches long can be cut from a rod whose length is 18 inches?

We can find the *quotient* by dividing 1800 by 102. Thus

$$\begin{array}{r} 102\,)\,1800\,(\,17 \\ 102 \\ \hline 780 \\ 714 \\ \hline 66 \end{array}$$

Hence there are 17 pieces; and since the original divisor and dividend were multiplied by 100, the remainder left over is

$(66 \div 100)$ inches $= \cdot 66$ inches.

EXAMPLES. VI. G.

Divide

1.	6·2 by ·01.	**2.**	·347 by ·001.	**3.**	12·3 by ·0001.
4.	3·5 by ·5.	**5.**	·75 by ·05.	**6.**	1·25 by ·005.
7.	62·5 by 2·5.	**8.**	·625 by ·025.	**9.**	625 by ·0025.
10.	1·1 by ·125.	**11.**	·019 by 1·25.	**12.**	170 by ·00125.
13.	1·5 by 2·4.	**14.**	5·76 by 4·8.	**15.**	8·1 by ·36.
16.	159·1 by 3·7.	**17.**	6·016 by ·94.	**18.**	70·992 by 8·7.

19. ·435435 by ·0065. **20.** 430·686 by ·0071.

21. 415·242 by ·0059. **22.** ·185745 by 3·05.

23. 4·07514744 by ·006724. **24.** ·9570744 by 159·3.

25. Find to 4 places of decimals $43 \cdot 21 \div 123 \cdot 4$, and $\cdot 0167 \div 3 \cdot 17$.

26. Simplify $360 \div 7 \cdot 2 \div \cdot 16$. **27.** Simplify $\cdot 0441 \div \cdot 21 \div \cdot 56$.

28. Simplify $1 \cdot 953 \div 8 \cdot 68 \times \cdot 035$.

29. How many lengths each 2·56 inches are there in a rod 10 feet long; and how much is left over?

30. How many packets of tea each containing 1·85 oz. can be made up out of a chest containing 150 lbs.; and how much is left over?

131. Reduction of vulgar fractions to decimals.

Ex. 1. Express $\frac{4}{25}$ as a decimal.

Since $\frac{4}{25}$ may be considered as the quotient obtained by dividing 4 by 25 [Art. 107], we have only to perform this division. Thus

$$\begin{array}{r} 25\,)\,4{\cdot}00\,(\,{\cdot}16 \\ \underline{25} \\ 150 \\ \underline{150} \end{array}$$

Ex. 2. Reduce, to 3 places of decimals, the vulgar fractions $\frac{3}{4}$, $\frac{41}{48}$ and $\frac{6}{7}$; and thus shew that the fractions are in ascending order of magnitude.

The decimals required are ·75, ·854... and ·857...

EXAMPLES. VI. H.

Reduce the following vulgar fractions to decimals:

1. $\frac{1}{2}$.	2. $\frac{1}{4}$.	3. $\frac{1}{8}$.	4. $\frac{1}{16}$.
5. $\frac{3}{4}$.	6. $\frac{3}{8}$.	7. $\frac{5}{16}$.	8. $\frac{9}{16}$.
9. $\frac{2}{5}$.	10. $\frac{11}{25}$.	11. $\frac{113}{125}$.	12. $\frac{3}{125}$.
13. $\frac{11}{40}$.	14. $\frac{21}{320}$.	15. $\frac{16}{125}$.	16. $\frac{11}{625}$.
17. $\frac{27}{512}$.	18. $3\frac{13}{5120}$.	19. $7\frac{113}{1024}$.	20. $13\frac{129}{312500}$.

Circulating Decimals.

132. We have hitherto considered examples of division of decimals in which by proceeding far enough an exact quotient is found with no remainder. This, however, is by no means always the case; in fact it is very rarely the case. [See Art. 136.]

Consider, for example, the division of 5 by 3.

$$\begin{array}{r|l} 3 & 5{\cdot}0000 \\ \hline & 1{\cdot}6666\ldots \end{array}.$$

We may here continue the process of division to any extent, but each figure of the quotient will be 6 and the remainder will always be 2.

Again, if we divide 2 by 7.

$$7\,|\,2\cdot000000000\ldots$$
$$\cdot285714285\ldots$$

Here the six digits 2, 8, 5, 7, 1, 4 come over and over again in the same order, and we shall never arrive at a stage at which there is no remainder.

When a decimal ends with digits which are repeated over and over again without end in the same order the decimal is called a **Recurring** or **Circulating** decimal, and the digit, or set of digits, which is repeated, is called the **circulating period.**

Thus 2·45555..., ·014141414..., and 5·1246246246... are circulating decimals with circulating periods of one, two and three figures respectively.

A circulating period is denoted by placing dots over the first and last of the figures which recur.

Thus $2\cdot4\dot{5}$ denotes 2·45555..., $\cdot0\dot{1}\dot{4}$ denotes ·014141414..., and $5\cdot1\dot{2}4\dot{6}$ denotes 5·1246246246...

A circulating decimal is said to be **pure** or **mixed** according as all the figures *after the decimal point* do or do not recur.

Thus $5\cdot\dot{6}$, $31\cdot\dot{2}\dot{4}$ and $14\cdot\dot{1}3\dot{5}$ are pure circulating decimals; and $\cdot5\dot{6}$, $3\cdot1\dot{2}\dot{4}$ and $\cdot14\dot{1}3\dot{5}$ are mixed circulating decimals.

A decimal which contains a definite number of figures is called a **terminating** decimal, to distinguish it from a circulating decimal, which contains an unlimited number of figures.

Note. Although it is not possible to reduce *any* vulgar fraction to a terminating decimal, it is always possible to find a decimal which is equal to the vulgar fraction *to any degree of accuracy that may be required.*

For example, $\frac{1}{3}$ lies between ·333 and ·334, so that the difference between $\frac{1}{3}$ and ·333 is less than *one-thousandth*, so also the difference btween $\frac{1}{3}$ and ·333333 is less than *one-millionth;* and so on.

Now there is no species of magnitude which can be measured with *perfect* accuracy. It would, for instance, be difficult to determine the length or the weight of a body without a possible error as great as *one-thousandth* of the whole. Hence the measure of any quantity can be expressed as accurately by means of decimals as by means of fractions.

EXAMPLES. VI. I.

Express the following quotients as circulating decimals:

1.	$1{\cdot}5 \div 2{\cdot}7$.	2.	$10 \div {\cdot}03$.	3.	$1{\cdot}7 \div {\cdot}09$.
4.	${\cdot}035 \div {\cdot}072$.	5.	${\cdot}316 \div 2{\cdot}4$.	6.	${\cdot}312 \div 8{\cdot}8$.
7.	$3{\cdot}1 \div 7$.	8.	$15{\cdot}6 \div {\cdot}07$.	9.	$1{\cdot}25 \div 13{\cdot}2$.
10.	$5{\cdot}193 \div {\cdot}0168$.	11.	${\cdot}0235 \div {\cdot}00616$.	12.	$16{\cdot}72 \div {\cdot}0143$.
13.	${\cdot}3157 \div {\cdot}259$.	14.	$27{\cdot}31 \div 6{\cdot}475$.	15.	$693{\cdot}11 \div {\cdot}011396$.

Reduce the following vulgar fractions to circulating decimals:

16.	$\frac{1}{3}$.	17.	$\frac{1}{7}$.	18.	$\frac{5}{9}$.	19.	$\frac{7}{11}$.
20.	$\frac{6}{13}$.	21.	$\frac{11}{14}$.	22.	$\frac{8}{15}$.	23.	$\frac{5}{17}$.
24.	$2\frac{7}{22}$.	25.	$3\frac{11}{21}$.	26.	$5\frac{15}{28}$.	27.	$7\frac{8}{31}$.
28.	$11\frac{3}{56}$.	29.	$13\frac{19}{168}$.	30.	$2\frac{615}{616}$.	31.	$5\frac{71}{518}$.

133. Reduction of a circulating decimal to an equivalent vulgar fraction. We have seen [Art. 122] that a terminating decimal can be at once expressed as a vulgar fraction. We have now to shew how to find a vulgar fraction equivalent to any circulating decimal.

CASE I. *To find a vulgar fraction equivalent to a pure circulating decimal.*

Consider the decimals (i) $\cdot\dot{3}\dot{1}$ and (ii) $\cdot\dot{5}21\dot{6}$.

(i) $\cdot\dot{3}\dot{1} = \cdot3131313131\ldots$

Multiply by that power of 10 which will move the point to the end of the first period, namely by 100. Then we have

$$\cdot\dot{3}\dot{1} \times 100 = 31{\cdot}31313131\ldots$$

Hence, by subtraction,

$$\cdot31 \times (100 - 1) = 31;$$

$$\therefore \cdot\dot{3}\dot{1} = \frac{31}{100 - 1} = \frac{31}{99}.$$

(ii) Since $\cdot\dot{5}21\dot{6} = \cdot 521652165216\ldots,$

$\cdot\dot{5}21\dot{6} \times 10000 = 5216\cdot52165216\ldots.$

Hence, by subtraction,

$$\cdot\dot{5}21\dot{6} \times (10000 - 1) = 5216.$$

Therefore $$\cdot\dot{5}21\dot{6} = \frac{5216}{10000 - 1} = \frac{5216}{9999}.$$

From the above examples it will be seen that **a pure recurring decimal is equivalent to a vulgar fraction whose numerator is the number formed by the recurring digits and whose denominator is made up of as many nines as there are digits in the recurring period.**

Thus we can write down at once

$$\cdot\dot{5} = \frac{5}{9}, \quad \cdot\dot{0}\dot{7} = \frac{07}{99} = \frac{7}{99}, \quad \cdot\dot{6}8\dot{4} = \frac{684}{999} = \frac{76}{111}.$$

It should be noticed that by the above rule $\cdot\dot{9} = \frac{9}{9} = 1$. This result can be seen independently; for the differences between 1 and the decimals ·9, ·99, ·999, &c. are respectively ·1, ·01, ·001, &c., each difference being *one-tenth* of the preceding, and therefore when a large number of nines is taken the difference between 1 and ·99999... becomes inconceivably small.

Since $\cdot\dot{9} = 1$, $\cdot 0\dot{9} = \cdot 1$, $\cdot 00\dot{9} = \cdot 01$, and so on. Thus 9 recurring can always be replaced by 1 in the next place to the left; for example $\cdot 7\dot{9} = \cdot 8$ and $\cdot 24\dot{9} = \cdot 25$.

CASE II. *To find a vulgar fraction equivalent to a mixed circulating decimal.*

Consider the decimals (i) $\cdot 3\dot{1}\dot{4}$ and (ii) $\cdot 15\dot{6}\dot{0}\dot{7}$.

(i) $\cdot 3\dot{1}\dot{4} = \cdot 314141414\ldots$

Multiply by that power of 10 which will move the decimal point first just before the first figure of the recurring period, and secondly just after the last figure of the period. Thus

$$\cdot 3\dot{1}\dot{4} \times 10 = 3\cdot 14141414\ldots,$$

and $$\cdot 3\dot{1}\dot{4} \times 1000 = 314\cdot 141414\ldots.$$

Hence, by subtraction,

$$\cdot 3\dot{1}\dot{4} \times (1000 - 10) = 314 - 3;$$

$$\therefore \cdot 3\dot{1}\dot{4} = \frac{314 - 3}{1000 - 10} = \frac{314 - 3}{990}.$$

(ii) Since $\cdot 15\dot{6}0\dot{7} = \cdot 15607607607\ldots,$

$\cdot 15\dot{6}0\dot{7} \times 100 = 15\cdot 607607607\ldots,$

and $\cdot 15\dot{6}0\dot{7} \times 100000 = 15607\cdot 607607\ldots.$

Hence, by subtraction,

$$\cdot 15\dot{6}0\dot{7} \times (100000 - 100) = 15607 - 15;$$

$$\therefore \cdot 15\dot{6}0\dot{7} = \frac{15607 - 15}{100000 - 100} = \frac{15607 - 15}{99900}.$$

From the above examples it will be seen that **a mixed recurring decimal is equivalent to a vulgar fraction whose numerator is the difference between the number formed by the digits down to the end of the first period and the number formed by the non-recurring digits, and whose denominator is made up of as many nines as there are recurring digits followed by as many noughts as there are non-recurring digits after the decimal point.**

Thus we can write down at once

$$\cdot 1\dot{5}\dot{6} = \frac{156 - 1}{990} = \frac{155}{990} = \frac{31}{198},$$

$$\cdot 130\dot{8}\dot{1} = \frac{13081 - 13}{99900} = \frac{13068}{99900} = \frac{121}{925}.$$

Also $3\cdot 3\dot{1}\dot{2} = 3\frac{312 - 3}{990} = 3\frac{309}{990} = 3\frac{103}{330};$

if, however, we wish to express the last fraction as an improper fraction we proceed

$$3\cdot 3\dot{1}\dot{2} = \frac{3312 - 33}{990} = \frac{3279}{990} = \frac{1093}{330}.$$

EXAMPLES. VI. J.

Find vulgar fractions in their lowest terms equivalent to the following circulating decimals:

1.	$\cdot\dot{3}$.	2.	$\cdot 0\dot{9}$.	3.	$\cdot\dot{2}\dot{7}$.	4.	$\cdot\dot{1}\dot{5}$.
5.	$\cdot 0\dot{2}\dot{7}$.	6.	$\cdot 0\dot{3}\dot{7}$.	7.	$\cdot\dot{1}8\dot{5}$.	8.	$\cdot\dot{3}9\dot{6}$.
9.	$\cdot\dot{1}42857\dot{1}$.	10.	$\cdot\dot{2}85714\dot{2}$.	11.	$\cdot\dot{4}2857\dot{1}\dot{4}$.	12.	$\cdot 0\dot{1}2987\dot{5}$.
13.	$\cdot 0\dot{4}87\dot{8}$.	14.	$\cdot 0\dot{7}31\dot{7}$.	15.	$\cdot\dot{2}\dot{3}$.	16.	$\cdot 7\dot{9}$.
17.	$\cdot 3\dot{6}$.	18.	$\cdot 3\dot{1}\dot{5}$.	19.	$\cdot 11\dot{6}$.	20.	$\cdot 025\dot{4}$.
21.	$\cdot 01\dot{6}$.	22.	$\cdot 7\dot{4}\dot{9}$.	23.	$\cdot 20\dot{2}\dot{7}$.	24.	$\cdot 193\dot{2}\dot{4}$.
25.	$\cdot 40\dot{2}43\dot{9}$.	26.	$\cdot 3\dot{0}487\dot{8}$.	27.	$\cdot 30\dot{2}1976$.	28.	$\cdot 5\dot{4}2857\dot{1}$.
29.	$\cdot 0\dot{1}2345679$.			30.	$\cdot\dot{1}35802469$.		

134. Addition and subtraction of circulating decimals.

To add circulating decimals they are placed so that the decimal points are in a vertical column, which ensures that units of the same order are throughout in vertical columns. The circulating periods are then repeated as often as may be necessary to shew the recurrence in the sum.

Ex. 1. Add together $\cdot 12\dot{3}$ and $\cdot 0\dot{5}\dot{6}$.

$$\begin{array}{r} \cdot 12\dot{3}3333\ldots \\ \cdot 0\dot{5}\dot{6}5656\ldots \\ \hline \cdot 17\dot{9}\dot{8}989\ldots \end{array}$$

Here it is obvious that the third and fourth columns are repeated, the corresponding figures in the sum must therefore be repeated.

Ex. 2. Find the sum of $7\cdot 31\dot{5}$, $19\cdot \dot{7}\dot{8}$ and $2\cdot 0\dot{4}9\dot{6}$.

$$\begin{array}{r|l} 7\cdot 31\dot{5}5555\dot{5} & 55\ldots \\ 19\cdot \dot{7}\dot{8}787878 & 78\ldots \\ 2\cdot 0\dot{4}9\dot{6}4964 & 96\ldots \\ \hline 29\cdot 15\dot{3}0839\dot{9} & \end{array}$$

In this case the first column in which all the figures belong to recurring periods is the third column down from the decimal point; and this column, and all succeeding ones, must be repeated after an interval equal to the **L. C. M. of the number of figures in the different recurring periods** of the decimals whose sum is required. Hence in the sum 6 figures [since 6 is the L. C. M. of 1, 2, 3] will recur, beginning at the third down. It is necessary to write one or two columns beyond the vertical line in order to determine what must be carried, namely 2 in the present instance: the addition of the figures in these columns should not, however, be written down.

Note. From the above example it will be seen that the number of recurring figures in the sum of several recurring decimals cannot be *greater* [it is, however, sometimes less] than the L. C. M. of the numbers in the circulating periods of the decimals to be added.

Ex. 3. Find the difference between $\cdot 3\dot{1}\dot{5}$ and $\cdot 24\dot{1}6\dot{3}$.

$$\begin{array}{r|l} \cdot 3\dot{1}\dot{5}15151 & 51\ldots \\ \cdot 24\dot{1}6\dot{3}163 & 16\ldots \\ \hline \cdot 07\dot{3}51988 & \end{array}$$

The first column which contains only figures which recur is the *third* down, and there are 2 and 3 figures respectively in the recurring periods of the given decimals. Hence there are 6 [the L. C. M. of 2 and 3] recurring figures in the difference, the recurrence beginning at the third place down.

EXAMPLES VI. K.

Simplify

1. $\cdot 3\dot{6} + \cdot 6\dot{3}$.
2. $14\cdot\dot{6}\dot{2} + 11\cdot 1\dot{7}\dot{3}$.
3. $\cdot 2\dot{1}\dot{5} + \cdot 0\dot{1}\dot{7}$.
4. $1\cdot 0\dot{1}\dot{8} + 21\cdot 5\dot{6}$.
5. $1\cdot 3\dot{5}\dot{7} + 2\cdot 3\dot{4} + \cdot 6\dot{1}\dot{6} + 1\cdot 4\dot{5}\dot{8}$.
6. $5\cdot 3\dot{0}\dot{7} + 10\cdot 0\dot{1}\dot{5} + 1\cdot 2\dot{9}\dot{8} + 7\cdot 0\dot{4}$.
7. $\cdot 0\dot{2} + \cdot 0\dot{3} + \cdot 3\dot{0}\dot{3}$.
8. $2\cdot\dot{3}\dot{2}\dot{4} + 3\cdot\dot{0}\dot{0}\dot{9} + 5\cdot 2\dot{1}\dot{8} + 4\cdot 2\dot{8}\dot{1}$.
9. $1\cdot 3\dot{1}\dot{2} + \cdot 0\dot{1}\dot{6}\dot{3} + 13\cdot\dot{0}\dot{5}\dot{7}\dot{4} + 2\cdot 3\dot{1}\dot{3}\dot{6}\dot{8}$.
10. $101\cdot 4\dot{1}\dot{6}\dot{3} + 2\cdot 1\dot{1}\dot{9} + 13\cdot 4\dot{1}\dot{5}\dot{2}\dot{3} + \cdot\dot{1}\dot{5}$.
11. $\cdot 2\dot{1}\dot{6}\dot{3} + \cdot 0\dot{1}\dot{2} + 1\cdot 4\dot{1}\dot{7}\dot{3} + 16\cdot 0\dot{0}\dot{5}\dot{2}$.
12. $3\cdot 6\dot{1}42857 + 1\cdot 00\dot{7}14285 + 9\cdot\dot{4}28571$.
13. $11\cdot 3\dot{1}\dot{5} + 3\cdot 0\dot{2}\dot{1}\dot{3} + 17\cdot\dot{1}63527$.
14. $2\cdot 1\dot{3} - 1\cdot 7\dot{2}\dot{4}$.
15. $13\cdot 3\dot{1}5\dot{4} - 11\cdot\dot{8}9\dot{6}$.
16. $1\cdot 6 - \cdot 21\dot{7}\dot{3} - \cdot 5\dot{6}\dot{4}$.
17. $2\cdot\dot{1}74\dot{5} - 1\cdot 72\dot{3}4\dot{5}$.
18. $3\cdot 409\dot{1} + 2\cdot 7\dot{1}9\dot{2} - 4\cdot\dot{3}$.
19. $12\cdot 3\dot{1}\dot{2} + 1\cdot 0\dot{1}\dot{5}\dot{3} - 6\cdot\dot{2}\dot{1}\dot{4} - 4\cdot 1\dot{2}\dot{0}\dot{3}$.

135. Multiplication and division of circulating decimals.

CASE I. *To multiply or divide a circulating decimal by a whole number or by a terminating decimal.*

The method will be seen from the following examples:

Ex. 1. Multiply $2\cdot 5\dot{3}\dot{4}$ by 6.

$2\cdot 5\dot{3}\dot{4} \mid 3434\ldots$
$\quad 6$
$15\cdot 2\dot{0}\dot{6}$

Multiply as in ordinary multiplication, but repeat the period in order to see what to carry.

Ex. 2. Multiply $2\cdot 5\dot{3}\dot{4}$ by 276.

$2\cdot 5\dot{3}\dot{4} \mid 3434\ldots$
$\quad 276$
$15\cdot 2\dot{0}\dot{6} \mid 06$
$177\cdot\dot{4}0\dot{4} \mid 04$
$506\cdot\dot{8}6\dot{8} \mid 68$
$699\cdot\dot{4}\dot{7}\dot{8}$

Multiply by each digit in succession as in Ex. 1, and repeat the recurring period in each line as often as may be necessary to ensure the correct sum of the partial products.

Ex. 3. Divide $21\cdot 71\dot{5}\dot{2}$ by 3.

$3 \mid 21\cdot 715252525252\ldots$
$7\cdot 23\dot{8}417\dot{5}0\dot{8}417\ldots$

The process is the same as in the division of a terminating decimal, except that the figures of the circulating period are repeated as often as may be necessary to shew the recurrence in the quotient.

Ex. 4. Divide $699{\cdot}4\dot{7}\dot{8}$ by 276.

$$
\begin{array}{r|l}
276 & 699{\cdot}4\dot{7}\dot{8}78\ldots\ (2{\cdot}5\dot{3}\dot{4} \\
 & \underline{552} \\
 & 1474 \\
 & \underline{1380} \\
 & 947 \\
 & \underline{828} \\
 & 1198 \\
 & \underline{1104} \\
 & 94
\end{array}
$$

After finding the fourth figure of the quotient, we notice that the remainder is the same as the remainder after the second figure of the quotient, and the figures of the dividend are also the same in order at the two stages. Hence the figures 3, 4 of the quotient must recur.

EXAMPLES VI. L.

Perform the operations indicated:

1.	$2{\cdot}1\dot{6}\times 2$.	**2.**	$3{\cdot}2\dot{5}\times 6$.	**3.**	$\cdot 12\dot{3}\times 3$	**4.**	$15{\cdot}\dot{7}\times 8$.
5.	$1{\cdot}7\dot{8}\times 8$.	**6.**	$\cdot 021\dot{7}\times 9$.	**7.**	$\cdot 0\dot{4}\dot{5}\times 9$.	**8.**	$30{\cdot}\dot{1}\dot{7}\dot{3}\times 6$.
9.	$2{\cdot}1\dot{3}\dot{5}\dot{7}\times 8$.	**10.**	$3{\cdot}1\dot{8}\dot{5}\times 75$.	**11.**	$\cdot 12\dot{3}\dot{4}\times 156$.		
12.	$15{\cdot}0\dot{9}\dot{7}\times\cdot 0274$.	**13.**	$1{\cdot}0\dot{8}2\dot{7}\times\cdot 0153$.	**14.**	$2{\cdot}1\dot{5}\dot{3}\dot{5}\times 1{\cdot}234$.		
15.	$2{\cdot}1\dot{5}\div 2$.	**16.**	$\cdot 05\dot{6}\div 3$.	**17.**	$70{\cdot}0\dot{8}\div 5$.		
18.	$\cdot 4\dot{0}\dot{9}\div 9$.	**19.**	$\cdot 2\dot{1}\dot{6}\dot{9}\div 4$.	**20.**	$150{\cdot}\dot{3}\dot{4}\div 12$.		
21.	$23{\cdot}8\dot{9}\dot{3}\div 7{\cdot}5$.	**22.**	$1{\cdot}66\dot{4}\dot{8}\div\cdot 123$.	**23.**	$201{\cdot}3\dot{9}\dot{7}\dot{2}\div\cdot 0421$.		
24.	$1{\cdot}9255\dot{7}\div\cdot 0156$.						

CASE II. *To multiply or divide one circulating decimal by another.*

When the multiplier or the divisor is a circulating decimal we must **reduce to equivalent vulgar fractions.**

Ex. 1. Multiply $\cdot 3\dot{6}$ by $\cdot 01\dot{3}$.

$$\cdot 3\dot{6}\times\cdot 01\dot{3}=\frac{36-3}{90}\times\frac{13-1}{900}=\frac{33}{90}\times\frac{12}{900}=\frac{44}{9000}=\cdot 00\dot{4}\dot{8}.$$

Ex. 2. Simplify $(1{\cdot}0\dot{5}+\cdot 1\dot{6}\dot{7})\div\cdot 4\dot{0}77441\dot{1}$.

$$
\begin{array}{r|l}
1{\cdot}0\dot{5}\dot{5} & 55\ldots \\
\cdot 1\dot{6}\dot{7} & 67\ldots \\
\hline
1{\cdot}2\dot{2}\dot{3} &
\end{array}
$$

Then $1{\cdot}2\dot{2}\dot{3}\div\cdot 4\dot{0}77441\dot{1}=\dfrac{1223-12}{990}\div\dfrac{4077441-4}{9999990}$

$$=\frac{\cancel{1211}}{\cancel{990}}\times\frac{\cancel{10101}\ 3}{\cancel{9999990}\ \cancel{4077437}\ \cancel{3367}}=3.$$

EXAMPLES VI. M.

Simplify, expressing the results in a decimal form :

1. $\dot{3}\times\dot{3}$. **2.** $1{\cdot}\dot{2}\times{\cdot}1\dot{6}$. **3.** ${\cdot}12\dot{3}\times{\cdot}52\dot{6}$. **4.** ${\cdot}\dot{5}\times{\cdot}\dot{5}$.

5. $9{\cdot}1\dot{6}\times19{\cdot}\dot{6}\dot{3}$. **6.** ${\cdot}2\dot{1}\dot{1}\times1{\cdot}0\dot{8}$. **7.** ${\cdot}\dot{2}85714\times{\cdot}11\dot{6}$.

8. ${\cdot}\dot{3}7\dot{0}\div{\cdot}\dot{5}$. **9.** ${\cdot}\dot{1}23\div{\cdot}\dot{4}$. **10.** $4{\cdot}\dot{5}\dot{4}\div2{\cdot}\dot{2}\dot{7}$.

11. ${\cdot}2\dot{7}\div75{\cdot}\dot{7}\dot{5}$. **12.** ${\cdot}37\dot{5}9\dot{2}\div{\cdot}0\dot{5}$.

13. $3{\cdot}\dot{7}14285\div5{\cdot}0\dot{1}42857$. **14.** $(2{\cdot}4+3{\cdot}5-5{\cdot}1\dot{2}9\dot{7})\div{\cdot}7\dot{2}9$.

15. $(1{\cdot}0\dot{5}+{\cdot}\dot{1}6\dot{7})\div{\cdot}0\dot{1}710\dot{1}$. **16.** $({\cdot}0\dot{6}+1{\cdot}45\dot{7})\div{\cdot}5\dot{0}7744\dot{1}$.

17. $\dfrac{5{\cdot}6\times1{\cdot}13\dot{6}}{31{\cdot}\dot{8}\dot{1}}$. **18.** $\dfrac{1{\cdot}\dot{1}8\dot{5}-{\cdot}6\dot{1}}{19{\cdot}\dot{5}-12{\cdot}\dot{8}}\times\dfrac{7}{31}$.

19. $\dfrac{6{\cdot}60416\dot{}}{33{\cdot}02083\dot{}}\div\dfrac{2{\cdot}77\dot{2}}{11{\cdot}\dot{0}\dot{9}}$. **20.** $\dfrac{1{\cdot}\dot{1}8\dot{5}-{\cdot}6\dot{1}}{19{\cdot}\dot{5}-12{\cdot}\dot{8}}\times\dfrac{241}{213}\div\dfrac{1{\cdot}55\times{\cdot}0\dot{3}\dot{7}}{4{\cdot}\dot{3}0}$.

136. In the operation by which a vulgar fraction is reduced to a decimal, the only remainders that can occur are 0, 1, 2, 3, &c...., up to one less than the denominator of the fraction. Hence, unless there is no remainder at some stage, in which case the fraction is equivalent to a terminating decimal, the remainders must come over and over again if we proceed far enough; but when any remainder occurs a second time, there must be a recurrence both of the remainders and of the corresponding figures of the quotient. Thus every fraction is equivalent either to a terminating or to a recurring decimal, and the number of figures in the recurring period cannot be greater than the number of the different possible remainders, which is one less than the denominator of the fraction.

For example, the number of figures in the circulating periods of the decimals equivalent to

$$\frac{1}{3},\ \frac{1}{7},\ \frac{1}{11},\ \frac{1}{13},\ \frac{1}{17},\ \frac{1}{37},\ \frac{1}{41}\ \text{respectively,}$$

cannot be *greater* than 2, 6, 10, 12, 16, 36 and 40 respectively. The actual number will be found to be 1, 6, 2, 6, 16, 3 and 5 respectively.

It should be noticed that *if a vulgar fraction in its lowest terms be equivalent to a terminating decimal, the denominator of the fraction can only contain the prime factors* 2 *and* 5.

For any fraction which is equal to a given fraction in its lowest terms must be obtained by multiplying the numerator and denominator of the given fraction by the same number. If then a fraction in its lowest terms is equivalent to a terminating decimal, some multiple of its denominator must be a power of 10, and this is impossible if the denominator contains any prime factor except 2 and 5, for these are the only prime factors of 10, or of any power of 10.

Recurring decimals which are equivalent to vulgar fractions have many interesting properties, which however require a knowledge of Algebra for their investigation*.

Decimals and Concrete Quantities.

137. The way in which one given quantity can be expressed as a decimal of another will be seen from the following examples:

Ex. 1. Express £1. 12*s*. $6\frac{3}{4}d$. as a decimal of £5.

Since £1. 12*s*. $6\frac{3}{4}d.$ = 1563 f., and £5 = 4800 f., the fraction is $\frac{1563}{4800}$, which will be found to be ·325625.

Or thus, Explanation: First $6\frac{3}{4}d. = 6{\cdot}75d.$

Divide by 12 to reduce to shillings, and add in the 12*s*. given, making 12·5625*s*.

12 | 6·75*d*.
20 | 12·5625*s*.
5 | 1·628125£.
·325625

Divide by 20 (to do this *divide by* 2 *and put the figures one place back*) to reduce to pounds and add in the £1, making £1·628125.

Now divide the pounds by 5, and we shall have the whole expressed in terms of £5 as unit.

* See C. Smith's *Treatise on Algebra*, Chapter XXVIII.

138. The way in which the value of a given decimal of a given concrete quantity can be found will be seen from the following examples:

Ex. 1. Find the value of £1·628125.

£**1**·628125
20
s. **12**·562500
12
d. **6**·7500
4
f. **3**·00

Explanation. Multiply the *fractional* part of the pounds by 20, to reduce to shillings; we thus get 12·5625*s.* Now multiply the fractional part of the shillings by 12, to reduce to pence; we thus get

$$6{\cdot}75d. = 6\tfrac{3}{4}d.$$

Hence the required value is

£1. 12*s.* $6\frac{3}{4}d.$

Ex. 2. Find the value of 3·275 of 1 cwt. 2 qrs. 12 lbs.

1 cwt. 2 qrs. 12 lbs. = 180 lbs.

Then 180 lbs. × 3·275 = 589·5 lbs.,

which is reduced to 5 cwt. 1 qr. 1 lb. 8 oz.

Ex. 3. Express ·00625 of 6*s.* 8*d.* + ·875 of 1*s.* 10*d.* + 3·16 of 9*s.* $10\frac{3}{4}d.$ as a decimal of £1. 6*s.* 4*d.*

·00625 of 6*s.* 8*d.* = ·00625 of 80*d.* = ·5*d.*

·875 of 1*s.* 10*d.* = ·875 of 22*d.* = 19·25*d.*

3·16 of 9*s.* $10\frac{3}{4}d.$ = 3·16 of 475*f.* = 1501*f.* = 375·25*d.*

Hence required sum = 395*d.*

Also £1. 6*s.* 4*s.* = 316*d.*;

and $\frac{395}{316} = \frac{5}{4} = 1{\cdot}25.$

EXAMPLES VI. N.

1. Express 1*d.*, 2*d.*, 3*d.*, 4*d.*, 5*d.* and 6*d.* as decimals of 1 shilling.

2. Express $1\frac{1}{2}d.$, $2\frac{1}{2}d.$, $2\frac{1}{4}d.$, $7\frac{1}{2}d.$ and $10\frac{1}{2}d.$ as decimals of 1 shilling.

3. Express 1*s.*, 2*s.*, 3*s.*, 4*s.*, 5*s.*, 6*s.* as decimals of £1.

4. Express 2*s.* 6*d.*, 1*s.* 3*d.*, 3*s.* 4*d.*, 7*s.* 6*d.*, 11*s.* 8*d.* as decimals of £1.

5. Express £3. 17*s.* $4\frac{1}{2}d.$, £1. 19*s.* $6\frac{3}{4}d.$ and £2. 0*s.* $11\frac{3}{4}d.$ as decimals of £1.

6. Express £2. 15*s.* $0\frac{1}{4}d.$ as a decimal of £10. 17*s.* $2\frac{1}{4}d.$

7. Express £4. 10*s.* $1\frac{1}{4}d.$ as a decimal of £8. 13*s.*

8. What decimal of 13 lbs. 6 oz. 8 dwts. 7 grs. is 11 lbs. 7 oz. 4 dwts. 6 grs.?

9. Express 9 tons 13 cwt. 2 lbs. as a decimal of 2 tons 13 cwt. 2 qrs. 13 lbs.

10. What decimal of a week is 14 hrs. 42 min.?

11. What decimal of an acre is 2 roods 14 poles?

12. What decimal of a lb. Troy is 3 oz. 6 dwts. 9 grs.?

13. What decimal of a day is 5 hrs. 6 min. 18 sec.?

14. Express 2 mi. 3 fur. 15 p. 2 yds. 1 ft. 6 in. in miles and decimals of a mile.

Find the value of

15. £·75. **16.** £·625. **17.** ·375*s*.

18. £·03125. **19.** £·88125. **20.** £·540625.

21. 10·3571428 guineas. **22.** ·46 of ·083 of £·5625.

23. ·2783203125 tons. **24.** 11·2765625 lbs. Troy.

25. 6·227083 days. **26.** ·4482954 miles.

27. Find in money the value of 2·6875 of £5. 10*s*., and of 15·425 of £441.

28. Find the value of 3·342857 of £·38.

29. What decimal of £100 is ·0625 of £495?

30. Find the value of

£·625 − ·75 crowns + 1·25 guineas − 15·375 shillings.

31. Subtract ·0254 of £134. 1*s*. 3*d*. from $\frac{13}{15}$ of £5. 3*s*. $1\frac{1}{2}d$., and express the result as a decimal of £25.

MISCELLANEOUS EXAMPLES IN DECIMALS.

1. Add together 31·057, 156·0083, 2·61759 and ·008347.

2. Subtract the difference between 3·14 and 1·0625 from the sum of 1·00172 and 2·127.

3. Multiply 31·025 by ·032, and ·0625 by ·00125.

4. Divide 7·0175 by 17·5, and 7·5 by ·00625.

5. Find the value of £2·725 + 15·375*s*.

6. By how much does the sum of 3·72 and 10·015 fall short of the sum of 7·216 and 6·52?

7. Express ·035, ·625 and ·12288 as vulgar fractions in their lowest terms.

8. Divide 2722·05 by ·345 and ·0272205 by 34·5.

9. Reduce $\frac{11}{16}$, $\frac{18}{125}$ and $\frac{602}{62500}$ to decimals.

10. Reduce £4. 17*s.* 10½*d.* to the decimal of £5.

---

11. Express as decimals $\frac{43}{100000}$, $\frac{27}{320}$ and $\frac{56}{1024}$.

12. Find the squares of 2·15 and ·0324.

13. Divide 990·6 by ·3048.

14. Simplify $\frac{9\cdot5 - 8\cdot625 + 1\cdot75}{1\cdot26 \div 5\cdot4}$.

15. Find the value of $2\frac{1}{7}$ of 7*s.* $3\frac{1}{2}d.$ + $\frac{2}{7}$ of $\frac{1}{3}$ of 2 guineas + £2·375.

---

16. After taking away ·75 and ·129 of a certain quantity, what decimal of the whole will remain?

17. Shew, by reducing the fractions to decimals, that $\frac{2}{3} + \frac{5}{7}$ is greater than 1·3809.

18. Divide the product of 37·5 and ·1248 by ·005625.

19. Express 2 cwt. 3 qrs. 13 lbs. 2 ozs. as a decimal of a ton.

20. Find the value of $\frac{\cdot321 \times \cdot321 - \cdot179 \times \cdot179}{\cdot321 - \cdot179}$ of £5.

---

21. Find the value of

$$\frac{1}{2\times3} + \frac{1}{2\times3\times4\times5} + \frac{1}{2\times3\times4\times5\times6\times7}$$

correct to 4 places of decimals.

22. Multiply ·02019 by 52·03.

23. Divide ·04312 by ·0044, and 9·0225 by ·225.

24. Add together £4·1625, £·817, £2·3585 and £2·662.

25. Express $\frac{1}{17}$ of £1. 0*s.* $2\frac{1}{4}d.$ + ·875 of 1*s.* 8*d.* as a decimal of 1*s.* 4*d.*

---

26. Subtract £1·740625 from the sum of £3·1375 and £·103125.

27. Express ·03125, ·7936 and ·14336 as vulgar fractions in their lowest terms.

28. Divide ·378 by 262·5 and 37·8 by ·02625.

29. Reduce 1 day 15 hours 54 min. to the decimal of a week.

30. Find the value of $\frac{·562 \times ·562 - ·188 \times ·188}{·562 - ·188}$ of £7. 10*s.* 6*d.*

---

31. Express as decimals $\frac{7}{16}$ of $\frac{3}{5}$ and $25\frac{5}{32}$ of $1\frac{7}{161}$.

32. Divide 2722·05 by ·345 and ·0272205 by 3·45.

33. Simplify $\frac{·035 \div ·00128}{·07 \div ·016}$.

34. Express the difference between ·2$\dot{2}$85714$\dot{2}$ and ·$\dot{4}$2857$\dot{1}$ as a vulgar fraction.

35. Express $\frac{1}{19}$ of £1. 2*s.* $6\frac{3}{4}d.$ + ·125 of 11*s.* 6*d.* as a decimal of 1*s.* 9*d.*

---

36. Express as decimals $\frac{3}{7}$, $\frac{15}{28}$ and $\frac{118}{875}$.

37. Add together 3·3$\dot{1}\dot{5}$, 5·0$\dot{1}\dot{8}$, 3·2$\dot{2}\dot{1}$ and 4·2$\dot{7}\dot{8}$.

38. Multiply 3·1$\dot{7}$ by 2·5 and divide 15·1$\dot{9}$ by 11.

39. Find the value of £·158$\dot{3}$ and of £1·6052083.

40. Reduce $\frac{3}{50}$ of £1. 2*s.* 6*d.* + $\frac{7}{8}$ of $\frac{11}{5}$ of 4*d.* − $\frac{4}{5}$ of $\frac{1}{80}$ of 3*s.* 1*d.* to the decimal of £49. 0*s.* 5*d.*

---

41. Simplify 1·702 × 2·9015 ÷ ·0005803.

42. Divide the product of ·0374 and ·0075 by the difference between ·675 and ·6375.

43. Express $\cdot 48\dot{3}2\dot{4}$ and $\cdot 29\dot{5}\dot{4}$ as vulgar fractions in their lowest terms.

44. Divide $3\cdot\dot{8}\dot{1}$ by $2\cdot 2\dot{2}\dot{7}$.

45. Add £$\cdot 43\dot{2}$ to £$\cdot 23\dot{4}$, and express the result as a fraction of 15*s*.

---

46. Multiply 6·25 by ·00512, and divide ·00125 by 62500.

47. How many pieces each ·17 inches long can be cut from a wire whose length is 21·09 inches, and how long will be the piece left over?

48. Reduce to decimals $\frac{15}{17}$ and $\frac{448}{925}$.

49. Express the difference between $\cdot 5\dot{3}8461\dot{5}$ and $\cdot\dot{0}7692\dot{3}$ as a vulgar fraction in its lowest term.

50. Add together $\cdot 7\dot{3}\dot{2}$ of 12*s*. $4\frac{1}{2}$*d*. and $\cdot\dot{0}2\dot{1}$ of £1. 0*s*. $9\frac{3}{4}$*d*., and express the result as a decimal of £1.

---

51. Divide ·03611 by 7·85 and 596·7 by ·0001887.

52. How many lengths each ·0375 of a foot are contained in 31·729687 feet, and what is the length of the remainder?

53. Reduce $\cdot 1\dot{2}\dot{7}$, $\cdot 00\dot{3}4\dot{5}6\dot{7}$ and $\cdot 3\dot{3}0487\dot{8}$ to vulgar fractions in their lowest terms.

54. Find $1\cdot\dot{0}2197\dot{8}$ of £6. 16*s*. 6*d*.

55. Add together ·5125 of a yard, ·62734 of a pole, and ·018325 of a furlong; subtract the result from ·0049 of a mile, and express the answer in yards.

---

56. Multiply 30·24 by ·0334, and divide 293·46 by ·0438.

57. Find $\cdot 12\dot{5}+4\cdot 16\dot{3}+1\cdot\dot{7}14\dot{3}+2\cdot\dot{5}\dot{4}$.

58. A cistern originally full of water lost ·09375 of its contents by leakage, and then 650 gallons were drawn off, after which it was just half full. How many gallons did it contain at first?

59. Express as a decimal in its simplest form

$$\frac{\cdot 1+\cdot\dot{0}\dot{1}+\cdot\dot{0}0\dot{1}}{\cdot 4\dot{1}} \div \frac{1}{\cdot 037}.$$

60. Find the difference between $\cdot 02\dot{7}$ of a guinea and $\cdot 0291\dot{6}$ of 15 shillings, and express this difference as a decimal of 14*s*. 7*d*.

CHAPTER VII.

Practice.

139. An **aliquot part** of any quantity is a *measure* of that quantity. An aliquot part can therefore be expressed as a fraction of the quantity, the numerator of the fraction being 1 and the denominator an integer.

Thus 10*s*., 5*s*. and 2*s*. 6*d*. are aliquot parts of £1, the fractions being $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ respectively.

140. **Practice** is a method of obtaining results by separating a whole into portions each of which is an easily calculated *aliquot part* of some particular unit or of one of the preceding portions. This separation can generally be effected in a variety of ways, and some experience is required to see the most convenient way.

EXAMPLES. VII. A.

Write down

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{16}$ and $\frac{1}{20}$ of £1.
2. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{15}$ and $\frac{1}{20}$ of 10*s*.
3. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{10}$ of 6*s*. 8*d*.
4. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{10}$ of 5*s*.
5. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{10}$ of 2*s*. 6*d*.
6. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ and $\frac{1}{12}$ of 1*s*.
7. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ and $\frac{1}{12}$ of 6*d*.
8. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{12}$ of 3*d*.
9. Write down the aliquot parts of 3*s*. 4*d*., 2*s*. 6*d*., 2*s*., 9*d*. and 8*d*., each of which is a whole number of pence.

141. There are two distinct kinds of question to which the method of Practice is applicable.

We may, for example, employ this method to find the cost of 3175 cwt. at £3. 17*s.* 6*d.* per cwt., or the weight of 1627 bales of goods each of which weighs 5 cwt. 3 qrs. 14 lbs. These are examples of **Simple Practice.**

We may also employ the method to find, for example, the cost of 15 cwt. 1 qr. 21 lbs. at £7. 11*s.* 9*d.* per cwt. This is an example of **Compound Practice.**

Simple Practice.

142. The method will be seen from the following examples.

Ex. 1. Find the cost of 1269 cwt. at £4. 6*s.* 8*d.* per cwt.

It is evident that the cost of any number of things at £4. 6*s.* 8*d.* each is the sum of what the cost would be at £4 each and at 6*s.* 8*d.* each.

Now the cost of 1269 things at £1 each would clearly be £1269; the cost at £4 each would be 4 times the cost at £1 each; also, since 6*s.* 8*d.* is $\frac{1}{3}$ of £1, the cost at 6*s.* 8*d.* each would be *one-third* of the cost at £1 each, that is $\frac{1}{3}$ of £1269. We can therefore obtain the required result as follows.

	£			
	1269	= cost at £1		each
	4			
	5076	=	£4	.....
6*s.* 8*d.* = $\frac{1}{3}$ of £1	423	=	6*s.* 8*d.*	.....
	£5499	=	£4. 6*s.* 8*d.*	.....

Ex. 2. Find the cost of 67214 things at £5. 17*s.* 6*d.* each.

		£				
		67214	= cost at £1			each
		5				
		336070	=	£5		.....
10*s.*	= $\frac{1}{2}$ of £1	33607	=		10*s.*	.....
5*s.*	= $\frac{1}{2}$ of 10*s.*	16803	. 10*s.* = ...		5*s.*	.....
2*s.* 6*d.*	= $\frac{1}{2}$ of 5*s.*	8401	. 15*s.* = ...		2*s.* 6*d.*	.....
		£394882	. 5*s.* = ...	£5. 17*s.* 6*d.*		.....

The cost at £1 each is multiplied by 5 to obtain the cost at £5 each. The cost at £1 is divided by 2 to obtain the cost at 10*s.*

each. The cost at 10*s.* each is divided by 2 to obtain the cost at 5*s.* each. Finally the cost at 5*s.* each is divided by 2 to obtain the cost at 2*s.* 6*d.* each. These results are then added to find the whole cost.

It should be noticed that in all cases of Simple Practice the results can be obtained by multiplication, as in Chapter III.

EXAMPLES VII. B.

Find the cost of

1. 3152 at 10*s.* each.
2. 7514 at 5*s.* each.
3. 1734 at 6*s.* 8*d.* each.
4. 1875 at 2*s.* 6*d.* each.
5. 2715 at 3*s.* 4*d.* each.
6. 3907 at 1*s.* 8*d.* each.
7. 365 at £2. 10*s.* each.
8. 479 at £3. 6*s.* 8*d.* each.
9. 1594 at £7. 5*s.* each.
10. 1374 at £7. 2*s.* 6*d.* each.
11. 6894 at £1. 2*s.* 6*d.* each.
12. 6078 at £6. 2*s.* each.
13. 797 at £3. 15*s.* each.
14. 3842 at £7. 12*s.* 6*d.* each.
15. 1234 at £9. 12*s.* each.
16. 1890 at £12. 7*s.* 6*d.* each.
17. 697 at £24. 6*s.* each.
18. 3874 at £35. 11*s.* each.
19. 3742 at £9. 7*s.* each.
20. 1894 at £24. 17*s.* 6*d.* each.
21. Find the total weight of 742 packages each of which weighs 1 cwt. 3 qrs.
22. Find the total weight of 1945 bales of goods each of which weighs 5 cwt. 3 qrs. 14 lbs.
23. Find the total length of string in 784 balls each of length 10 yards 1 ft. 6 in.
24. Find the total length of string in 1964 balls each of length 3 yards 2 ft. 3 in.

143. The following examples are somewhat more difficult.

Ex. 1. Find the cost of 1392 things at £8. 9*s.* each.

	£			
	1392	= cost at £1		each
	8			
	11136	=	£8	.....
5*s.* = $\frac{1}{4}$ of £1	348	=	5*s.*	.....
4*s.* = $\frac{1}{5}$ of £1	278 . 8*s.*	=	4*s.*	.....
	£11762 . 8*s.*	=	£8. 9*s.*	

Ex. 2. Find the cost of 1275 things at £3. 19*s*. 4*d*. each.

	£				
	1275		= cost at £1		
	3				
	3825		=	£3	
10*s*. = $\frac{1}{2}$ of £1	637	.	10*s*. = ...		10*s*.
5*s*. = $\frac{1}{2}$ of 10*s*.	318	.	15*s*. = ...		5*s*.
4*s*. = $\frac{1}{5}$ of £1	255		=		4*s*.
4*d*. = $\frac{1}{12}$ of 4*s*.	21	.	5*s*. = ...		4*d*.
	£5057	.	10*s*. = ...	£3. 19*s*.	4*d*.

We may also proceed thus:

	£					
	1275		= cost at £1			each.
	4					
	5100		=	£4		.....
8*d*. = $\frac{1}{30}$ of £1	42	.	10*s*. = ...		8*d*.	.....
	£5057	.	10*s*. = ...	£3	19*s*.	4*d*. .....

Ex. 3. Find the cost of 1798$\frac{1}{3}$ cwt. at £3. 16*s*. 10$\frac{1}{2}$*d*. per cwt.

Since $\frac{1}{3}$ £ = 6*s*. 8*d*., the cost of 1798$\frac{1}{3}$ cwt. at £1 each is £1798. 6*s*. 8*d*.

	£.		*s*.		*d*.			
	1798	.	6	.	8	= cost at £1		
					3			
	5395	.	0	.	0	=	£3	
10*s*. = $\frac{1}{2}$ £	899	.	3	.	4	=	10*s*.	
4*s*. = $\frac{1}{5}$ £	359	.	13	.	4	=	4*s*.	
2*s*. = $\frac{1}{2}$ of 4*s*.	179	.	16	.	8	=	2*s*.	
6*d*. = $\frac{1}{4}$ of 2*s*.	44	.	19	.	2	=		6*d*.
3*d*. = $\frac{1}{2}$ of 6*d*.	22	.	9	.	7	=		3*d*.
1$\frac{1}{2}$*d*. = $\frac{1}{2}$ of 3*d*.	11	.	4	.	9$\frac{1}{2}$	=		1$\frac{1}{2}$*d*.
	6912	.	6	.	10$\frac{1}{2}$	=	£3. 16*s*.	10$\frac{1}{2}$*d*.

EXAMPLES VII. C.

Find the cost of

1. 349 at £9. 16*s*. 8*d*.
2. 3189 at £7. 9*s*.
3. 1795 at £11. 14*s*.
4. 7284 at £12. 16*s*.
5. 857 at £25. 6*s*. 6*d*.
6. 1897 at £30. 3*s*. 6*d*.
7. 3584 at £20. 8*s*.
8. 3874 at £5. 17*s*.
9. 1695 at £15. 13*s*.
10. 2678 at £25. 13*s*.
11. 2697 at £12. 18*s*.
12. 1697 at £35. 19*s*.
13. 1987 at £30. 18*s*. 9*d*.
14. 3856 at £7. 15*s*. 6*d*.
15. 934 at £8. 14*s*. 6*d*.
16. 1924 at £20. 6*s*. 6*d*.
17. 3957 at £24. 8*s*. 11$\frac{1}{2}$*d*.
18. 1248 at £3. 6*s*. 10$\frac{1}{2}$*d*.

19. 4321 at £1. 16*s.* $9\frac{1}{2}d.$

20. 617 at £3. 7*s.* $5\frac{1}{2}d.$

21. 1875 at £4. 13*s.* $6\frac{1}{2}d.$

22. 297 at £1. 3*s.* $11\frac{3}{4}d.$

23. 6864 at £4. 3*s.* $9\frac{1}{4}d.$

24. 1357 at 7*s.* $8\frac{1}{2}d.$

25. 2814 at 11*s.* $8\frac{3}{4}d.$

26. 6741 at £1. 18*s.* $7\frac{1}{2}d.$

27. 3927 at 15*s.* $10\frac{3}{4}d.$

28. 2345 at £2. 9*s.* $7\frac{1}{2}d.$

29. 1864 at £5. 17*s.* $11\frac{3}{4}d.$

30. 1891 at £1. 9*s.* $11\frac{1}{2}d.$

31. $7925\frac{1}{2}$ at £1. 8*s.* 10*d.*

32. $1846\frac{1}{4}$ at £6. 12*s.* 6*d.*

33. $6481\frac{2}{3}$ at £8. 5*s.* 6*d.*

34. $5555\frac{1}{3}$ at £1. 6*s.* $10\frac{1}{2}d.$

Compound Practice.

144. The method will be seen from the following examples.

Ex. 1. Find the rent of 5 A. 3 R. 20 P. at £1. 5*s.* 6*d.* an acre.

	£.	*s.*	*d.*		
	1	5	6	= rent of 1 acre	
			5		
	6	7	6	=	5 A.
2 R. = $\frac{1}{2}$ of 1 A.		12	9	=	2 R.
1 R. = $\frac{1}{2}$ of 2 R.		6	$4\frac{1}{2}$	=	1 R.
20 P. = $\frac{1}{2}$ of 1 R.		3	$2\frac{1}{4}$	=	20 P.
	7	9	$9\frac{3}{4}$	=	5 A. 3 R. 20 P.

Ex. 2. Find the value of 11 oz. 13 dwt. 8 gr. of gold at £3. 17*s.* $10\frac{1}{2}d.$ per oz.

	£.	*s.*	*d.*		
	3	17	$10\frac{1}{2}$	= value of 1 oz.	
			11		
	42	16	$7\frac{1}{2}$	=	11 oz.
10 dwts. = $\frac{1}{2}$ of 1 oz.	1	18	$11\frac{1}{4}$	=	10 dwts.
2 dwts. = $\frac{1}{5}$ of 10 dwts.		7	$9\frac{9}{20}$	=	2 dwts.
1 dwt. = $\frac{1}{2}$ of 2 dwts.		3	$10\frac{29}{40}$	=	1 dwt.
8 grs. = $\frac{1}{3}$ of 1 dwt.		1	$3\frac{23}{40}$	=	8 grs.
	45	8	$6\frac{1}{2}$	=	11 oz. 13 dwts. 8 grs.

The sum of $\frac{23}{40} + \frac{29}{40} + \frac{9}{20} + \frac{1}{4} + \frac{1}{2}$ will be found to be $2\frac{1}{4}$.

In the above example fractions of a penny less than $\frac{1}{4}d.$ are introduced in the course of the work but do not occur in the final sum; in many cases, however, the final answer contains fractions of a penny which it would be impossible to pay. The student should in general find the *exact* answer, as an exercise in accuracy, although in all real transactions the money to be paid would be either the farthing (or, when the total amount is not small, the penny) next *below*, or the farthing (or the penny) next *above* the true value.

The above example may also be treated as follows. It will be found [see Art. 137] that £3. 17*s*. 10½*d*. = £3·89375.

	£		
	3·89375	= value of 1 ounce	
	11		
	42·83125	=	11 ounces
10 dwts. = ½ of 1 oz.	1·946875	=	10 dwts.
2 dwts. = ⅕ of 10 dwts.	·389375	=	2 dwts.
1 dwt. = ½ of 2 dwts.	·1946875	=	1 dwt.
8 grs. = ⅛ of 1 dwt.	·06489583	=	8 grs.
	£45·42708333	=	11 oz. 13 dwts. 8 grs.

Then £45·427083 will be found to be £45. 8*s*. 6½*d*., by the method of Art. 138.

Ex. 3. Find the cost of 3 A. 2 R. 2 P. 8 sq. yds. at £80. 13*s*. 4*d*. an acre.

	£.	*s*.	*d*.		
	80	13	4	= cost of 1 acre	
			3		
	242	0	0	=	3 A.
2 R. = ½ of 1 A.	40	6	8	=	2 R.
2 P. = 1/40 of 2 R.	1	0	2	=	2 P.
5½ yards = 1/11 of 2 P.		1	10	=	5½ yds.
½ yard = 1/11 of 5½ yds.			2		
∴ 2½ yards = 5/11 of 5½ yds.			10	=	2½ yds.
	283	9	6	=	3 A. 2 R. 2 P. 8 yds.

Ex. 4. A bankrupt pays 6*s*. 5½*d*. in the pound, how much will a creditor receive to whom he owes £560. 10*s*.?

	£.	*s*.	*d*.		
	560	10	0	= total amount of debt	
5*s*. = ¼ of 1£	140	2	6	= amount paid at 5*s*.	for 1£
1*s*. 3*d*. = ¼ of 5*s*.	35	0	7½	=	1*s*. 3*d*. .....
2½*d*. = 1/6 of 1*s*. 3*d*.	5	16	9¼	=	2½*d*. .....
	180	19	10¾	=	6*s*. 5½*d*. .....

EXAMPLES VII. D.

1. Find the cost of 5 cwt. 3 qrs. 14 lbs. at £2. 6*s*. 8*d*. per cwt.
2. Find the cost of 25 cwt. 1 qr. 21 lbs. at £3. 2*s*. 8*d*. per cwt.
3. Find the cost of 8 oz. 12 dwts. 12 grs. at £1. 6*s*. 8*d*. per oz.
4. Find the cost of 9 yds. 2 ft. 9 in. at £2. 3*s*. 6*d*. per yard.
5. Find the rent of 86 A. 1 R. 24 P. at £3. 10*s*. an acre.

6. The rent of a farm of 138 A. 1 R. 24 P. is at the rate of 35*s.* an acre. Find the whole rent.

7. Find the cost of 18 sq. yds. 4 sq. ft. 96 sq. in. at 16*s.* 10½*d.* per sq. yd.

8. Find the cost of 3 qrs. 3 bus. 1 pk. of oats at 34*s.* 8*d.* a quarter.

9. Find the cost of 3 qrs. 7 bus. 2 pks. 1 gal. at £2. 14*s.* 8*d.* per quarter.

10. Find the cost of 21 qrs. 3 bus. 2 pks. 1 gal. at £1. 17*s.* 4*d.* a quarter.

11. Find the cost of 3 tons 3 cwt. 3 qrs. 27 lbs. 12 oz. at £7 per cwt.

12. Find the cost of 24 tons 6 cwt. 2 qrs. 8 lbs. at 17*s.* 6*d.* a ton.

13. Find the cost of 21 tons 3 cwt. 3 qrs. 7 lbs. 12 oz. at £6. 1*s.* 4*d.* per cwt.

14. Find the cost of 4 tons 13 cwt. 1 qr. 16 lbs. at £5. 3*s.* 3*d.* per cwt.

15. Find the value of a bar of gold weighing 61 lbs. 8 oz. 12 dwt. 12 grs. at £4. 4*s.* 8*d.* an oz.

16. Find, to the nearest farthing, the cost of 10 cwt. 3 qrs. 22 lbs. at £6. 6*s.* 6*d.* per cwt.

17. Find the value of a silver cup weighing 2 lbs. 5 oz. 17 dwt. 6 grs. at 8*s.* 4*d.* an ounce.

18. Find the value of a gold cup weighing 1 lb. 6 oz. 15 dwt. 5 grs. at £6. 10*s.* an ounce.

19. Find the cost of 235 tons 6 cwt. 3 qrs. 6 lbs. 8 oz. at £2. 2*s.* per cwt.

20. Find the cost of 114 tons 3 cwt. 16 lbs. at £844. 13*s.* 4*d.* per ton.

21. Find the rent of 76 A. 3 R. 8 P. at £2. 0*s.* 7½*d.* an acre.

22. Find the rent of 132 A. 3 R. 25 P. at £3. 7*s.* 4*d.* an acre.

23. Find the rent of a farm of 325 A. 3 R. 8 P. at £1. 15*s.* 2½*d.* per acre.

24. Find the value of 13 A. 3 R. 25 P. at £232. 16*s.* 8*d.* per acre.

25. If a bankrupt pays 13*s.* 2*d.* in the pound, how much will a creditor receive to whom he owes £563. 15*s.*?

26. A bankrupt's liabilities are £3758. 10*s.*, what are his assets if he can pay 13*s.* 7½*d.* in the pound?

27. A bankrupt whose debts amount to £8428 can pay 13*s.* 10½*d.* in the pound; what are his assets?

28. What would be the amount of a tax on £575 at 1*s.* 1¾*d.* in the pound?

29. The whole taxation for a certain district is at the rate of 2*s.* 7½*d.* in the pound; how much does a property pay whose annual rateable value is £3567. 6*s.* 8*d.*?

30. Find the rent of 4 acres 110 sq. yds. at £5. 15*s.* 6*d.* per acre.

31. Find the yearly cost of keeping in order a road 29 miles 6 fur. 200 yds. long at £6. 1*s.* per mile.

32. Find the cost of making a railway 17 miles 3 fur. 53 yds. long at £3476 per mile.

33. Find the cost of 6 fur. 12 poles 2 yds. at £14. 13*s.* 4*d.* a furlong.

34. Find the cost of making a fence 3 fur. 11 poles 3¼ yds. long at £183. 6*s.* 8*d.* per mile.

35. Find the rent of 25 A. 3 R. 15 P. 2¾ yds. at £2. 15*s.* an acre.

36. Find the cost of 1 A. 3 R. 2 P. 17 yds. at £40. 6*s.* 8*d.* per acre.

INVOICES.

145. When purchases are made a written list shewing the quantity and the cost of each article is made out by the tradesman and sent with the goods. This written list is called a **Bill** or an **Invoice**, and each separate entry in it is called an **Item**.

If bills are not paid at once, a statement of the total cost of the goods supplied on particular days is made up at the end of a certain period; and this statement is called an **Account**. If all the different items are given in the account it is called a **Detailed Account**.

Invoices and Accounts are usually made out in the following forms.

Specimen of a Bill or Invoice.

Cambridge, *May 10th*, 1891.

Mrs W. Wilson.

Bought of J. Woods and Co.
Grocers.

	£.	*s.*	*d.*
5 lbs. tea at 1*s.* 8*d.* per lb.		8	4
1 stone sugar at 3½*d.* per lb.		4	1
4 lbs. butter at 1*s.* 5*d.* per lb.		5	8
3 lbs. cheese at 11*d.* per lb.		2	9
	£1	0	10

Specimen of an Account.

London, *April 1st*, 1891.

J. Woods and Co., Cambridge.

Dr to Parker, Watson and Co.

		£.	*s.*	*d.*
Jany 6	To Goods as per Invoice	59	17	4
Jany 29	To Goods as per Invoice	157	12	9
Feby 23	To Goods as per Invoice	294	11	1
		£512	1	2

EXAMPLES VII. E.

Make out the following bills with names and dates. When the exact amount of one of the items cannot be expressed in money, *the farthing next above* should be charged, as is done by tradesmen.

1. 7½ yds. of flannel at 11*d.* per yd.; 7½ yds. brown holland at 6¾*d.* per yd.; 3½ yds. ribbon at 7½*d.* per yd.; and 4 reels of cotton at 2½*d.* each.

2. 2½ lbs. tea at 1*s.* 10*d.* per lb.; 7 lbs. biscuits at 9*d.* per lb.; 14 lbs. rice at 2½*d.* a lb.; and 3 lbs. butter at 1*s.* 2*d.* per lb.

3. 7 lbs. 3 oz. beef at 11*d.* per lb.; 2 lbs. 4 oz. steak at 13*d.* per lb.; 4 lbs. 5 oz. lamb at 11½*d.* per lb.; and 2 lbs. suet at 7*d.* per lb.

4. 17 yds. calico at $8\frac{1}{2}d.$ per yd.; $25\frac{1}{2}$ yds. linen at 1*s.* $6\frac{3}{4}d.$ per yd.; 40 yards huckaback at $9\frac{1}{2}d.$ per yd.; and 3 cases of needles at 1*s.* each.

5. 11 lbs. 7 oz. mutton at $10\frac{1}{2}d.$ per lb.; 10 kidneys at 2*d.* each; 3 lbs. 5 oz. steak at 13*d.* per lb.; 7 lbs. 2 oz. veal at $11\frac{1}{2}d.$ per lb.

6. $11\frac{1}{2}$ doz. copybooks at 2*s.* 3*d.* per doz.; 15 quires paper at $7\frac{1}{2}d.$ per quire; 10 books at 1*s.* $4\frac{1}{2}d.$ each; and 3 boxes of steel pens at 8*d.* each.

7. 5 lbs. tea at 1*s.* 9*d.* per lb.; 21 lbs. sugar at $2\frac{3}{4}d.$ per lb.; 7 lbs. jam at $5\frac{1}{2}d.$ per lb.; $4\frac{1}{2}$ lbs. currants at $3\frac{1}{2}d.$ per lb.; and 5 lbs. 7 oz. bacon at 1*s.* 1*d.* per lb.

MISCELLANEOUS EXAMPLES IV.

1. What is the least number which must be added to a million that the sum may be exactly divisible by 573?

2. Reduce £17. 15*s.* $9\frac{1}{2}d.$ to farthings and 2 tons 5 cwt. 13 lbs. to ounces.

3. How many times is £6. 17*s.* $9\frac{1}{4}d.$ contained in £943. 14*s.* $7\frac{1}{4}d.$?

4. Find the G.C.M. and the L.C.M. of 936 and 2925.

5. Divide $4\frac{3}{5}$ of $1\frac{7}{8}$ by $5\frac{1}{9}$, and the result by $2\frac{1}{4}$.

6. Divide 1·516515 by ·303 and $(\cdot00242)^2$ by 3·125.

7. Multiply £14. 6*s.* 4*d.* by 6·875 and divide £147 by 47·04.

8. Find the cost of 58 cwt. 3 qrs. 21 lbs. at £5. 9*s.* 4*d.* per cwt.

---

9. A certain number was divided by 105, by 'short' divisions; the quotient was 192, the first remainder was 1, the second was 4 and the third was 6. What was the dividend?

10. Multiply £1388. 10*s.* 5*d.* by 37, and divide the result by 83.

11. Reduce 3 lbs. 5 oz. 16 dwts. to grains, and express 1 oz. 16 dwts. 11 grains in Avoirdupois weight.

12. Subtract $16\frac{1}{2} \times \dfrac{1\frac{3}{5}}{3+\dfrac{1}{3\frac{1}{3}}}$ from $\dfrac{5\frac{2}{5} \text{ of } 7\frac{2}{9}}{8\frac{7}{24}-3\frac{5}{12}}$.

13. A man after spending $\frac{2}{5}$ and $\frac{1}{9}$ of his money, found that he had £33 left. How much had he at first?

14. Simplify $\cdot\dot{4}28571\dot{4} \times 12\cdot6 - 2\cdot34 \times 2\cdot2\dot{7}$.

15. From a rod a yard long, portions each ·083 of an inch are cut off. How many portions can be cut off, and what will be the length of the remaining piece?

16. Find the cost of 25 cwt. 3 qrs. 25 lbs. 12 oz. at £2. 6*s*. 8*d*. per cwt.

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17. Divide five million seven hundred and ninety-one thousand seven hundred and fifty-five by one hundred and twenty-one.

18. Find the value of 51 things, any four of which are worth £19. 3*s*. 1*d*.

19. The circumferences of the large and the small wheels of a bicycle are 143 inches and 40 inches respectively. How many more turns will the latter have made than the former in a distance of 13 miles?

20. Find the G.C.M. of 2079 and 2898; and the L.C.M. of 12, 15, 18, 21, 25, 35 and 210.

21. Arrange $\frac{3}{62}$, $\frac{7}{144}$ and $\frac{4}{83}$ in ascending order of magnitude.

22. Divide ·0663 by ·00204; also divide 3·6 by ·37.

23. Express $\cdot3\dot{8}$, $\cdot6\dot{7}61904\dot{4}$ and $6\cdot75 \times \cdot0\dot{3}\dot{7}$ as vulgar fractions in their lowest terms.

24. Find the cost of 3 roods 17 poles at £256. 10*s*. per acre.

---

25. How many numbers each 567 must be added together that the sum may be greater than a million?

26. Reduce 772642 square yards to acres, roods, poles and yards.

27. A bill of £30. 19*s*. 1*d*. was paid with an equal number of sovereigns, half-sovereigns, half-crowns and pennies. How many were there of each?

28. Find the greatest number which will exactly divide both 618234 and 712923; find also the smallest number which is exactly divisible by both the numbers.

29. Find the proper fraction which added to $\frac{7}{16}$, $\frac{3}{20}$ and $\frac{9}{35}$ will make the result an integer.

30. Simplify $1\frac{2}{3}$ of $\dfrac{6\frac{3}{4} - 2\frac{7}{9}}{3\frac{1}{2} - 1\frac{1}{18}} \div \left\{2\frac{5}{6} - \frac{3}{5} \text{ of } \frac{1}{4\frac{4}{5}}\right\}$.

31. Find in hours, minutes and seconds ·6575 of a day.

32. Find the cost of 3 tons 13 cwt. 1 qr. 17 lbs. at £2. 6*s.* 8*d.* per ton.

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33. What is the greatest number of Sundays there can be in a year? On what day of the week will the first of February fall when the number of Sundays in a year of 365 days is greatest?

34. How many times can £17. 5*s.* $9\frac{1}{2}d.$ be subtracted from £100, and what will be the last remainder?

35. A grocer buys 15 cwt. of goods for £24. 10*s.*; at what rate per lb. must he sell to gain £7 on the whole?

36. A bar of metal weighing 100 oz. 16 dwts. is made into coins weighing 1 oz. 8 dwts. each. How many coins are there?

37. Simplify $2\frac{2}{3}$ of $1\frac{1}{13} \times 3\frac{5}{7}$ of $1\frac{4}{11}$, and divide $3\frac{3}{5}$ of $\frac{1}{6}$ by $5\frac{3}{3}$.

38. Add $\frac{4\frac{2}{3}}{6\frac{2}{3}}$ of $\frac{1\frac{1}{2}-\frac{5}{6}}{1\frac{1}{2}+\frac{5}{6}}$ of £2. 10*s.* to $\frac{6\frac{9}{11}}{11\frac{4}{11}}$ of $\frac{2\frac{1}{4}+1\frac{1}{8}}{2\frac{1}{4}-1\frac{1}{8}}$ of 7*s.* 6*d.*

39. Express as a decimal $\cdot 1\dot{3}\dot{6} \times 7\cdot\dot{3} \div \cdot 4\dot{3}$.

40. Find ·025 of £2 + ·75 of a crown + $1\cdot\dot{6}$ of sixpence + ·25 of half a guinea.

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41. The heights of 6 brothers are 6 ft. 1 in., 5 ft. $11\frac{1}{2}$ in., 5 ft. 11 in., 5 ft. $10\frac{1}{2}$ in., 5 ft. $10\frac{1}{4}$ in, and 5 ft. $9\frac{3}{4}$ in. respectively. What is their average height?

42. Reduce 10 miles 2 fur. 13 poles to inches, and 2 acres 3 roods 13 poles 4 sq. yds. to yards.

43. What is the least number which when divided by 15 leaves a remainder 3, when divided by 18 leaves a remainder 6, and when divided by 24 leaves a remainder 12?

44. Find the greatest length of which both 1 m. 4 fur. 16 p. 2 yds. and 1 m. 1 fur. 10 p. 2 yds. are multiples.

45. How much is required to make the difference between $2\frac{3}{7}$ and $1\frac{3}{4}$ equal the sum of $\frac{1}{14}$, $1\frac{5}{6}$ and $\frac{3}{56}$?

46. Simplify $2\frac{2}{3}$ of $1\frac{1}{4} \div 5\frac{1}{3}$ of $3\frac{3}{4}$, and $\cfrac{7}{5+\cfrac{3}{1-\cfrac{1}{3-\frac{5}{7}}}}$.

47. What decimal of £4. 11*s.* is $\frac{3}{8}$ of $\frac{5\frac{1}{11}}{2-\frac{5}{13}}$ of £·77?

48. Find the annual cost of repairing a road 9 miles 3 fur. 177 yds. long at £16. 10*s.* per mile.

CHAPTER VIII.

AREAS AND VOLUMES.

146. A **square** is a plane figure bounded by four equal sides, and having all its angles right angles.

A **rectangle** is a plane figure bounded by four sides, and having all its angles right angles.

Opposite sides of a rectangle are equal to one another.

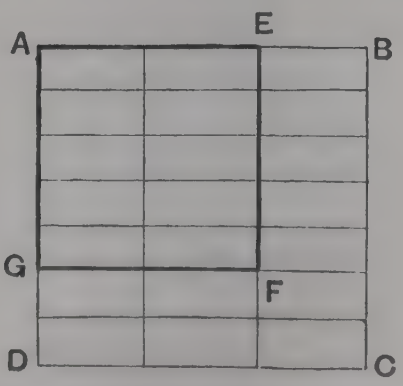
The **area** of a plane figure is the amount of surface included within its bounding lines, and this area is measured by the number of times it contains some standard unit of area.

Different units in terms of which an area can be expressed have already been given in Chapter III.

A **square inch** is a square each of whose sides is one inch, and similarly for a *square foot, a square yard,* &c.

The problem of finding the area of a plane figure, except in the simple case of a rectangle, is beyond the limits of this book.

147. To find the area of a rectangle. Let *ABCD* be the rectangle whose area is required.



Suppose, for example, that AB is 4 inches and that AD is 3 inches. Divide AB into 4 equal parts and AD into 3 equal parts, and draw lines parallel to the sides as in the figure on the left.

Then the rectangle is divided up into squares each of which is a square inch; and the number of these squares is clearly the product of the number of inches in AB by the number of inches in AD.

The above reasoning clearly applies to all cases, both the length and the breadth of the rectangle being an integral number of inches.

Now suppose, for example, that in the figure on the right AB is $\frac{3}{2}$ inches, and that AD is $\frac{7}{5}$ inches.

Let $AEFG$ be one square inch. Divide AE into two equal parts, and AG into five equal parts and GD into two equal parts. Then the subdivisions of AB will be all equal, as also those of AD. Hence, if lines be drawn as in the figure, $ABCD$ will be divided into 3×7 equal rectangles, such that the square inch $AEFG$ will contain 2×5 of these rectangles. Hence AB will contain $\dfrac{3 \times 7}{2 \times 5}$ square inches, that is $\dfrac{3}{2} \times \dfrac{7}{5}$ square inches.

From the above it follows that **the number of square inches (or square feet, &c.) in a rectangle is equal to the product of the number of inches (or feet, &c.) in the length by the number of inches (or feet, &c.) in the breadth.**

It should be noticed that the length and breadth must both be expressed in terms of *the same unit*.

For example, the area of a rectangle whose length is 2 feet and breadth 6 inches is $(2 \times {}_1^6)$ square feet, or (24×6) square inches.

The above rule for finding the area of a rectangle is often expressed shortly by the statement that **area = length × breadth**; this being understood to mean that (the number of units of) *area* = (the number of units in the) *length* × (the number of units in the) *breadth*.

148. Now that we can find the area of a rectangle, we can see that the relations between the different units given in the Table for Square Measure on page 42 follow at once from the relations between the corresponding units in linear measure.

For, since 12 inches make 1 foot, (12×12) square inches make 1 square foot.

Since 3 feet make 1 yard, (3×3) square feet make 1 sq. yd.

Since $5\frac{1}{2}$ yards make 1 pole, $(5\frac{1}{2} \times 5\frac{1}{2})$ sq. yds. make 1 sq. pole.

Again 22 yards make 1 chain, therefore (22×22) sq. yds. $= 484$ sq. yds. make 1 square chain.

Thus 4840 sq. yds. $= 10$ sq. chains $= 1$ acre.

Also 1 sq. mile $= (1760 \times 1760)$ sq. yds. $= 1760 \times 1760 \div 4840$ acres $= 640$ acres.

Ex. Find the acreage of a rectangular field whose length is 132 yards and whose breadth is $38\frac{1}{2}$ yards.

The area $= (132 \times 38\frac{1}{2})$ sq. yds.
$= 5082$ sq. yds. $= \frac{5082}{4840}$ acres
$= \frac{21}{20}$ acres $= 1$ a. 8 p.

149. If the area of a rectangle be known and also the length (or the breadth), the breadth (or the length) can be at once found.

For example, to find the breadth of a rectangle whose length is 15 feet and whose area is 200 sq. ft.

Since the product of the number of feet in the breadth by the number of feet in the length is equal to the number of square feet in the area, we have

breadth $= (200 \div 15)$ ft. $= 13\frac{1}{3}$ ft. $= 13$ ft. 4 in.

EXAMPLES VIII. A.

Find the areas of the rectangles whose lengths and breadths are as under:

1.	12 ft., 14 ft.	**2.**	22 ft., 17 ft.
3.	25 yds., 17 yds.	**4.**	114 in., 122 inches.
5.	5 feet, 17 inches.	**6.**	10 yds., 23 ft.
7.	3 yds. 2 ft., 5 yds. 1 ft.	**8.**	21 yds. 2 ft., 18 yds.
9.	13 ft. 4 in., 9 ft. 2 in.	**10.**	11 ft. 9 in., 8 ft. 7 in.

Find the acreage of the rectangular fields whose lengths and breadths are as under:

11.	319 yds., 275 yds.	**12.**	363 yds., 240 yds.
13.	400 yds., $214\frac{1}{2}$ yds.	**14.**	$178\frac{3}{4}$ yds., $162\frac{4}{5}$ yds.
15.	550 yds., 400 yds.	**16.**	125 yds., $49\frac{1}{2}$ yds.

17. Find the area of a rectangular field whose length is 9 chains 12 links and whose breadth is 6 chains 25 links.

18. Find the area of a rectangular field whose length is 9 chains 25 links and whose breadth is 7 chains 75 links.

19. The area of a rectangle is 925 square inches and its breadth is 25 inches; what is its length?

20. What is the length of a rectangular table the area of whose top is 71 sq. ft. 16 sq. in., and the breadth 6 ft. 8 in.?

21. The area of a rectangular courtyard is 52 sq. yds. 2 sq. ft. 36 sq. in., and its length is 14 yds. 9 in.; what is its breadth?

22. What will it cost to paint the ceiling of a room whose length is 24 ft. 6 in. and breadth 16 ft. 6 in. at 3*s.* per square yard?

23. The area of a rectangular field is 1 a. 1 r. 10 p., and its breadth is $82\frac{1}{2}$ yds.; what is its length?

24. What is the cost of a rectangular plot of building land 55 ft. wide and 165 ft. long at the rate of £1000 an acre?

25. What is the cost of a rectangular plot of building ground 250 ft. long by 60 ft. wide at £1210 an acre?

Carpeting Floors and Papering Walls.

150. The following examples are of frequent occurrence :—

Ex. 1. How much will it cost to carpet a room 16 ft. by 20 ft. 3 in. with carpet 27 inches wide at 3*s.* 6*d.* a yard?

Area of floor $= (16 \times 20\frac{1}{4})$ sq. ft. $= 324$ sq. ft.

Since the carpet is to cover the whole floor, the whole area of the carpet is to be 324 sq. ft., and its width is $\frac{27}{12}$ ft.

Hence length of carpet $= \left(324 \div \frac{27}{12}\right)$ ft. $= 144$ ft. $= 48$ yds.

Therefore cost of carpet $= 3s.\ 6d. \times 48 = £8.\ 8s.$

Ex. 2. How much will it cost to paper the walls of a room 19 ft. 3 in. long, 15 ft. 9 in. wide and 12 ft. high, with paper 21 inches wide which costs $2\frac{1}{2}d.$ per yard?

The area of each of the two sides of a room = length × height.

The area of each of the two ends of a room = breadth × height.

Hence the total area of the four walls = 2 (length + breadth) × height.

Hence the total area of the four walls of the room in question

$$= \{2\,(19\frac{1}{4} + 15\frac{3}{4}) \times 12\} \text{ sq. ft.} = 840 \text{ sq. ft.}$$

Hence the length of paper required

$$= \left(840 \div \frac{21}{12}\right) \text{ ft.} = 480 \text{ ft.} = 160 \text{ yds.}$$

Since the paper costs $2\frac{1}{2}d.$ per yard, the whole cost will be

$$2\frac{1}{2}d. \times 160 = £1.\ 13s.\ 4d.$$

Note. In the preceding questions we have found the quantity of carpet or wall-paper which would be required if it were of one uniform colour throughout. When, as is almost invariably the case, there is a pattern on the carpet or paper, there must be a certain amount of waste, if the different lengths are properly fitted together. Moreover wall-papers are only sold in lengths of 12 yards called '**pieces**'; if therefore, as in Ex. 2, 160 yards of paper were required, 14 pieces = 168 yards would have to be bought. English wall papers are generally 21 inches wide.

Ex. 3. A room 21 ft. by 19 ft. has a Turkey carpet in it, a border 3 ft. wide all round being left uncovered by the carpet. The border was stained at a cost of 1*s.* 6*d.* a square yard, and the carpet cost 18*s.* a square yard: what was the total cost?

Since the border is 3 ft. wide all round the room, the length of the carpet must be 21 ft. − 3 ft. × 2 = 15 ft., and the breadth must be 19 ft. − 3 ft. × 2 = 13 ft.

Hence area of carpet = 15×13 sq. ft. = $\frac{65}{3}$ sq. yds.

Hence price of carpet = 18*s.* × $\frac{65}{3}$ = £19. 10*s.*

The area of the border is *the difference between the whole area of the floor and the area covered by the carpet*, and is therefore

$$(21 \times 19 - 15 \times 13) \text{ sq. ft.} = 204 \text{ sq. ft.}$$

Hence cost of staining at 2*d.* a sq. foot = 408*d.* = 34*s.*

Thus total cost = £19. 10*s.* + £1. 14*s.* = £21. 4*s.*

EXAMPLES VIII. B.

1. A carriage drive is 69 ft. 9 in. long and 16 ft. wide, find the cost of gravelling it at 1*s.* 3*d.* per square yard.

2. How much carpet 27 in. wide will cover a room 22 ft. 6 in. long and 15 ft. 9 in. wide, and what will the carpet cost at 4*s.* 3*d.* per yard?

3. A room is 10 yds. 2 ft. long and 7 yards 1 ft. 6 in. wide; find the cost of covering it with carpet at 4*s.* 6*d.* a square yard.

4. Find the cost of carpeting a room $8\frac{1}{4}$ yards long by 6 yds. 2 ft. broad with carpet $2\frac{1}{4}$ ft. wide at 3*s.* 6*d.* a yard.

5. The outer and inner boundaries of a gravel path are squares, and the path is 4 ft. wide. The side of the square enclosed by the path is 50 yards. How much would it cost to gravel the path at 1*s.* 6*d.* a superficial yard?

6. Find the cost of turfing a lawn tennis court which is 78 feet long and 39 feet wide making a margin of grass 12 feet wide at each end and 6 feet wide at each side. The turf costing 4*d.* a square yard.

7. Supposing a postage stamp to be 1 inch long and $\frac{4}{5}$ of an inch broad, how many stamps would be required to cover a wall which is 15 ft. 6 in. long and 10 ft. 8 in. high?

8. What would be the expense of carpeting a room 24 ft. 6 in. by 18 ft. with carpet 27 in. wide, and which costs 4*s.* 6*d.* a yard?

9. How much carpet 27 inches wide would be required for a room 32 feet by 23 feet, a margin 4 feet wide being left uncovered?

10. What will be the cost of carpeting a room 33 feet long and 24 feet wide with carpet 27 inches wide and costing 3*s.* 6*d.* a yard, a border one yard wide being left uncovered?

11. A hall is 103·23 feet long and 83·25 feet broad and it is to be paved with equal square tiles: what is the size of the largest tile which will exactly fit, and how many of them will be required?

12. Find the cost of covering the floor of a hall 39 ft. $4\frac{1}{2}$ in. long by 20 ft. $11\frac{3}{4}$ in. wide with tiles each $5\frac{1}{4}$ inches by $4\frac{3}{4}$ inches, and costing (including the labour of laying them) £1 a hundred.

13. It cost 4*s.* 1*d.* to paint a board 2 ft. 4 in. square. How much was charged per square foot?

14. It cost £3. 17*s.* 6*d.* to gravel a path 20 yds. 2 ft. long and 5 ft. wide. How much did it cost per square yard?

15. Find the area of the four walls of a room 15 ft. long, 14 ft. wide and 10 ft. high.

16. Find the area of the four walls of a room 16 ft. 4 in. long, 13 ft. 8 in. wide and 11 ft. 4 in. high.

17. Find the area of the four walls of a room 14 ft. 6 in. long, 13 ft. 10 in. wide and 10 ft. 8 in. high.

18. A room is 18 feet long, 13 ft. 6 in. wide, and 12 feet high. How much paper 21 inches wide will be required to cover the walls, and what will be the cost at 2*s.* 6*d.* per piece of 12 yards?

19. How much will it cost to paper a room 17 ft. 6 in. square, and 14 ft. 3 in. high, with paper 1 ft. 9 in. wide at 8*d.* a yard?

20. A room 22 ft. 3 in. long, 17 ft. 9 in. wide and 12 ft. 6 in. high, has two windows each 5 ft. 3 in. by 3 ft. 4 in., a door 7 ft. by 3 ft. 9 in., and a fire-place 5 ft. 3 in. by 4 ft. 4 in. How many pieces (each 12 yards long) of paper 21 inches wide would have to be bought to paper the room?

VOLUME OF SOLIDS.

151. A solid or body has three dimensions, namely length, breadth and thickness.

A **Rectangular Solid** is a body bounded by six plane faces which are all rectangles.

A **Cube** is a body bounded by six equal square faces.

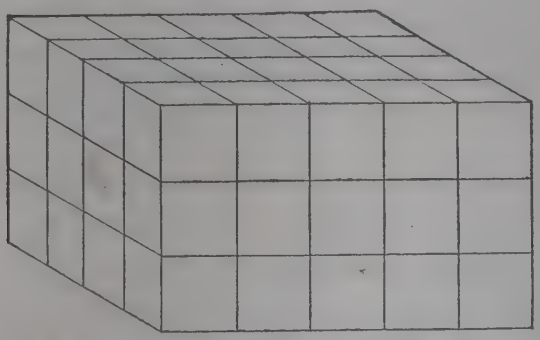
The **Volume** or **Capacity** of a Solid is the space included within its bounding surfaces, and this volume is measured by the number of times it contains some standard volume.

Different units in terms of which a volume can be expressed have already been given in Chapter III.

A **cubic inch** is a cube each of whose sides is one inch; and similarly for a *cubic foot*, &c.

152. To find the volume of a rectangular solid.

Suppose, for example, that the dimensions of the solid are 5 in. by 4 in. by 3 in. We can divide the edges respectively into 5, 4 and 3 parts each being one inch; and if planes be drawn through the points of division parallel to the outer faces of the solid, as in the figure, the whole solid will be divided up into equal cubes each of which is a cubic inch.



There will be as many layers of cubes as there are inches in the height of the solid, and the number of cubes in each layer will be the product of the number of inches in the length by the number of inches in the breadth.

Thus **the number of cubic inches (or cubic feet, &c.) in a rectangular solid is equal to the continued product of the number of inches (or feet, &c.) in its length, breadth and thickness.**

153. Now that we can find the volume of a rectangular solid, we can find the relations between the cubic yard, the cubic foot and the cubic inch.

For 1 cubic yard $= (3 \times 3 \times 3)$ cubic feet,

and 1 cubic foot $= (12 \times 12 \times 12)$ cubic inches.

Ex. 1. Find the volume of a rectangular block of stone 12 ft. long, 7 ft. wide and 1 ft. 6 in. high.

Volume $= (12 \times 7 \times 1\frac{1}{2})$ cubic feet $= 126$ cubic feet.

Ex. 2. A beam 1 ft. 6 in. wide and 1 ft. 3 in. high contains $46\frac{1}{4}$ cubic feet of timber; what is its length?

Since volume = length × breadth × thickness,

$$\text{length} = \frac{\text{volume}}{\text{breadth} \times \text{thickness}}.$$

Hence length required $= \dfrac{46\frac{1}{4}}{1\frac{1}{2} \times 1\frac{1}{4}}$ ft. $= 24\frac{2}{3}$ ft.

Ex. 3. The Imperial Gallon is the volume occupied by 10 lbs. of water, and a cubic foot of water weighs 1000 oz. nearly; find, on this supposition, the number of cubic inches in a gallon.

Since 1728 cubic inches weigh 1000 oz., $\frac{1728}{1000}$ cubic inches weigh 1 oz., and therefore $\frac{1728}{1000} \times 160$ cubic inches weigh 160 oz., that is 10 lbs., which is the weight of a gallon.

Hence a gallon contains $\frac{1728}{1000} \times 160$ cubic inches, that is 276·48 cubic inches.

Ex. 4. How many gallons will a cistern 6 ft. long, 4 feet wide and 3 feet high hold, it being given that a gallon of water weighs 10 lbs. and that a cubic foot of water weighs 1000 oz.

The cistern will hold $(6 \times 4 \times 3)$ cubic feet.

Hence the weight of the water which will fill the cistern is 1000 ounces $\times 6 \times 4 \times 3$.

A gallon weighs 160 ounces. Hence the number of gallons required

$$= \frac{6 \times 4 \times 3 \times 1000}{160} = 450.$$

Ex. 5. The external dimensions of a rectangular stone tank are length 12 ft. 6 in., breadth 8 ft., and height 4 ft. The interior is also rectangular, and the sides and bottom are 3 inches thick. Find the number of cubic feet of stone in the tank.

The internal length $= 12$ ft. 6 in. $- 3$ in. $\times 2 = 12$ ft.,

the internal breadth $= 8$ ft. $- 3$ in. $\times 2 = 7$ ft. 6 in.,

and the internal height $= 4$ ft. $- 3$ in. $= 3$ ft. 9 in.

Now the volume of the stone is the difference between the volumes given by the external and internal dimensions.

Hence volume required $= (12\frac{1}{2} \times 8 \times 4 - 12 \times 7\frac{1}{2} \times 3\frac{3}{4})$ cubic feet

$= (400 - 337\frac{1}{2})$ cubic feet $= 62\frac{1}{2}$ cubic feet.

EXAMPLES VIII. C.

Find the volumes of the rectangular solids whose dimensions are as under :

1. 5 ft. by 4 ft. by 2 ft. **2.** 12 ft. by 6 ft. by 4 ft.
3. 3 yds. by $1\frac{1}{2}$ yds. by 2 ft. **4.** 5 yds. by $2\frac{1}{2}$ yds. by 4 ft.
5. 6 ft. 4 in. by 4 ft. 3 in. by 2 ft. 6 in.
6. 7 ft. 9 in. by 5 ft. 3 in by 3 ft. 6 in.
7. 5 yds. 1 ft. by 3 yds. 2 ft. by 2 ft. 9 in.
8. 6 yds. 9 in. by 2 yds. 1 ft. by 2 ft. 7 in.

9. A rectangular block of stone 4 ft. long and 2 ft. 6 in. broad contains $17\frac{1}{2}$ cubic feet of stone ; what is its height ?

10. Find the length of a rectangular beam which contains 98 cubic feet of timber and whose cross section is 2 feet square.

11. How many loads (cubic yards) of gravel would be required to cover a path 150 yards long and 4 feet wide to a depth of 2 inches ?

12. A school room whose floor is 60 ft. by 40 ft. has accommodation for 360 children, allowing 100 cubic feet of air for each child. What must be the height of the room ?

13. If 1 pint = 34·66 cubic inches and 1 gallon of water weighs 160 ounces Avoir., find the number of gallons and the weight of the water which would fall on an area of an acre during a rainfall of one inch.

14. A tank is 21 ft. 4 in. long, 3 ft. wide and 2 ft. deep ; it is filled with water to within 3 inches of the top. What is the volume of the water, and what is its weight ? [A cubic foot of water weighs 1000 ounces.]

15. If a cubic foot of water weigh 1000 oz. what weight of water will fall on an acre of ground during a rainfall of an inch ?

16. A level tract of land 20 miles long and $\frac{3}{4}$ of a mile broad is flooded to a depth of 4 feet. Given that a cubic foot of water weighs 62 lbs., find in tons the weight of the water on the land.

17. A cubical cistern open at the top, costs £16. 6*s*. 8*d*. to line with lead at 1*s*. 4*d*. per square foot. How many cubic feet will it contain ?

18. What would be the weight of a beam of oak 18 inches square and 20 feet long, on the supposition that oak weighs 845 ounces per cubic foot ?

19. How many cubic inches of wood are there in a wooden box whose external dimensions are 4 ft. 4 in. by 3 ft. 10 in. by 3 ft. 6 in., the wood being everywhere one inch thick ?

20. An iron safe is everywhere $1\frac{1}{2}$ in. thick, and its external dimensions are 6 ft. by 4 ft. 6 in. by 3 ft. 6 in. How much does the iron weigh, assuming that a cubic foot of iron weighs about 480 lbs. ?

CHAPTER IX.

APPROXIMATION.

154. No continuous magnitude can be measured with *perfect* accuracy. When, for example, we endeavour to make two pieces of wire equally long, all that we can ensure is that they shall be of the same length *so far as the eye, or other instrument, can judge;* they however may, and probably will, really differ by some thousandths or even hundredths of an inch.

In all questions involving continuous magnitude, such as length, weight, &c., we must therefore be content with *approximations* more or less accurate to the true measure. It therefore follows that calculations dependent upon measurement can only give approximately accurate results.

For example, if we are told that a slab of stone is 17·6 inches long and 12·4 inches wide, we are not to conclude that these are *perfectly accurate* measurements, but only that the measurements are near enough for *practical* purposes, the real length and breadth being at any rate less than 17·7 and 12·5 respectively.

If the given measurements were quite accurate the area of the slab would be $(17{\cdot}6 \times 12{\cdot}4)$ square inches. The actual area may however have any value between $(17{\cdot}6 \times 12{\cdot}4)$ square inches and $(17{\cdot}7 \times 12{\cdot}5)$ square inches, that is between 218·24 square inches and 221·25 square inches.

155. When the measure of any quantity is given, for example as 3·628, it generally means that the measure is not less than 3·628 and not greater than 3·629, the possible error made by stopping at the third

decimal place being an error in defect less than *one-thousandth* of the unit. Now if the above measure had to be given as far only as hundredths of the unit, 3·63 would be more accurate than 3·62. This principle is often employed when approximate measures are given. Thus the quantity whose measure is 6·57684 would be most accurately given by 6·5768, 6·577, or 6·58 to 4, 3, or 2 decimal places respectively, the possible error in excess or defect being now not greater than *half* the unit represented by the last decimal place retained.

156. To find the sum of any numbers to any given number of decimal places it would be necessary to consider the figures *two* places beyond in order to see what had to be carried.

Ex. 1. Find to 3 places of decimals the sum of 14·61825, 3·17924, ·518479 and 154·017235.

14·618	25
3·179	24
·518	479
154·017	235
172·333	

Ex. 2. Find, to within one-thousandth, the sum of 5·3184, 27·5162, 18·4196 and 23·0135

5·31	84
27·51	62
18·41	96
23·01	35
74·27	

Here we have to find the sum correct to the first *four* figures. The sum of the numbers in the fifth column is 27, which is nearer to 30 than to 20. Hence the most accurate sum to four figures will be 74·27.

157. The method of finding a product or a quotient to any required degree of accuracy will be seen from the following examples.

Ex. 1. Find, to 2 places of decimals, the product of 4·163 and 5·784.

4·16	3	
	5·7	84
20·81	5	
2·91	4	1
·33	3	0
·01	6	6
24·08		

Arrange as in Art. 126, and begin the multiplication from the left of the multiplier. The vertical line on the left gives the figures which are to be finally retained; it is, however, necessary to go two places lower down to see what should be carried to the last column retained.

Multiply as usual so long as all the figures are to be retained, in the present case all the figures in the first two rows are to be retained.

Before multiplying by 8, cross out the last figure of the multiplicand, namely 3; then multiply 416 by 8, putting down the first figure of the product (adding in mentally what would be carried from the multiplication of the figure crossed out) in the last column. Now cross out another figure of the multiplicand, and multiply what remains by 4, again putting down the first figure of the product (with what must be carried from the multiplication of the last figure crossed out) in the last column. Proceed in this way to the end.

Since the sum of the figures in the fifth column is 18, the most accurate product we can give to two places of decimals is 24·08.

Ex. 2. Find, to within *one-millionth* of the whole, the product of 51·6243 and 112·4167.

51·624	3	
11	2·4	167
5162·43		
516·243		
103·248	6	
20·649	7	2
·516	2	4
·309	7	4
·036	1	3
5803·433		

Here we have to find the product correct to the first 7 figures.

Ex. 3. Find to within *one-millionth*, the quotient
$516{\cdot}24175 \div 123{\cdot}456$.

$$
\begin{array}{r}
123{\cdot}456 \,)\, 516{\cdot}24175 \,(\, 4{\cdot}181585 \\
493\ 824 \\
\hline
22\ 4177 \\
12\ 3456 \\
\hline
10\ 07215 \\
9\ 87648 \\
\hline
19567 \\
12345 \\
\hline
7222 \\
6172 \\
\hline
1050 \\
987 \\
\hline
63 \\
61 \\
\hline
\end{array}
$$

We have here to find the first 7 figures of the quotient. Having found the first 3 figures in the ordinary way, the remaining 4 figures, being less by *two* than the number of figures in the divisor, can be found by a shortened process. Namely, instead of annexing a nought at every stage on the right of the remainder as usual, we strike out the last figure on the right of the divisor instead, taking care, however, to use the last figure struck out to see what should be carried.

Ex. 5. Find, to the nearest penny, the value of
£51·3125 × 17·1874.

Since $\frac{1}{4}d. = £{\cdot}001$ nearly, it will be unnecessary to retain more than 4 decimal places in the product.

Thus

£	
51·3125	
17·1874	
513·125	
359·1875	
5·1312	5
4·1050	0
·3591	8
205	2
£881·9285	
20	
*s.***18**·5700	
12	
*d.***6**·84	

Thus the value required is £881. 18*s*. 7*d*.

EXAMPLES IX.

Find the following to the nearest *one-thousandth* of the whole:

1. $14{\cdot}625 \times 31{\cdot}857$.

2. $15{\cdot}816 \times 19{\cdot}714$.

3. $156{\cdot}423 \times 175{\cdot}45$.

4. $138{\cdot}714 \times 89{\cdot}47$.

5. $314{\cdot}2108 \div 18{\cdot}306$.

6. $81{\cdot}4623 \div 129{\cdot}54$.

7. $15{\cdot}8193 \times 6{\cdot}7149 \div 1{\cdot}3425$.

8. $115{\cdot}416 \times 123{\cdot}518 \div 119{\cdot}417$.

Find to within a millionth of the whole:

9. $198{\cdot}4653 \times 5{\cdot}194238$.

10. $8{\cdot}10976429 \div 15{\cdot}623$.

Find to within one-thousandth of the whole the areas of the rectangles whose dimensions are:

11. 17·215 inches by 34·827 inches.

12. 184·27 yards by 112·53 yards.

13. Find to 4 places of decimals:

(i) $1+\frac{1}{1}+\frac{1}{1\times 2}+\frac{1}{1\times 2\times 3}+\frac{1}{1\times 2\times 3\times 4}+\ldots\ldots$

(ii) $1-\frac{1}{1}+\frac{1}{1\times 2}-\frac{1}{1\times 2\times 3}+\frac{1}{1\times 2\times 3\times 4}-\ldots\ldots$

Find the value in money, to the nearest farthing, of

14. £31·625 × 12·8743.

15. £119·48125 × ·46127.

16. £3·43125 × 118·274.

17. £·41623 × 179·198.

18. £111·845 ÷ 314·627.

CHAPTER X.

DECIMAL MEASURES. THE METRIC SYSTEM.

158. In almost all civilised countries, England being unfortunately an exception, the coinage and the different weights and measures have been arranged on the decimal system.

In France, Belgium and Switzerland all sums of money are expressed in terms of the **Franc**, with its sub-unit the **Centime** ($\frac{1}{100}$th of a franc). In Italy, Spain and Greece the standard unit of money is of exactly the same value as the franc, but is called by different names.

In Germany the standard unit is the **Mark**, with its sub-unit the **Pfennig** ($\frac{1}{100}$th of a mark).

In Austria the standard unit is the **Gulden**, with its sub-unit the **Kreutzer** ($\frac{1}{100}$th of a gulden).

Approximately £1 = 25·15 francs = 20·4 marks
= 10·2 gulden.

The Metric System.

159. In the **Metric System** of weights and measures, which is used throughout nearly the whole continent of Europe, the fundamental unit is the unit of length called a **metre**, which is approximately 39·3708 inches.

The standard units of *area*, *volume*, *capacity* and *weight* are derived from the metre.

160. Decimal sub-multiples of a standard unit are distinguished by the **Latin** prefixes *deci-*, *centi-*, *milli-*.

Decimal multiples of a standard unit are distinguished by the **Greek** prefixes *deca-*, *hecto-*, *kilo-*, *myria-*.

Thus 1 Metre (m.) = 10 deci-metres (dm.) = 100 centi-metres (cm.) = 1000 milli-metres (mm.).

Also 1 Deca-metre (Dm.) = 10 metres,
1 Hecto-metre (Hm.) = 100 metres,
1 Kilo-metre (Km.) = 1000 metres,
1 Myria-metre (Mm.) = 10000 metres.

161. The fundamental unit of *surface* is a **square metre**, that is a square each of whose sides is a metre.

In measuring land it is usual to take for unit a *square decametre*, called an **are**, with the sub-units

1 hectare (Ha.) = 100 ares,
1 centiare (ca.) = $\frac{1}{100}$ are.

162. The unit of *volume* is a **cubic metre**. The cubic metre is sometimes called a **stere**.

The unit of *capacity* is a *cubic decimetre*, which is called a **litre**.

Thus 1 cubic metre = 1000 litres.

163. The unit of *weight* is the **Gramme**, which is the weight of a *cubic centimetre* of distilled water at its greatest density.

1 Kilo-gramme (Kg.) = 1000 grammes.
100 Kg. = 1 Quintal.
1000 Kg. = 1 Tonneau (T.).

Since a litre is a cubic *deci*-metre = 1000 cubic *centi*-metres, it follows that a litre of water weighs 1000 grammes.

164. It would be a good exercise for the student to verify, by the contracted method of Chapter IX., Art. 157, the following relations between English weights

and measures and those of the Metric System, having given

1 metre = 39·3708 inches,
1 gallon = 277·274 cubic inches,
weight of 1 gallon of water at 60° F. = 10 lbs.*

TABLE.

	1 metre	= 39·3708 inches,
	1 Kilo-metre	= 39370·8 inches = 1093·63 yards.
Thus	8 Km.	= 5 miles, nearly.
	1 sq. metre	= 1550·0598 sq. in.
	1 Ha.	= 2·4711 acres.
1 cubic metre		= 61027·1 cubic inches = 220·097 gallons.
1 litre		= ·2201 gallons = 1·76 pints.
	1 gramme	= 15·432 grains.
	1 kilogram	= 2·2046 lbs. Av.
	1 tonneau	= ·948 tons.

165. By the adoption of a decimal system of weights and measures calculations are very much simplified.

Ex. 1. Reduce 15624·5 metres to kilometres.

We have only to divide by 1000; thus

15624·5 m. = 15·6245 Km.

The above length could be read

1 Mm. 5 Km. 6 Hm. 2 Dm. 4 m. 5 dm.,

but this is not usual.

Ex. 2. Find the cost of 4·625 metres at 3·25 francs per metre.

3·25 fr. × 4·625 = 15·03125 fr.
= 15 fr. 5 centimes,

to the *nearest* coin. [A five-centime piece is the smallest coin in use, although centime and two-centime pieces are really coined in France.]

* Weight of one gallon of water at its greatest density is 10·016 lbs.

EXAMPLES X.

1. A man spends 7·75 francs a day; how much does he save in a year (of 365 days) out of a yearly income of 3000 fr.?

2. A man spends 9·35 marks a day; how much in English money does he spend in a year (of 365 days), taking a mark to be worth $11\frac{3}{4}d.$?

3. Having given that a metre is 39·37079 inches, prove that the difference between 5 miles and 8 kilometres is nearly 51 yards.

4. Having given that a metre is 39·37079 inches, express an inch as a decimal of a metre.

5. How many hectares are there in a square mile, given that a metre is 39·37079 inches?

6. Having given that 1 metre = 39·37079 inches, and that a gallon contains 277·27 cubic inches; shew that a gallon = 4·543 litres.

7. Having given that 1 m. = 39·37079 in., and that a cubic foot of water weighs 62·42 lbs.; shew that 1 lb. = 453·59 grammes.

8. If a cubic centimetre of iron weighs 7·788 g., what will be the weight of a cubic foot? Having given that
1 metre = 39·3708 inches, and 1 kilogram = 2·2406 lbs. Avoir.

9. If 146·8 Kg. of sugar cost 110·1 fr.; how much will 78·6 Kg. cost at the same rate?

10. A field is 192 metres long and 57·75 metres wide. How many hectares does it contain, and what would it cost at 7500 francs per hectare?

11. How much would it cost to put gravel to a depth of a decimetre all over a court-yard 7·5 metres by 5·75 metres, the gravel and labour costing 8 francs per stere?

12. A tradesman sold goods for 310·5 fr. at a gain of 15 per cent. What was the cost?

13. Compare the cost of posting at a shilling a mile and at 75 centimes per kilometre. [1 metre = 39·37 inches and £1 = 25 francs.]

14. A vessel steams 18 knots an hour. How many kilometres an hour is this equivalent to, taking 1 knot = 6080 feet and 1 metre = 3·281 feet?

CHAPTER XI.

RATIO AND PROPORTION. THE UNITARY METHOD.

166. THE relation between one quantity and another of the same kind with respect to magnitude, measured by the number of times the first contains the second, is called the **ratio** of the first to the second.

We may also say that the ratio of one quantity to another is *the fraction the first is of the second*, or that it is *the fraction which expresses the first in terms of the second as unit.*

Thus the ratio of 6 ft. to 2 ft. is 3, the ratio of 62 cwt. to 16 cwt. is $\frac{62}{16}=3\frac{7}{8}$.

The ratio of one quantity to another, for example of 8 lbs. to 12 lbs., is expressed by the notation

$$8 \text{ lbs.} : 12 \text{ lbs.},$$

and the ratio is equal to

$$8 \text{ lbs.} \div 12 \text{ lbs.} = \frac{8}{12} = \frac{2}{3}.$$

167. The two quantities compared in a ratio are called the **terms** of the ratio.

The old names **antecedent** and **consequent** for the first and second terms respectively of a ratio are still sometimes used.

The terms of a ratio must be of the same kind of magnitude, for we cannot compare, for example, tons with weeks, or acres with gallons.

168. When one term of a given ratio is given the other can at once be found.

Ex. 1. What sum of money has to £3 the ratio 5 : 8?

Here $\frac{\text{sum required}}{£3} = \frac{5}{8}$;

$\therefore$ sum required $= \frac{5}{8}$ of £3 $=$ £1. 17*s.* 6*d.*

Ex. 2. Find, in its simplest form, the ratio £3. 13*s.* 4*d.* : £4. 6*s.* 8*d.*

The ratio $= \frac{£3.\ 13s.\ 4d.}{£4.\ 6s.\ 8d.} = \frac{3\frac{2}{3}£}{4\frac{1}{3}£} = \frac{11}{13}$.

EXAMPLES XI. A.

Express in their simplest forms the following ratios.

1. 24 : 36. **2.** 169 : 91. **3.** 315 : 357. **4.** 792 : 924.

5. £5 : £7. 10*s.* **6.** £12. 13*s.* 4*d.* : £14. 6*s.* 8*d.*

7. £3. 14*s.* 3*d.* : £5. 1*s.* 9*d.* **8.** 3 tons 5 cwt. : 1 ton 15 cwt.

9. 3 cwt. 14 lbs. : 4 cwt. 1 qr. 14 lbs.

10. 3 lbs. 2 oz. Avoir. : 4 lbs. 11 oz. Avoir.

11. 9 hrs. 24 min. : 21 hrs. 9 min.

12. 7 miles 5 fur. 8 poles : 4 miles 6 fur. 37 po. 4 yds. 1 ft. 6 in.

13. Find what has to £1. 10*s.* the ratio 2 : 3.

14. .....£17. 11*s.* 4*d.* ..... 3 : 4.

15. .....11 cwt. 3 qrs. 15 lbs. ..... 7 : 11.

16. .....6 hrs. 18 min. 40 secs..... 25 : 16.

PROPORTION.

169. When four quantities are such that the ratio of the first to the second is equal to the ratio of the third to the fourth, the four quantities are said to be **proportionals.**

For example the ratio £5 : £15 is equal to the ratio 3 tons : 9 tons. Hence the four quantities £5, £15, 3 tons and 9 tons, in the order named, form a *proportion.* This is expressed by the notation £5 : £15 = 3 tons : 9 tons, or £5 : £15 :: 3 tons : 9 tons, the latter form being generally read £5 *is to* £15 *as* 3 tons *is to* 9 tons.

Since the two terms of any ratio must be of the same kind of magnitude, the first and second terms of a

proportion must be of the same kind, and the third and fourth terms must be of the same kind.

170. The first and fourth, of four quantities in proportion, are called the **extremes**, and the second and third are called the **means.**

In the above case the ratios are $\frac{5}{15}$ and $\frac{3}{9}$ respectively; and since $\frac{5}{15}=\frac{3}{9}$, it follows that $5\times 9=15\times 3$.

Thus in this, and similarly in other cases, **the product of the** (measures of) **the extremes is equal to the product of the** (measures of) **the means.**

Each pair of quantities must be expressed in terms of the *same unit.*

171. When any three terms of a proportion are known, the remaining term can be found.

Ex. 1. What quantity has to 3 cwt. 1 qr. the same ratio as 6*s.* : 3*s.* 3*d.*?

$$\frac{\text{Required quantity}}{3\text{ cwt. }1\text{ qr.}}=\frac{6s.}{3s.\ 3d.}=\frac{72}{39};$$

$$\therefore \text{ required quantity}=\left(\frac{72}{39}\times 13\right)\text{ qrs.}=24\text{ qrs.}=6\text{ cwt.}$$

Ex. 2. Fill up the blank in the proportion £1. 12*s.* 6*d.* : — :: 1 cwt. 18 lbs. : 1 cwt. 2 qrs. 2 lbs.

$$\frac{£1.\ 12s.\ 6d.}{\text{Required sum}}=\frac{1\text{ cwt. }18\text{ lbs.}}{1\text{ cwt. }2\text{ qrs. }2\text{ lbs.}}=\frac{130\text{ lbs.}}{170\text{ lbs.}}.$$

$$\therefore \text{ required sum}=\frac{170}{130}\text{ of £1. 12}s.\text{ 6}d.=£2.\ 2s.\ 6d.$$

172. Three numbers, or three quantities of the same kind, are said to be in **continued proportion** when the ratio of the first to the second is equal to the ratio of the second to the third; also the second is called the **mean proportional** between the first and third.

Thus 4, 8, 16 are in continued proportion, since 4 : 8 = 8 : 16.

EXAMPLES XI. B.

1. Find the fourth proportional to 4, 7 and 12.
2. Find the fourth proportional to 9, 8 and 3.
3. Find the fourth proportional to $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{5}{6}$.

4. Find the mean proportional to 4 and 9.

5. Find the mean proportional to 8 and 18.

6. Find the mean proportional to $\frac{2}{3}$ and $\frac{3}{2}$.

7. What quantity has to 5*s.* 6*d.* the same ratio as 4 ft. 3 in. has to 2 ft. 9 in.?

8. To what sum has 1*s.* 9*d.* the same ratio as 6 days 10 hrs. has to 7 days 8 hrs.?

9. Fill up the blank in the proportion
£1. 12*s.* 6*d.* : £2. 2*s.* 6*d.* :: 1 cwt. 18 lbs. : ————.

10. Fill up the blank in the proportion
£1. 12*s.* 6*d.* : £2. 2*s.* 6*d.* :: ———— : 1 cwt. 2 qrs. 2 lbs.

173. Questions in which a missing term of a proportion has to be found, and other questions of a similar nature, are best treated by a method which we proceed to consider which is called the *unitary method.*

THE UNITARY METHOD.

174. The method will be seen from the following examples :—

Ex. 1. If 5 lbs. of tea cost 9*s.* 2*d.*, how much will 8 lbs. cost at the same rate?

Since cost of 5 lbs. = 9*s.* 2*d.*,
..... **1** lb. = 9*s.* 2*d.* ÷ 5;
∴ ..... 8 lbs. = 9*s.* 2*d.* ÷ 5 × 8 = 14*s.* 8*d.*

Ex. 2. If 1 cwt. 2 qr. 21 lbs. of sugar cost 10*s.* 6*d.*, how much will 2 cwt. 1 qr. cost at the same rate?

Since cost of 1 cwt. 2 qr. 21 lbs. (189 lbs.) = 10*s.* 6*d.*;
∴ ..... **1** lb. ..... = 10*s.* 6*d.* ÷ 189;
∴ .....2 cwt. 1 qr. ..... (252 lbs.) = 10*s.* 6*d.* ÷ 189 × 252
= 10*s.* 6*d.* × $\frac{252}{189}$ = 10*s.* 6*d.* × $\frac{4}{3}$
= 14*s.*

Ex. 3. If 27 men can mow a field in 4 days, how many men would it take to mow the same field in 18 days?

Since 27 men take 4 days,
27 × 4 men would take **1** day;
∴ 27 × 4 ÷ 18 ..... 18 ...
Thus the number required = 27 × 4 ÷ 18 = 6.

Ex. 4. A bankrupt's debts amount to £2400, and his assets to £1380. How much can he pay in the pound?

For a debt of £2400 he can pay £1380;

$\therefore$ ..... **£1** ..... $£1380 \div 2400 = £\frac{1380}{2400} = 11s.\ 6d.$

Ex. 5. A bankrupt's debts amount to £1375. 18*s.* 9*d.* and his assets to £1009. 0*s.* 5*d.* What dividend can he pay in the pound?

For a debt of £1375. 18*s.* 9*d.* he can pay £1009. 0*s.* 5*d.*;

$\therefore$ ..... **£1** ..... $\frac{\text{£1009. 0s. 5d.}}{\text{£1375. 18s. 9d.}}$ of £1.

And $\frac{\text{£1009. 0s. 5d.}}{\text{£1375. 18s. 9d.}}$ of $£1 = \frac{24216\,5}{33022\,5}£ = \frac{11}{15}£ = 14s.\ 8d.$

EXAMPLES XI. C.

1. If 15 tons cost £36, what will 50 tons cost at the same rate?

2. If 76 articles cost £11. 8*s.*, how much will 85 cost at the same rate?

3. A man walked 12 miles in 3 hours; how far would he travel in $1\frac{1}{2}$ hrs. at the same rate?

4. If a certain quantity of food would be consumed by 1100 men in 45 days, how many days would it last 500 men?

5. If 30 bushels of oats last 20 horses for 8 days, how long would the same quantity last 16 horses?

6. If 27 men can mow a field in 8 hours, how long will 36 men take to mow the same field?

7. If 18 yards are bought for 16*s.* 6*d.*, find the price of 111 yards.

8. How far should 100 tons be carried for the money charged for carrying 75 tons a distance of 120 miles?

9. If 19 men do a certain piece of work in 117 days, how long will it take 13 men to do the same work?

10. If 19 horses can be bought for £475, how many can be bought for £700 at the same rate?

11. If 25 cows cost £387. 10*s.*, how much will 6 cost at the same rate?

12. If a train run 704 yards in 12 seconds, how long will it take to go half a mile?

13. How many men would do in 20 days the same amount of work as 15 men can do in 16 days?

14. How long would 75 horses take to consume the same quantity of food that 40 horses eat in 15 days?

15. If I lend a man £100 for 14 weeks, how long ought he to lend me £175 in return?

16. If 7 cwt. 4 lbs. of steel cost £27. 1*s.* 9*d.*, what will 3 cwt. 4 lbs. cost at the same rate?

17. If 1 cwt. 19 lbs. of coffee cost £8. 14*s.* 8*d.*, how much will 5 cwt. cost at the same rate?

18. If 123 yards of silk cost £36. 18*s.*, how much can be bought for £45. 6*s.* at the same rate?

19. A man walks 9 miles in 2 hours; how long will he take to walk 12 miles at the same rate?

20. If I lend a man £350 for 34 weeks, how long ought he to lend me £170 in return?

21. If gold is worth £4. 3*s.* 4*d.* per ounce, what is the value of a cup weighing 7 oz. 5 dwts. 12 grs.?

22. Find the value of 12 things any 7 of which are worth £26. 18*s.* 10$\frac{1}{4}$*d.*

23. If 3$\frac{1}{2}$ lbs. can be bought for £1. 17*s.* 4*d.*, how much can be bought for £106. 2*s.* 8*d.*?

24. If a ton of sugar cost £11. 13*s.* 4*d.*, how much will 8 cwt. 3 qrs. 20 lbs. cost at the same rate?

25. Find the cost of 5 cwt. 1 qr. 22 lbs. at £3. 19*s.* 4*d.* per cwt.

26. Find the cost of 6 cwt. 2 qrs. 14 lbs. at £5. 7*s.* 4*d.* per cwt.

27. If 32 cwt. 14 lbs. cost £29. 19*s.* 8*d.*, what will be the cost of 46 cwt. 8 lbs.?

28. If 13 yds. 1 ft. of silk cost £3. 15*s.*, what will be the cost of 23 yds. 1 ft.?

29. If 108 qrs. 1 bus. 1 pk. of barley cost £201. 17*s.* 10*d.*, what will 75 qrs. 2 bus. 3 pks. cost?

30. If 5 cwt. 2 qrs. 17 lbs. cost £211, find the cost of a ton.

31. If 17 acres 3 roods 33 poles cost £1236. 6*s.*, how much will 56 acres 2 roods 21 poles cost at the same rate?

32. A rate of 13$\frac{1}{2}$*d.* in the pound brings in £1013. 5*s.* 9*d.*; how much will a rate of 17*d.* in the pound bring in?

33. 7 tons 13 cwt. 42 lbs. of an article cost £79. 4*s.* 10$\frac{1}{2}$*d.*; what will 7 cwt. 7 lbs. cost at the same rate?

34. If 4 cwt. 1 qr. 26 lbs. of tobacco can be bought for £56. 9*s.* 6*d.*, how much can be bought for £14. 19*s.* 3*d.*?

35. A creditor received only 2*s.* 8*d.* in the pound due to him, and thereby lost £433. 6*s.* 8*d.* How much was due to him?

36. A bankrupt's liabilities are £5374. 15*s.* 9*d.* (including £117. 12*s.* 3*d.* for rent, rates and wages, which have to be paid in full). His assets are £4498. 11*s.* 10*d.* What dividend in the pound can he pay his creditors?

37. If a bankrupt pays 14*s.* 8*d.* in the pound, how much will a creditor receive to whom he owes £363. 15*s.*?

38. A bankrupt's assets are £3363. 15*s.* 3*d.*, and he owes £12814. 6*s.* 8*d.* What will his estate pay in the pound?

39. The rates on a certain estate amount to £487. 8*s.* $1\frac{3}{4}d.$ at 2*s.* $2\frac{3}{4}d.$ in the pound. What is the value of the property?

40. A bankrupt's liabilities amount to £9653. 10*s.*; he can pay a dividend of 15*s.* $4\frac{1}{2}d.$ in the pound. What is the amount of his assets?

41. Find the rateable value of a parish when a rate of 1*s.* 11*d.* in the pound produces £7297. 18*s.*

42. A bankrupt's liabilities are £3758. 9*s.* 4*d.*, and he can pay 13*s.* 9*d.* in the pound. What are his assets?

43. If the rates on a house, the rent of which is £52. 10*s.*, amount to £7. 6*s.* $6\frac{3}{4}d.$, what is the rent of a house the rates on which are £11. 14*s.* 6*d.*?

44. After paying income tax at 6*d.* in the pound, a man had £536. 5*s.* left. What was his income?

45. A wall 100 yards long, $8\frac{1}{2}$ feet high and 14 inches thick cost £131. 5*s.* Calculate the cost at the same rate of building a wall of the same height and thickness so as to enclose a space 53 feet by 44 ft. 8 in.

175. We now proceed to cases of greater complexity.

Ex. 1. If 5 men can mow 15 acres of grass in 4 days, how many men could mow 36 acres in 12 days?

5 men mow 15 acres in 4 days;

$\therefore 5 \times 4$ ..... 15 ..... 1 .....;

$\therefore 5 \times \frac{4}{12}$ ..... 15 ..... 12 .....;

$\therefore 5 \times \frac{4}{12} \div 15$ ..... 1 ..... 12 .....;

$\therefore 5 \times \frac{4}{12} \div 15 \times 36$ ..... 36 ..... 12 .....

Thus the number of men required $= 5 \times \frac{4}{12} \times \frac{36}{15} = 4$.

After some practice in questions of this nature the reasoning could be done mentally, and only the last line need be written down.

Ex. 2. If 4 men can dig a trench 100 ft. long, 4 ft. wide and 9 ft. deep in 6 days, how long a trench 5 ft. wide and 8 ft. deep would 5 men dig in 8 days?

We have to consider changes in the number of *men*, in the *days*, in the *width* of the trench and in its *length*.

First, with the other particulars unaltered, if 4 men dig a trench 100 feet long, 1 man will dig 100 ft. $\div 4$, and 5 men will dig 100 ft. $\div 4 \times 5 =$ 100 ft. $\times \frac{5}{4}$.

Now consider the alteration in the days. Since 100 ft. $\times \frac{5}{4}$ is dug in 6 days, 100 ft. $\times \frac{5}{4} \div 6$ will be dug in 1 day, and therefore

$$100 \text{ ft.} \times \frac{5}{4} \div 6 \times 8, \text{ that is } 100 \text{ ft.} \times \frac{5}{4} \times \frac{8}{6},$$

will be dug in 8 days.

Next consider the alteration in the width of the trench. Since a length of 100 ft. $\times \frac{5}{4} \times \frac{8}{6}$ is dug when the trench is 4 ft. wide,

$$100 \text{ ft.} \times \frac{5}{4} \times \frac{8}{6} \times 4 \div 5, \text{ that is } 100 \text{ ft.} \times \frac{5}{4} \times \frac{8}{6} \times \frac{4}{5},$$

will be dug when the trench is 5 ft. wide.

Lastly, since a length of 100 ft. $\times \frac{5}{4} \times \frac{8}{6} \times \frac{4}{5}$ is dug when the trench is 9 ft. deep, a length of 100 ft. $\times \frac{5}{4} \times \frac{8}{6} \times \frac{4}{5} \times 9 \div 8$ will be dug when the trench is 8 ft. deep.

Thus the length required $= 100$ ft. $\times \frac{5}{4} \times \frac{8}{6} \times \frac{4}{5} \times \frac{9}{8} = 150$ ft.

The reasoning may be done mentally and only the last line need be written down.

EXAMPLES XI. D.

1. If 10 men can mow 25 acres of grass in 3 days, how many men would be required to mow 35 acres in 6 days?

2. A man performs a journey of 180 miles in 3 days, travelling 8 hours a day. How long would he be in performing a journey of 630 miles if he travelled 7 hours a day at the same rate?

3. If 3 men can mow 20 acres of grass in 8 days, how many acres will 16 men mow in 12 days?

4. If 15 men are fed for 7 days at a certain cost when wheat is 4*s.* per bushel; what must be the cost per bushel when 10 men can be fed for 8 days at the same cost?

5. If 30 men, working 8 hours per day at a uniform rate, earn £72 in 9 days; how much will 63 men earn in 10 days, who work for 13 hours per day at two-thirds of the same uniform rate. Wages being proportional to the amount of work done.

6. If 10 men working 8 hours a day can do a piece of work in 24 days, how many men would be required to do the same piece of work in 6 days, working 10 hours a day?

7. The mason's bill for a certain piece of work is £110. 16*s.* 8*d.* when they work 9 hours a day and receive 4*s.* 9*d.* a day for wages: what will the same work cost when the men work 8 hours a day and receive 5*s.* a day for wages?

8. If 7 men, working 10 hours a day, mow 21 acres of grass in 4 days, in how many days will 5 men, working 8 hours a day, mow 33 acres?

9. How many days will it take 150 men to excavate a canal 1 mile long, 21 ft. wide and 9 ft. deep, 75 days having been required by 210 men to excavate a canal 700 yards long, 24 feet wide and 11 ft. deep?

10. If the wages of 11 labourers for 15 weeks amount to £103. 2*s.* 6*d.*, in what time will the wages of 13 labourers amount to £170. 12*s.* 6*d.*?

11. If 6 compositors in 16 days of $10\frac{1}{2}$ hours each can set in type 720 pages, each of 60 lines with 40 letters in a line; in how many days of 7 hours each will 9 compositors set 960 pages, each of 45 lines with 50 letters in a line?

12. If 12 men can do a piece of work in 11 days of 10 hours each, how many men must be employed to do three times the amount in 20 days of 11 hours?

13. If 24 oxen require 6 acres of turnips to supply them for 10 weeks, how many acres would supply 6 score of sheep for 15 weeks, 3 oxen eating as much as 10 sheep?

14. If the cost of provisioning a gun-boat carrying 84 men be £598. 10*s.* when the ship is at sea for 95 days, what will it cost to provision for 33 days a ship carrying a crew of 110 men?

15. If the carriage of 41 cwt. 1 lb. for 49 miles cost £20. 9*s.* 6*d.*, what must be paid for the carriage of 13 cwt. 2 qrs. 19 lbs. for 35 miles?

16. A MS. of 250 pages, each containing 28 lines with an average of 12 words in a line, takes 30 days to copy; how long would it take to copy a MS. of 100 pages, each containing 32 lines, and with an average of 14 words in a line?

17. If 5 men can reap a field 800 ft. long and 720 ft. broad in 3 days of 10 hours each, in how many days of 12 hours each can 6 men reap a field 768 feet long and 360 feet wide?

18. If 10 men can build a wall 225 feet long and 12 ft. high in 18 days, working 8 hours a day, how many hours a day must 12 men work to build a wall 150 feet long and 9 feet high in 8 days?

19. If it takes £22. 10*s.* to provide oats for 14 horses for 10 days when oats are 16*s.* 6*d.* per quarter; how much will it cost for 110 horses for 3 weeks when oats are 13*s.* 6*d.* per quarter?

20. If a package containing $7\frac{1}{2}$ cwt. be carried 125 miles for 14*s.* 7*d.*, how much should be charged for the carriage of 3 tons 15 cwt. for a distance of 200 miles?

21. If 8 bricklayers build a wall 40 ft. long and 3 ft. high in 8 days, how long would it take 14 men to build a wall of the same thickness 30 feet long and 7 feet high?

22. If 100 men dig a trench 200 yards long, 3 yds. wide and 2 yds. deep in 6 days of 10 hours each, in how many days of 8 hours each can 180 men dig a trench 360 yds. long, 4 yds. wide and 3 yds. deep?

23. If 7 men can mow 84 acres in 12 days working $8\frac{1}{4}$ hours a day each, how many hours per day must they work when there are 208 acres to be finished by 20 men in 11 days?

24. If ·056 of a trench be excavated in ·14 of a day by 24 men, in how many days will 20 men excavate a trench of the same size?

25. A contractor engaged to make $7\frac{1}{2}$ miles of road in 252 days, but after employing 54 men for 180 days he found they had only finished $4\frac{1}{2}$ miles: how many additional men must he engage, that the work may be finished in the prescribed time?

CHAPTER XII.

PERCENTAGES. PROFIT AND LOSS.

176. In many cases the ratio of one number to another, or of one quantity to another of the same kind, is expressed by the *number of hundredths* the first is of the second, and this is called the **percentage** the first is of the second.

For example, since $\frac{1}{4} = \frac{25}{100}$, $\frac{1}{4}$ is 25 *per cent.*

Also $2s. = \frac{1}{10}$ of £1 $= \frac{10}{100}$ of £1; thus 2*s.* is 10 *per cent.* of £1.

Again, if in a town whose population is 46000 there are 115 deaths in a year, the death rate is $\frac{115}{460}$, that is ·25, for every 100 of population. Thus the death rate is said to be ·25 *per cent.* of the population.

177. The following examples will shew how to express any given quantity as a percentage of any other given quantity of the same kind.

Ex. 1. How much per cent is 5*s.* of £2?

$$5s. = \frac{5}{40} \text{ of } £2 = \frac{1}{8} \text{ of } £2 = \frac{\frac{100}{8}}{100} \text{ of } £2 = \frac{12\frac{1}{2}}{100} \text{ of } £2.$$

Thus the required rate per cent. is $12\frac{1}{2}$.

Ex. 2. In a town whose population was 243200 there were 15504 children born in a year. Find the percentage the number of births was of the population.

Number of births for every 100 of population

$$= \frac{15504}{243200 \div 100} = \frac{15504}{2432} = 6{\cdot}375.$$

178. The following examples will shew how to find a given percentage of a given quantity.

Ex. 1. What is $12\frac{1}{2}$ per cent. of £2?

$$12\frac{1}{2} \text{ per cent. of } £2 = \frac{12\frac{1}{2}}{100} \text{ of } £2 = \frac{25}{200} \text{ of } £2 = 5s.$$

Ex. 2. In a town whose population was 243200 the birth rate in a year was 6·375 per cent. of the population. How many children were born in the year?

$$6{\cdot}375 \text{ per cent. of } 243200 = \frac{6{\cdot}375}{100} \text{ of } 243200$$

$$= \frac{6375}{100000} \text{ of } 243200 = 15504.$$

Ex. 3. Of what sum is $22\frac{1}{2}$ per cent. equal to $4\frac{1}{2}d.$?

$$22\frac{1}{2} \text{ per cent.} = \frac{22\frac{1}{2}}{100} = \frac{45}{200} = \frac{9}{40}.$$

$$\text{Hence sum required} \times \frac{9}{40} = 4\tfrac{1}{2}d.;$$

$$\therefore \text{ sum required} = \frac{9}{2}d. \times \frac{40}{9} = 20d. = 1s.\ 8d.$$

179. It should be noticed that 5 per cent. $= \frac{5}{100} = \frac{1}{20}$, and is therefore equivalent to *a shilling in the pound.*

To calculate mentally 5 per cent. of any sum of money, take a shilling for every pound, $1\frac{1}{2}d.$ for every half-crown and $\frac{1}{4}d.$ for every $5d.$ left over.

Thus 5 per cent. on £113. $17s.$ $4d.$, that is on £113 + 6 half-crowns $+ 28d.$, is $113s. + 9d. + 1\frac{1}{4}d.$, neglecting a fraction of a farthing.

180. An agent who is employed to let or to collect rents for the landlord is usually paid a *percentage* of the total rental, which is called his **commission.**

The agent who is employed to buy or sell stocks or shares is called a **broker**, and his commission is called **brokerage.**

To ensure against damage by fire, a certain percentage of the total value of the property is paid to a Fire Insurance Company; and, in consideration of this payment, the Company undertakes to make good any damage caused by fire. In this and in other cases of insurance, the payment is called a **premium.**

Ex. 1. The total rental of an estate is £1765. 10*s.* and the agent is paid a commission of 5 per cent. What does the commission amount to?

Commission = 1765*s.* 6*d.* = £88. 5*s.* 6*d.*

Ex. 2. What is the annual premium of fire insurance for a building worth £1500, at the rate of 2*s.* per £100?

$$\text{Premium} = \frac{2s.}{100£} \text{ of } £1500 = \frac{1}{1000} \text{ of } £1500 = £1.\ 10s.$$

Ex. 3. The cost of repairs and other outgoings amounted to 20 per cent. of the gross rent of an estate, and the net rent received was £580: what was the gross rent?

The landlord received 80 per cent. of the gross rent; hence

$$\frac{80}{100} \text{ of gross rent} = £580;\ \therefore \text{ gross rent} = \frac{100}{80} \text{ of } £580 = £725.$$

EXAMPLES XII. A.

What fractions are denoted by the following rates per cent.?

1. 50.	**2.** 25.	**3.** 20.	**4.** 10.
5. 5.	**6.** $2\frac{1}{2}$.	**7.** $12\frac{1}{2}$.	**8.** $16\frac{2}{3}$.
9. $33\frac{1}{3}$.	**10.** $3\frac{1}{3}$.	**11.** $6\frac{1}{4}$.	**12.** $5\frac{5}{9}$.

What rates per cent. are equivalent to the following fractions?

13. $\frac{1}{2}$.	**14.** $\frac{3}{4}$.	**15.** $\frac{2}{3}$.	**16.** $\frac{1}{3}$.
17. $\frac{1}{10}$.	**18.** $\frac{1}{20}$.	**19.** $\frac{3}{40}$.	**20.** $\frac{11}{25}$.
21. $\frac{4}{15}$.	**22.** $\frac{5}{16}$.	**23.** $\frac{8}{30}$.	**24.** $\frac{11}{12}$.

Find the percentage the first of the following pairs of quantities is of the second:

26. 5*s.*, £1.	**27.** 7*s.* 6*d.*, £2.
28. 3*s.* 4*d.*, £1. 10*s.*	**29.** 2*s.*, 6*s.* 8*d.*
30. 1*s.* $1\frac{1}{2}$*d.*, £1. 2*s.* 6*d.*	**31.** 7 lbs., 1 cwt.
32. 1 hr. 12 min., 1 day.	**33.** 3 oz. 15 dwts., 1 lb. 10 dwts.

34. Write down 5 per cent. of the following sums, £7. 10*s.*, £17. 15*s.*, £23. 17*s.* 6*d.*, £115. 3*s.* 4*d.*, £127. 11*s.* 8*d.*, and £314. 14*s.* 2*d.*

35. How much is 10 per cent. of £17. 14*s.* 2*d.*?

36. How much is $12\frac{1}{2}$ per cent. of £31. 17*s.* 8*d.*?

37. How much is 6 per cent. of 3 tons 2 cwt. 3 qrs. 22 lbs.?

38. The rental of an estate was £1215. 12*s.* 6*d.*; what was the agent's commission at 8 per cent.?

39. After paying 5 per cent. to his agent a man received £299. 5*s.* What was the agent's commission?

40. What is the amount of the annual premium for the insurance of a building for £1725 at $\frac{1}{10}$ per cent.?

41. A landlord allowed to all his tenants 20 per cent. reduction from their rents. What was the nominal rent of a tenant whose reduced rent was £360?

42. The population of a certain town increased 50 per cent. in the 10 years from 1881 to 1891, and the population in 1891 was 34617; what was the population in 1881?

43. Find the percentage of error in the statement that 1 oz. Troy is equal to 1·1 oz. Avoir.

44. Assuming that a gallon of water weighs 10 lbs., and contains 277·274 cubic inches, find the percentage of error in the statement that a cubic foot of water weighs 1000 oz.

45. What was the gross income of a man whose net income, after paying an income-tax of 6*d.* in the pound, was £569. 8*s.*?

46. The outgoings were 3*s.* 6½*d.* in the pound of the gross income of an estate, and the net income was £425. 18*s.* 10*d.* What was the gross income?

47. The cost of repairs and other outgoings were 18 per cent. of the gross rent of an estate, and the net rent received was £1011. 10*s.* 1*d.*: what was the gross rent?

48. Fill up the following Table by giving the percentages of the 1889 amounts to the nearest *tenth*.

Receipts from—	1890	1889	Increase	
			Amount	Per Cent.
Ordinary passengers:	£	£	£	
First class	3,194,000	3,188,000	6,000	
Second class	2,646,000	2,705,000	59,000*	
Third class	21,143,000	19,785,000	1,358,000	
Season-ticket holders	2,316,000	2,196,000	120,000	
Excess luggage, mails, &c.	5,029,000	4,757,000	272,000	
Total	34,328,000	32,631,000	1,697,000	

* Decrease.

49. Fill up the following Table by giving the percentages of the 1889 numbers to the nearest *tenth*.

—	1890	1889	Increase	
			Number	Per Cent.
	No.	No.		
First class	30,187,000	30,075,000	112,000	
Second class	62,860,000	62,688,000	172,000	
Third class	724,697,000	682,420,000	42,277,000	
Total	817,744,000	775,183,000	42,561,000	

50. Fill up the following Table by giving the percentages of the 1889 amounts to the nearest *tenth*.

Receipts from—	1890	1889	Increase	
			Amount	Per Cent.
	£	£	£	
Mineral traffic	17,543,000	17,052,000	491,000	
General merchandise traffic	23,300,000	22,694,000	606,000	
Live stock	1,377,000	1,340,000	37,000	
Total	42,220,000	41,086,000	1,134,000	

PROFIT AND LOSS.

181. When anything is sold for more than it cost it is said to be sold *at a profit*, and when it is sold for less than it cost it is said to be sold *at a loss*. Profit or loss is often expressed as a *percentage* and this percentage is always to be reckoned on the **cost price**.

Thus, if goods which cost £50 are sold for £60, the profit is £10. And £10 is $(\frac{10}{50} =) \frac{20}{100}$ of the original outlay, so that the profit is 20 per cent. of the outlay.

182. The following examples will shew how to treat questions involving profit or loss.

Ex. 1. A house was bought for £400 and sold for £480; what was the profit per cent.?

The total profit = £480 − £400 = £80.

And £80 = $\frac{80}{400}$ of £400 = $\frac{20}{100}$ of £400.

Hence the gain was 20 per cent.

Ex. 2. An article cost £2. 3*s*. 4*d*. and was sold at a loss of 15 per cent. For how much was it sold?

Selling price = cost price − $\frac{15}{100}$ of cost price;

∴ selling price = $\frac{85}{100}$ of cost price = $\frac{85}{100}$ of £2. 3*s*. 4*d*.

= $\frac{17}{20}$ of 520*d*. = 442*d*. = £1. 16*s*. 10*d*.

Ex. 3. What was the cost price of goods which were sold for £56 at a gain of 12 per cent.?

Selling price = cost price + $\frac{12}{100}$ of cost price = $\frac{112}{100}$ of cost price;

∴ cost price = $\frac{100}{112}$ of selling price = $\frac{100}{112}$ of £56 = £50.

Ex. 4. By selling tea at 2*s*. 1*d*. a pound a grocer would gain 5 per cent. more than by selling the tea at 2*s*. a pound. What was the cost price?

The difference between 2*s*. 1*d*. and 2*s*. is 5 per cent. of the cost price. Hence 1*d*. = $\frac{5}{100}$ of cost price; ∴ 20*d*. = cost price.

Ex. 5. A manufacturer sells at a profit of 20 per cent. to a wholesale dealer, who sells at a profit of 15 per cent. to a retail dealer, and the retail dealer sells for 11*s*. 6*d*. and makes a profit of 25 per cent. Find the cost of manufacture.

It cost the retail dealer $\frac{100}{125}$ of 11*s*. 6*d*.

,, ,, wholesale dealer $\frac{100}{115}$ of $\frac{100}{125}$ of 11*s*. 6*d*.

,, ,, manufacturer $\frac{100}{120}$ of $\frac{100}{115}$ of $\frac{100}{125}$ of 11*s*. 6*d*.

Thus required cost = $\frac{100}{120} \times \frac{100}{115} \times \frac{100}{125}$ of 138*d*. = 80*d*. = 6*s*. 8*d*.

EXAMPLES XII. B.

What was the gain or loss per cent. in the following cases?

1. Cost price £20, selling price £24.
2. ..... 8*s.* 4*d.*, ... 9*s.* 6*d.*
3. ..... 1*s.* 8*d.*, ... 1*s.* 10*d.*
4. ..... 12*s.* 6*d.*, ... 15*s.*
5. ..... £140, ... £130.
6. ..... 5*s.*, ... 6*s.* 9*d.*
7. ..... 3*s.* 6*d.*, ... 4*s.* 1*d.*
8. ..... £1. 12*s.* 6*d.*, ... £1. 17*s.* $4\frac{1}{2}d.$
9. ..... £15. 8*s.* 4*d.*, ... £14. 12*s.* 11*d.*
10. ..... £3. 3*s.* 4*d.*, ... £4. 5*s.* 6*d.*
11. ..... £1. 5*s.* 10*d.*, ... £1. 1*s.* $3\frac{3}{4}d.$
12. ..... £15. 6*s.* 3*d.*, ... £12. 14*s.* $2\frac{1}{4}d.$

13. If an article be bought for 17*s.* 6*d.* and sold for 27*s.* 6*d.*, what is the gain per cent.?

14. What was the cost price of tea which is sold at a gain of 25 per cent. for 3*s.* 4*d.* a pound?

15. If a grocer buy 60 lbs. of tea for £4. 7*s.* 6*d.*, at what price per lb. must he sell it so as to make 20 per cent. profit?

16. An article was sold for 2*s.* 4*d.* at a gain of 12 per cent. What did it cost?

17. A grocer buys sugar at 30*s.* per cwt. and sells it at $3\frac{1}{2}d.$ per lb. How much per cent. does he gain?

18. The profit on an article sold for 12*s.* 6*d.* is 25 per cent.; what would be the profit if it were sold for 12*s.*?

19. By selling a house for £759 a builder gained 10 per cent. What would he have lost per cent. if he had sold for £621?

20. If a profit of $22\frac{1}{2}$ per cent. is made by selling an article for 12*s.* 3*d.*, what would be the selling price if the profit were only 5 per cent.?

21. A person bought a carriage and sold it for £7. 17*s.* 6*d.* more than he gave for it, thereby clearing 7 per cent. What did he give for it?

22. A house is sold for £4000 and 25 per cent. profit is made. How much per cent. profit would be made by selling for £3360?

23. A tradesman by selling an article for 6*s.* 9*d.* gains 35 per cent. What would he have gained per cent. if he had sold it for 8*s.* 3*d.*?

24. A man bought apples at the rate of 6 for 1*d.* and an equal number at the rate of 10 for 1*d.*; and he sold the whole at the rate of 5 for 1*d.* What profit per cent. did he make?

25. If 5 per cent. more be gained by selling an article for 1*s.* than by selling it for 11½*d.*, what was the original price?

26. If 3 per cent. more be gained by selling a horse for £83. 5*s.* than by selling for £81, what must have been the original cost?

27. If a woman by selling 5 herrings for 7*d.* gains 12 per cent., what per cent. would she gain by selling them at 6 for 9*d.*?

28. If a woman buys eggs at 10*d.* a dozen, how many ought she to sell for 9*d.* in order to gain 8 per cent.?

29. A spirit merchant mixes 4 gallons of whiskey which cost 15*s.* 3*d.* per gallon with 6 gallons which cost 16*s.* 11*d.* per gallon, and sells the mixture so as to gain 33⅓ per cent. At what price per gallon does he sell?

30. A man who had been paying 5 guineas for 4 tons of coal changed his coal merchant and then got 5 tons for 4 guineas. How much did he save per cent.?

31. A draper bought 240 yards of silk. He sold ¼ at a gain of 25 per cent., ⅓ at a gain of 20 per cent., and the remainder at a loss of 15 per cent., and received £160 in all. What was the cost price per yard?

32. A draper bought a piece of silk 35 yds. long; and, after cutting off 2 yards which were damaged, he sold the remainder so as to clear 10 per cent. on his outlay. How much per cent. was the selling price of a yard higher than the cost price?

33. A manufacturer sold at a profit of 25 per cent. to a wholesale dealer, who sold at a profit of 12 per cent. to a retail dealer, and the retail dealer sold for 13*s.* 5*d.* and made a profit of 15 per cent. What was the cost of manufacture?

34. A quantity of wheat was sold in succession by three dealers each of whom made a profit of 5 per cent. The last of the three sold for £3087; how much did it cost the first?

35. A house was sold by the builder at a profit of 30 per cent., and the purchaser sold it again at an advance of £117 in the price and gained 20 per cent. on his outlay. How much did the house cost the builder?

CHAPTER XIII.

INTEREST.

183. **Interest** is the payment made for the use of money. A *percentage* of the sum of money lent is generally charged as interest, and the percentage payable for each year is called the **Rate per cent.** per annum.

Thus, if money be borrowed at 5 per cent. per annum interest, then £5 will be charged each year for every £100 borrowed.

If nothing is said to the contrary, the percentage is always understood to be *per annum.*

The sum on which interest is charged is called the **Principal**, or the **Capital.**

The sum of the Principal and the Interest together is called the **Amount.**

When interest is supposed to be paid as it falls due the principal is said to be lent at **Simple Interest.**

184. We shall for the present consider cases of Simple Interest, that is cases in which the Interest is either paid as it falls due, or in which *nothing is paid as interest on the overdue interest itself.*

The way in which the simple interest is found when the principal, the rate per cent. and the time are given will be seen from the following examples.

Ex. 1. Find the simple interest paid in 5 years on £375 at 4 per cent.

Since the interest paid each year is 4 per cent., that is $\frac{4}{100}$ of the Principal,

$$\text{interest for 1 year} = £375 \times \frac{4}{100};$$

$$\therefore \text{ ..... 4 .....} = £375 \times \frac{4}{100} \times 5 = £\frac{375 \times 4 \times 5}{100} = £75.$$

It is generally best to multiply by the rate per cent. and by the number of years before dividing by 100.

From the above it will be seen that to find simple interest we **multiply the principal by the rate per cent. and by the number of years, and divide the product by 100.**

Ex. 2. Find the simple interest payable in 4 years for the loan of £273. 12*s.* 6*d.* at 5 per cent. per annum.

£.	*s.*	*d.*
273 .	12 .	6
		5
1368 .	2 .	6
		4
£54,72 .	10 .	0
20		
*s.*14,50		
12		
*d.*6,00		

Or thus: £273. 12*s.* 6*d.* = 273·625.

£273·625
5
1368·125
4
100) 5472·5
£54·725
20
*s.*14·500
12
*d.*6·0

Ex. 3. Find the interest on £160. 12*s.* 6*d.* for $\frac{3}{4}$ year at $2\frac{2}{3}$ per cent.

Here it is better to multiply by the product of the rate and time, and not by $2\frac{2}{3}$ and by $\frac{3}{4}$ separately.

EXAMPLES XIII. A.

1. Find the simple interest on £400 for 3 years at 4 per cent.
2. ..... £600 ... 4 ..... 3 .....
3. ..... £750 ... 6 ..... 5 .....
4. ..... £1230 ... 7 ..... 5 .....
5. ..... £125 ... 3 ..... 4 .....
6. ..... £745 ... 5 ..... 4 .....

7. Find simple interest on £315. 12*s.* 6*d.* for 2 years at 4 per cent.

8. ..... £172. 5*s.* ... 4 ..... 5 .....

9. ..... £744. 7*s.* 6*d.* ... $4\frac{1}{2}$ ..... 6 .....

10. ..... £561. 13*s.* 4*d.* ... $5\frac{1}{2}$ ..... 6 .....

11. ..... £318. 14*s.* 2*d.* ... 7 ..... $2\frac{1}{2}$.....

12. ..... £1172. 11*s.* 8*d.* ... 6 ..... $2\frac{1}{2}$.....

13. Find the amount of £500 in 4 years at 3 per cent. simple interest.

14. Find the amount of £382. 10*s.* in 6 years at 4 per cent. simple interest.

15. Find the amount of £385. 4*s.* 2*d.* in $4\frac{1}{2}$ years at 5 per cent. simple interest.

16. Find the simple interest on £4069. 0*s.* $2\frac{1}{2}$*d.* for 4 years at $3\frac{1}{5}$ per cent.

17. Find the simple interest on £3245. 10*s.* for $2\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.

18. Find the interest on £245. 16*s.* 8*d.* for 73 days at 4 per cent.

185. We have hitherto given only examples in which the interest can be exactly paid in money. This is by no means always the case; and, *as a penny is the smallest coin recognised by bankers, fractions of a penny may generally be disregarded.*

Ex. 1. Find the interest on £420. 12*s.* 6*d.* for 15 months at 5 per cent.

£.	*s.*	*d.*	
420 .	12 .	6	
		5	
2103 .	2 .	6	
		$1\frac{1}{4}$	
2103 .	2 .	6	
525 .	15 .	$7\frac{1}{2}$	
£26,28 .	18 .	$1\frac{1}{2}$	
	20		
s. 5,78			
	12		
d. 9,37			

Or thus: Interest $= \frac{5}{100} \times 1\frac{1}{4}$ of £420. 12*s.* 6*d.*
$= \frac{1}{16}$ of £420. 12*s.* 6*d.*
$=$ £26. 5*s.* $9\frac{3}{8}$*d.*

Thus, rejecting fractions of a penny, the interest required is £26. 5*s.* 9*d.*

Ex. 2. Find the interest on £357. 12*s.* 6*d.* from May 11 to July 20 at 3 per cent.

From May 11 to July 20 is 70 days*, that is $\frac{70}{365} = \frac{14}{73}$ of a year.

£357. 12*s.* 6*d.* = £357·625

Interest = £357·625 × 3 × $\frac{14}{73}$ ÷ 100

```
                £
           357·625
                 3
          1072·875
                14
          4291·500
         10728·75
73 ) 15020·25 ( 205·75
     146
     ---
      420
      365
      ---
       552
       511
       ---
        415
        365
        ---
         50
```

The division by 73 is only carried to *two* places of decimals, for when divided by 100 this will give the interest to within one *ten-thousandth* of a pound, and one penny is very nearly four *thousandths* of a pound.

Thus interest = £205·75 ÷ 100 = £2·0575 = £2. 1*s.* 1*d.*, rejecting fractions of a penny.

It may in some cases be required to find the interest to the *nearest* penny. It will be found that £2·0575 = £2. 1*s.* 1·8*d.*, and the interest to the *nearest* penny is £2. 1*s.* 2*d.*

EXAMPLES XIII. B.

Find the interest in the following cases, rejecting fractions of a penny :

1. On £722. 11*s.* 4*d.* for $1\frac{1}{4}$ years at 3 per cent.
2. „ £1562 for 4 years at 3 per cent.
3. „ £1850. 10*s.* for 3 years at 3 per cent.
4. „ £140 for 5 months at 3 per cent.
5. „ £875. 10*s.* for 3 years 4 months at 4 per cent.
6. „ £1035 for $10\frac{1}{2}$ months at 4 per cent.
7. „ £756. 12*s.* 6*d.* for 146 days at 4 per cent.
8. „ £380. 4*s.* 2*d.* for 120 days at $4\frac{1}{2}$ per cent.
9. „ £1167. 12*s.* 4*d.* from Jan. 1 to Feb. 20 at 5 per cent.
10. „ £1172. 13*s.* 10*d.* from May 13 to June 12 at 4 per cent.

* The first of the days is not counted; from January 1 to January 2, for example, is *one* day.

11. Find, to the *nearest* penny, the interest on £240. 12*s.* 6*d.* at $2\frac{1}{2}$ per cent. for 9 months.

12. Find to the *nearest* penny the simple interest on £886. 10*s.* 9*d.* for 4 years 2 months at $5\frac{1}{5}$ per cent.

186. Of the four quantities, Principal, Rate per cent., Time, and Simple Interest, any one can be found when the other three are given.

The following are examples of such questions.

Ex. 1. In what *time* will the simple interest on £350 amount to £21 at $4\frac{1}{2}$ per cent.?

Find the interest at the given rate per cent. for **one year;** this will be

$$£350 \times 4\tfrac{1}{2} \div 100 = £\tfrac{63}{4}.$$

Then number of years $= \dfrac{\text{whole interest}}{\text{interest for } \textit{one year}}$

$$= £21 \div £\frac{63}{4} = \frac{21 \times 4}{63} = \frac{4}{3}.$$

Thus the time required $= 1\frac{1}{3}$ years $= 1$ year 4 months.

Ex. 2. At what rate per cent. will the interest on £350 amount to £21 in 1 year 4 months?

Find the interest for the given time at **one per cent.;** this will be

$$£350 \times 1 \times 1\tfrac{1}{3} \div 100 = £\tfrac{14}{3}.$$

Then rate per cent. $= \dfrac{\text{whole interest}}{\text{interest at } \textit{one per cent.}}$

$$= £21 \div £\frac{14}{3} = \frac{21 \times 3}{14} = 4\tfrac{1}{2}.$$

Ex. 3. On what sum will the interest for 1 year 4 months at $4\frac{1}{2}$ per cent. amount to £21?

Find the interest on £100 *in the given time and at the given rate;* this will be $£100 \times 4\frac{1}{2} \times 1\frac{1}{3} \div 100 = £\frac{3}{2} \times \frac{4}{3} = £6.$

Then
£6 = interest on £100;
∴ £1 = ...£100 ÷ 6;
∴ £21 = ...£100 ÷ 6 × 21
= .....£350.

187. The Amount is equal to the sum of the Principal and Interest; and therefore if any two of these three quantities be given, the third can be at once found.

Ex. 1. In what time will £350 amount to £371 at $4\frac{1}{2}$ per cent.?
Simple Interest = Amount − Principal
= £371 − £350 = £21.
The question is therefore reduced to Ex. 1, Art. 186.

Ex. 2. What sum will amount to £371 in 1 year 4 months at $4\frac{1}{2}$ per cent.

Here, as in the examples in the previous article, we take a known quantity of the same nature as that required.

Find the amount of £100 *in the given time and at the given rate;* this will be £100 + £100 × $4\frac{1}{2}$ × $1\frac{1}{3}$ ÷ 100 = £100 + £6 = £106.

Then £106 = amount of £100;
∴ £ 1 = ... £100 ÷ 106;
∴ £371 = ... £100 ÷ 106 × 371
= ..... £350.

EXAMPLES XIII. C.

Fill up the blank spaces below:

	Principal,	Rate per cent,	Time,	Simple Interest.
1.	£350,	4,	,	£42.
2.	£215,	5,	,	£43.
3.	£178,	5,	,	£44. 10*s.*
4.	£123. 2*s.* 6*d.*,	4,	,	£7. 7*s.* 9*d.*
5.	£350,	,	3 yrs.,	£42.
6.	£178,	,	5 yrs.,	£44. 10*s.*
7.	£370,	,	$2\frac{1}{4}$ yrs.,	£27. 15*s.*
8.	£314. 12*s.* 6*d.*,	,	2 yrs. 8 m.,	£31. 9*s.* 3*d.*
9.	,	$2\frac{1}{2}$,	4 yrs.,	£37.
10.	,	$3\frac{1}{4}$,	3 yrs.,	£20. 16*s.*
11.	,	$2\frac{1}{2}$,	5 months,	£1. 10*s.*
12.	,	$3\frac{1}{3}$,	9 months,	£19. 6*s.* 3*d.*

13. What sum will amount to £495 in 2 years at 5 per cent. simple interest?

14. What sum will amount to £1262. 13*s.* 8*d.* in 3 years at 4 per cent. simple interest?

15. What sum will amount to £378. 8*s.* 9*d.* in $3\frac{1}{4}$ years at $2\frac{1}{2}$ per cent. simple interest?

16. In what time will £6755 amount to £7759. 16*s.* $1\frac{1}{2}$*d.* at $3\frac{1}{2}$ per cent. simple interest?

17. A money-lender charged £1 for a loan of £10 for a month; how much per cent. per annum did he get?

18. After lying out at interest for 73 days £500 became £504. 10*s.* Find the rate of interest per cent.

19. In what time will £1240 amount to £1336. 12*s.* 4*d.* at $4\frac{1}{4}$ per cent. simple interest?

20. A certain sum of money was deposited for 3 months in a bank paying $2\frac{1}{2}$ per cent., and the amount at the end of the time was £241. 10*s.* How much was deposited?

21. On what sum of money will the interest for 9 months at 6 per cent. amount to £3. 7*s.* 6*d.*?

22. If the interest on £410 for 2 months be £1. 14*s.* 2*d.*, what is the rate per cent.?

23. For how long must £535 be put out at $3\frac{3}{4}$ per cent. simple interest to produce £80. 5*s.* interest?

24. In what time will the interest on £666. 13*s.* 4*d.* amount to £20, 8*s.* 4*d.* at $5\frac{1}{4}$ per cent.?

25. On what capital will the interest for 219 days at 4 per cent. amount to £14. 2*s.* 6*d.*?

26. If the simple interest on £265. 8*s.* 9*d.* for 11 years be £145. 19*s.* $9\frac{3}{4}$*d.*, find the simple interest on £1178. 11*s.* 8*d.* for 3 years at the same rate.

27. A man lent a certain sum at 5 per cent., and the yearly interest, after deducting income-tax at 8*d.* in the pound, was £108. 15*s.* How much did he lend?

28. A person has £4800 invested at a certain rate per cent. and £3600 invested at $\frac{1}{4}$ per cent. lower, and his total income from these investments is £327. Determine the rates of interest.

29. A man lends £1000 in four sums. If he gets 3 per cent. for £200, $3\frac{1}{2}$ per cent. for £400, and 5 per cent. for £250, how much per cent. must he get for the remainder in order that his average interest may be £3. 17*s.* per cent.?

COMPOUND INTEREST.

188. When the interest on a sum of money as it falls due at the end of each year, or other period agreed upon, is added to the Principal, the money is said to be put out at Compound Interest.

The interest as it falls due is added to the principal, and the sum gives the principal on which interest becomes due for the next period; and so on to the end.

For example, to find the amount of £100 for 3 years at 5 per cent. per annum.

The interest due at the end of the first year is £5. The principal for the second year is £100 + £5, the interest on which for the second year is £5. 5*s*. The principal for the third year is £105 + £5. 5*s*. = £110. 5*s*., the interest on which for the third year is £5. 10*s*. 3*d*. Hence the total amount due at the end of the 3 years is £110. 5*s*. + £5. 10*s*. 3*d*. = £115. 15*s*. 3*d*.

To find the *interest* we must subtract the original capital, so that the interest = £115. 15*s*. 3*d*. − £100 = £15. 15*s*. 3*d*.

189. The following examples will shew how to deal with questions in Compound Interest.

Ex. 1. Find the compound interest on £1562. 10*s*. for 3 years at 4 per cent.

£.	*s*.
1562 .	10
	4
£62,50 .	0
	20
s.10,00	

£.	*s*.	
1562 .	10	= 1st Prin.
62 .	10	= 1st year's Int.
1625 .	0	= 2nd Prin.
	4	
£65,00 .	0	

£.	*s*.	
1625 .	0	= 2nd Prin.
65 .	0	= 2nd year's Int.
1690 .	0	= 3rd Prin.
	4	
£67,60 .	0	
	20	
s.12,00		

£.	*s*.	
1690 .	0	= 3rd Prin.
67 .	12	= 3rd year's Int.
1757 .	12	= Final Amount
1562 .	10	= Original Capital
195 .	2	= Interest reqd.

Or thus: £1562. 10*s*.

£.	
= 1562·5	= 1st Prin.
62·500	= 1st year's Int.
1625·0	= 2nd Prin.
65·000	= 2nd year's Int.
1690·0	= 3rd Prin.
67·600	= 3rd year's Int.
1757·6	= Final Amount
1562·5	= Original Capital
£195·1	= Interest reqd.

In the second method, we multiply by 4, the rate per cent., and *put the figures two places back*, which is equivalent to dividing by 100; this gives the interest at once.

The second method should always be adopted.

Ex. 2. Find, to the nearest penny, the amount at compound interest of £270. 12*s*. 6*d*. in 4 years at 5 per cent.

£	
£270. 12*s*. 6*d*. = 270·625	= 1st Prin.
13·53125	= 1st year's Int.
284·15625	= 2nd Prin.
14·20781	= 2nd year's Int.
298·36406	= 3rd Prin.
14·91820	= 3rd year's Int.
313·28226	= 4th Prin.
15·66411	= 4th year's Int.
£328·94637	= Final Amount.
20	
s. 18·92740	
12	
d. 11·12880	

Thus the amount to the nearest penny is £328. 18*s*. 11*d*.

To find the interest at 5 per cent., we may either multiply by 5 and put the figures two places back, or divide by 20, which we do by dividing by 2 and putting the figures one place back. **The rate per cent. need never be written down.**

Since 1*d*. is nearly equal to 4 *thousandths* of a pound, we only require the result to be true to 3 places of decimals, and to ensure this result it is only necessary to retain 5 places in the work, figures beyond this point are therefore rejected throughout.

Ex. 3. Find, to the *nearest* penny, the amount of £1156 in $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. compound interest.

£	
1156·	1st Principal
34·68	1st year's Interest
5·78	1st year's Interest
1196·46	2nd Principal
35·8938	2nd year's Interest
5·9823	2nd year's Interest
1238·3361	3rd Principal
12·38336	Half-year's Interest
6·19168	Half-year's Interest
3·09584	Half-year's Interest
£1260·00698	Final Amount
20	
s. 0·13960	
12	
d. 1·67520	

Thus the amount to the nearest penny is £1260. 0*s*. 2*d*.

The interest for the half year is $\frac{1}{2}$ of $3\frac{1}{2}$ per cent. $1\frac{3}{4}$ per cent. $= (1 + \frac{1}{2} + \frac{1}{4})$ per cent.

Ex. 4. Find, to the nearest penny, the compound interest on £3000 for 2 years at 5 per cent., the interest being payable half-yearly.

Since 5 per cent. for a year is $2\frac{1}{2}$ per cent. for a half year, we thus have 4 periods, the interest for each period being $2\frac{1}{2}$ per cent. of the capital at the beginning of that period.

Since $2\frac{1}{2}$ per cent. $=\frac{1}{40}$, the interest will be found by dividing by 4 and putting the figures one place back.

190. We have seen [Art. 186] that we can easily obtain any one of the four quantities Principal, Rate per cent., Time and Simple Interest, when the other three are given; we now give examples of this nature when the interest is compound.

Ex. 1. What sum will amount to £405. 12*s.* in 2 years at 4 per cent. compound interest?

Find the amount of £100 *in the given time and at the given rate;* this will be £108·16.

Then £100 will amount to £108·16;

$\therefore \frac{£100}{108{\cdot}16}$ £1;

$\therefore \frac{£100}{108{\cdot}16} \times 405{\cdot}6$..... £405·6.

Thus the sum required $= \frac{£100 \times 405{\cdot}6}{108{\cdot}16} = £375.$

Or thus: The amount at the end of a year at 4 per cent. is found by multiplying the principal at the beginning of the year by the fraction $\frac{104}{100}$. Hence the amount of any sum at the end of 2 years at 4 per cent. compound interest will be given by

$$\text{Amount} = \text{Principal} \times \left(\frac{104}{100}\right)^2;$$

$$\therefore \text{Principal} = \left(\frac{100}{104}\right)^2 \text{ of Amount} = \left(\frac{25}{26}\right)^2 \text{ of } £405.\ 12s. = £375.$$

Ex. 2. In what time will £3125 amount to £3425. 1*s.* 4*d.* at 4 per cent. compound interest?

Here we can proceed to find the amount of £3125 at 4 per cent. compound interest for 1, 2, 3, &c. years in order until we find an amount equal to that given, which we shall certainly do if the time required is an exact number of years.

In the present case it will be found that the amount at the end of 2 years would be £3380, and the amount at the end of 3 years would be £3515. 4*s.* Thus the time required is between 2 and 3 years. Now the interest for the time beyond 2 years is £3425. 1*s.* 4*d.* − £3380 = £45. 1*s.* 4*d.*, and the interest on £3380 for 1 year is £135. 4*s.*

Hence time beyond 2 years $= \frac{£45.\ 1s.\ 4d.}{£135.\ 4s.}$ of 1 year $= \frac{1}{3}$ of a year.

Thus time required $= 2\frac{1}{3}$ years.

Questions in which the rate per cent. is required will be considered later on. [See Art. 214, Ex. 4.]

EXAMPLES XIII. D.

Find, to the *nearest* penny, the compound interest on :

1. £150 for 2 years at 5 per cent.
2. £1250 for 2 years at 4 per cent.
3. £625 for 3 years at 4 per cent.
4. £480 for 3 years at 5 per cent.
5. £1500 for 3 years at 3 per cent.
6. £1800 for 3 years at 2 per cent.
7. £560. 10*s.* for 3 years at $2\frac{1}{2}$ per cent.
8. £1364. 5*s.* for 3 years at $3\frac{1}{2}$ per cent.
9. £1275. 12*s.* 6*d.* for 3 years at $4\frac{1}{2}$ per cent.
10. £1500 for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.
11. £3600 for $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.
12. £271. 15*s.* 3*d.* for $2\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.
13. £845. 14*s.* 6*d.* for $2\frac{1}{4}$ years at $1\frac{1}{4}$ per cent.
14. £666. 13*s.* 4*d.* for $2\frac{1}{2}$ years at $3\frac{1}{4}$ per cent.
15. £1875. 11*s.* 8*d.* for $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.

Find, to the nearest penny, the compound interest in the following cases, in each of which the interest is payable half-yearly :

16. £500 for 2 years at 5 per cent.
17. £850 for $1\frac{1}{2}$ years at 4 per cent.
18. £1200 for 2 years at $2\frac{1}{2}$ per cent.
19. £360. 12*s.* 6*d.* for 2 years at $3\frac{1}{2}$ per cent.
20. £1184. 3*s.* 4*d.* for 2 years at $4\frac{1}{2}$ per cent.
21. Find the difference between the simple and the compound interest on £1350 for 3 years at 5 per cent.
22. Find the difference between the amount of £1562. 10*s.* at simple and at compound interest for 3 years at 4 per cent.
23. Find the difference between the simple and the compound interest on £7500 at 4 per cent. for $2\frac{1}{2}$ years, the interest being payable half-yearly.

24. Find the difference between the compound interest on £500 for 2 years at 4 per cent. (1) when the interest is payable yearly, and (2) when the interest is payable half-yearly.

25. What sum of money lent at 5 per cent. per annum compound interest will amount in 2 years to £882?

26. What principal will amount to £6552. 3*s.* in 2 years at $4\frac{1}{2}$ per cent. per annum compound interest?

27. What sum will amount to £3515. 4*s.* in 3 years at 4 per cent. compound interest?

28. The compound interest on a certain sum of money for 3 years at 5 per cent. is £1324. 1*s.* What is the sum?

29. What sum invested for $1\frac{1}{2}$ years at 4 per cent. compound interest payable half-yearly will amount to £2210. 17*s.*?

30. The amount of £825 after $2\frac{1}{2}$ years at simple interest is £907. 10*s.* What would have been the amount, to the nearest penny, of the same sum at compound interest for the same time and at the same rate per cent.?

31. The compound interest on a certain sum for 2 years at 5 per cent. is greater than the simple interest by 7*s.* 8*d.* Find the principal.

32. The simple interest on a certain sum of money for 2 years would be £42. 10*s.*, and the compound interest for 2 years on the same sum at the same rate would be £43. 7*s.* Find the sum and the rate per cent.

33. Assuming that £1 will amount to £1·27628 in 5 years at 5 per cent., find the approximate amount in 10 years and also in 15 years.

CHAPTER XIV.

PRESENT WORTH AND DISCOUNT.

191. SINCE interest can always be obtained for the use of money, it follows that any sum of money paid at once is more valuable than the same sum which is to be paid at some future time; and that the actual value in cash of a sum of money due some time hence is that sum which together with the interest which could be obtained for its use for the given time would amount to the debt in question.

192. The actual value in cash of a sum of money due at some future time is called its **Present Value.**

The deduction from the nominal value of a debt, made when the debt is paid before it is due, is called **Discount.**

Since the amount of a debt due at some future time is equal to its Present Value together with the interest on the Present Value for the given time, it follows that

Discount is equal to the Interest on the Present Value.

193. The following are examples of the different questions which may arise.

Ex. 1. Find the present value of £410 due 6 months hence, money being supposed to be worth 5 per cent.

The interest on £100 for 6 months at 5 per cent. is £2. 10*s*.

Hence £100 cash is worth £102. 10*s*. in six months;

$\therefore \frac{£100}{102\frac{1}{2}}$ ..... £1 ..... ;

$\therefore \frac{£100}{102\frac{1}{2}} \times 410$ .....£410 .....

Thus the required present worth $= \frac{£100 \times 410}{102\frac{1}{2}} = £400.$

Ex. 2. Find the present value of £439. 8*s.* due 3 years hence, compound interest being reckoned at 4 per cent.

The amount of £1 at compound interest for 3 years at 4 per cent. will be found to be £1·124864.

Hence £1 cash is worth £1·124864 at the end of 3 years;

$\therefore$ £$\frac{439\cdot4}{1\cdot124864}$ ..... £439·4 .....

Hence the required present worth = £$\frac{439\cdot4}{1\cdot124864}$

= £390·625 = £390. 12*s.* 6*d.*

Ex. 3. On what sum due at the end of 6 months does the discount amount to £75, money being worth $2\frac{1}{2}$ per cent.?

Since £100 will amount to £$101\frac{1}{4}$ in 6 months at $2\frac{1}{2}$ per cent.;

£100 cash is worth £$101\frac{1}{4}$ at the end of 6 months;

$\therefore$ discount on £$101\frac{1}{4}$ is £$1\frac{1}{4}$;

$\therefore$ ..... £$101\frac{1}{4} \times \frac{£75}{£1\frac{1}{4}}$ is £75.

Hence sum required = £$101\frac{1}{4} \times \frac{£75}{£1\frac{1}{4}}$ = £6075.

EXAMPLES XIV. A.

Find the present value of

1.	£520	due 1 year	hence at 4	per cent.
2.	£406	... 6 months	 3	.....
3.	£257. 10*s.*	... 9 months	 4	.....
4.	£93. 2*s.* 7*d.*	... 8 months	..... $4\frac{1}{2}$	.....
5.	£1645. 17*s.* 6*d.*	...10 months	..... $3\frac{3}{4}$	.....
6.	£1013. 2*s.* 6*d.*	...$5\frac{1}{4}$ months	 3	.....

Find the discount on

7.	£210	due 1 year	hence at 5	per cent.
8.	£328	... 6 months	 5	.....
9.	£906. 15*s.*	... 3 months	 3	.....
10.	£2599. 16*s.* 8*d.*	... 7 months	..... $4\frac{1}{2}$	.....
11.	£539. 10*s.*	... 10 months	..... $4\frac{1}{2}$	.....
12.	£416	.... 219 days	 4	.....

Find the present value (reckoning compound interest) of

13. £514. 10*s.* due 2 years hence at 5 per cent.

14. £4553. 0*s.* 7*d.* ... 3 ..... 3 .....

15. £56243. 4*s.* ... 3 ..... 4 .....

16. £8857. 7*s.* due $1\frac{1}{2}$ years hence at $2\frac{1}{2}$ per cent., payable half-yearly.

17. £2000 due 2 years hence at 5 per cent., payable half-yearly.

18. Of what sum due 4 months hence is £275 the present value, the interest on money being $4\frac{1}{2}$ per cent.?

19. If the present value of £347. 12*s.* 6*d.* due 8 months hence be £337. 10*s.*, how much per cent. per annum is money supposed to be worth?

20. The discount on a certain sum due 5 months hence at 3 per cent. per annum is £2. 19*s.* 1*d.* What is the sum?

Bills of Exchange. Banker's Discount.

194. It is usual for merchants to pay for goods by giving what is called a **Bill of Exchange**, which is a document authorizing the person to whom the bill is given to receive a specified sum at the end of a stated time.

If the original holder of the Bill does not wish to wait until the expiration of the time specified, he goes to a Banker and sells to him the right to receive the amount of the Bill when it falls due.

This sale of a bill for cash is called **discounting** the bill, and the amount of discount charged is the difference between the amount of the bill, called its **face value,** and the cash which is given for it.

We have already seen that the actual cash value of a bill is that sum which with the interest upon it would amount to the face value of the bill. The cash value and corresponding discount calculated in this way are called the **true** or the **theoretical** present worth and discount respectively, to distinguish them from the cash value and discount of actual commerce.

195. Banker's Discount, which is the discount of actual commerce, is the *interest*, at the rate per cent. agreed upon, on the face value of the bill for the time it still has to run.

Thus, if a bill for £100 were discounted 6 months before it fell due at 5 per cent., the discount on it would be £2. 10*s*., namely the interest on £100 for 6 months at 5 per cent.

N.B. The theoretically true present value of a sum of money due after a long interval would have to be calculated, *compound* interest being of course reckoned; but in the case of a Bill, whose period is always short, the real cash value is its face value less the Commercial Discount.

196. Since True discount is the interest on the Present Value of the bill, and Banker's discount is the interest on the full amount, which exceeds the Present Value by the True discount, it follows that

Banker's discount = True discount + Interest on True discount.

197. By English Law if a bill of exchange is nominally due on a certain day three more days are allowed before it need be paid, which are called **days of grace.**

Ex. 1. Find the cash value of a bill for £560 drawn on May 6 for 6 months and discounted at 3 per cent. on July 12.

The bill is legally due on Nov. 9; therefore the number of days it has to run on July 12

$= 19 + 31 + 30 + 31 + 9 = 120$. [See Note, page 189.]

Hence discount charged is the interest on £560 for 120 days at 3 per cent. $= £560 \times 3 \times \frac{120}{365} \div 100 = £\frac{2016}{365} =$ £5. 10*s*. 6*d*. to the *nearest* penny.

Thus the cash value = £560 − £5. 10*s*. 6*d*. = £554. 9*s*. 6*d*.

It is clear that Banker's discount is more easily calculated than the theoretically true discount would be; moreover Tables of Interest are printed and are used in Offices to facilitate the calculations.

Ex. 2. A bill was discounted 3 months before it fell due at $2\frac{1}{2}$ per cent., and the actual discount charged exceeded the theoretically true discount by 15*s*. What was the amount of the bill?

The interest on £100 for 3 months at $2\frac{1}{2}$ per cent. $= £\frac{5}{8}$. This is the discount charged on £100.

The *true* discount on £100 for the given time and rate would be

$$\frac{\frac{5}{8}}{100+\frac{5}{8}} \text{ of } £100 = £\frac{500}{805} = £\frac{100}{161}.$$

Hence the excess on £100 $= £\frac{5}{8} - £\frac{100}{161} = £\frac{5}{1288}$;

$\therefore$ .....£100 $\times \frac{1288}{5} = £1$;

$\therefore$ .....£100 $\times \frac{1288}{5} \times \frac{3}{4} = 15s.$

Thus the amount of the bill $= £100 \times \frac{1288}{5} \times \frac{3}{4} = £19320.$

TRADE DISCOUNT.

198. Sometimes a tradesman makes a deduction from the price of goods when they are paid for in ready money, or when large quantities are taken, or for other reasons. This deduction is called *discount*; it, however, is very different from the discount we have hitherto considered, for the amount of the deduction does not depend upon *time*.

Ex. A tradesman gives a discount of 5 per cent. on ready money purchases; how much will be taken off a bill for £15. 17*s*. 6*d*.?

Five per cent. on £15. 17*s*. 6*d*. is $15s. + 1\frac{1}{2}d. \times 7 = 15s.\ 10\frac{1}{2}d.$

EXAMPLES XIV. B.

In the following questions discount is to be taken to mean the **real discount of actual commerce** unless *theoretical discount* is asked for. Calculations are to be made to the nearest penny.

1. Find the discount on a bill for £500 drawn on May 20 at 3 months and discounted on June 11 at 5 per cent.

2. Find the discount on a bill for £1575 drawn on Sept. 11, 1891, at 6 months and discounted on Oct. 19 at 4 per cent.

3. Find the discount on a bill for £1257. 10*s*. drawn on March 9 at 3 months and discounted on April 2 at $2\frac{1}{2}$ per cent.

4. Find the discount on a bill for £1180 drawn on Feb. 7 at 6 months and discounted on March 3 at $2\frac{1}{4}$ per cent.

5. Find the cash values of the following bills which were all discounted at 3 per cent. on May 11:

(i)	£1000	at 3 months drawn May 1,
(ii)	£1500	..... April 15,
(iii)	£1862. 10*s.*	..... May 5,
(iv)	£2746. 15*s.*	..... March 1.

6. What is the *true* present worth of a bill for £151. 10*s.* due in 80 days, the rate of interest being $4\frac{9}{16}$ per cent.?

7. What is the *true* discount on £76. 15*s.* due eight months hence, at $3\frac{1}{2}$ per cent.?

8. Find the *true* discount on £328. 13*s.* 5*d.* due three months hence at 4 per cent.

9. Find the *true* discount, allowing the usual 3 days' grace, upon a bill for £603, drawn Oct. 4 at 4 months, and discounted on Nov. 26 at $2\frac{1}{2}$ per cent.

10. Find the difference between the true discount and banker's discount on £500 due in 4 months at 3 per cent.

11. Find the difference between the true discount and banker's discount on a bill for £2500. 3*s.* 5*d.* due 3 months hence at 4 per cent.

12. The *true* discount on a bill for £554. 1*s.* 3*d.* at $3\frac{1}{2}$ per cent. is £6. 11*s.* 3*d.* How many days has the bill to run?

13. A banker gains 3*s.* 9*d.* by deducting interest instead of true discount from a bill due 10 months hence. What is the amount of the bill, money being supposed to be worth 3 per cent.?

14. If the banker's discount and the true discount on a certain sum due a year hence be £6. 10*s.* and £6. 5*s.* respectively, what is the sum?

15. The banker's discount on a bill due 1 year hence is $\frac{16}{15}$ of the theoretical discount. What is the rate per cent.?

16. Find the present value of £1000 due at the end of 5 years, supposing that money is worth 5 per cent., having given that £1 amounts to £1·276282 in 5 years at 5 per cent.

17. Find the present value of £1000 due at the end of 20 years, supposing that money is worth 4 per cent., having given that £1 will amount to £2·191123 in 20 years at 4 per cent.

CHAPTER XV.

SHARES AND STOCKS.

199. THERE are many business undertakings, such as railways, banks, gas works, &c., which are on so large a scale that many persons must combine to provide the money necessary to carry on the business. This is generally done by dividing up the whole sum required into '**shares**' of definite amount, say of £1, or £10, or £100 each.

The whole body of partners is called a **Company,** and the individual partners are called **Shareholders.**

The total amount of money raised to carry on the business of the Company is called its **Capital.**

The affairs of a Company are managed by a small number of elected shareholders called **Directors.**

The profits made by the Company are periodically divided among the shareholders; and the **dividend** is declared as a percentage on the capital.

200. A shareholder in a Company cannot demand the return of the money he paid for his shares; he can, however, *sell* the shares.

If the dividends of the Company are high, and are likely to continue to be high, the shares will sell for more than they originally cost; if, however, the Company is not prosperous, the shares would have to be sold for less than they originally cost.

Thus the shareholders in a Company are continually changing, and different shareholders may have bought their shares at very different prices.

201. The most important point to notice is that the amount of dividend paid to a shareholder *does not depend on the price at which his shares were bought, but simply on their nominal value.*

Thus two men who had the same number of £10 shares in a Company would be entitled to the same amount of dividend, although one may have bought, for example, £10 shares for £18 and the other for £5 each.

202. Shares are said to be above or below '**par**' according as they are sold for more or for less than their nominal value.

Thus if £10 shares sell for £11 each, since £11 is $\frac{110}{100}$ of £10, the shares are 10 *per cent. above par.*

When the price of shares is more than their nominal value they are said to be '**at a premium**,' and when the price is less than their nominal value the shares are '**at a discount**.'

203. The following are examples of the different questions which may have to be considered.

Ex. 1. £10 shares in a gas company sell for £24 each. How much will 70 shares cost?

Each £10 share costs £24 cash;

$\therefore$ 70 shares cost £24 × 70 = £1680.

Ex. 2. A man bought £10 shares in a gas company for £1680, giving £24 for each £10 share. How many shares did he buy?

Since each share cost £24,

the number of shares = £1680 ÷ £24 = 70.

Ex. 3. A gas company pays a dividend of 8 per cent. per annum, how much does a man receive who holds 70 shares of £10 each?

His share of the capital is £10 × 70 = £700, and he receives 8 per cent. on this.

Hence his yearly income = £700 × $\frac{8}{100}$ = £56.

Ex. 4. A man invests money in the shares of a gas company, each £10 share costing £24, and the company pays a dividend of 8 per cent. per annum. What percentage does he get on his outlay?

He is paid £10 × $\frac{8}{100}$, that is £$\frac{4}{5}$, for each share; and he pays £24 for the share.

He therefore gets £$\frac{4}{5}$ income for £24,

or £$\frac{4}{5} \div 24 \times 100$ ..... £100.

Thus percentage required $= \frac{4}{5} \div 24 \times 100 = 3\frac{1}{3}$.

204. Sometimes a Company does not need its full capital to carry on its business; and in that case only a certain fraction of the nominal amount of the shares is '*paid up*'; the shareholders are however bound to pay the rest if it should become necessary. When a dividend is declared at so much per cent., this percentage is paid only on the amount paid up on the shares, and not on their full nominal value.

Ex. What income will be obtained by investing £1008 in the purchase of £20 bank shares, on each of which £5 is paid up, at £24 each share, the bank paying a dividend of 18 per cent.?

Since £24 buys one share, £1008 will buy £1008 ÷ £24 = 42 shares. These 42 shares, on each of which £5 is paid, make up a capital of £5 × 42 = £210. On this capital of £210 a dividend of 18 per cent. is paid; hence income required = £210 × $\frac{18}{100}$ = £37. 16*s*.

EXAMPLES XV. A.

1. If £10 gas shares sell for £15 each, how much will 25 shares cost?

2. If £5 shares in a water-works company sell for £6. 10*s*. each, how much will 30 shares cost?

3. If £1 shares in a mining company sell for 15*s*. each, how much will 350 shares cost?

4. If £100 shares sell for £127 each, how many shares can be bought for £1651?

5. If £1 shares sell for 35*s*. each, how many can be bought for £927. 10*s*.?

6. Mining shares of £1 are sold at 5*s*. discount; how much will 100 shares cost?

7. £2 shares in a company are sold at 7*s*. 6*d*. premium; how much will 80 shares cost?

8. The shares in a certain mine are sold at 10 per cent. above par; how much must be paid for fifty £1 shares?

9. A company pays a dividend of 8 per cent.; how much will be paid to a man who has fifty £5 shares?

10. A man holds 350 shares of £5 each in a company which pays a dividend of 7 per cent. What is his total dividend?

11. A man sells sixty-three £10 gas shares at £18 each, and buys with the proceeds £5 tramway shares at £3. 10*s.* How many tramway shares does he buy?

12. A man sold seventy-five £5 shares at £6. 10*s.* each, and bought with the proceeds £10 shares in another company at £12. 10*s.* each. How many shares did he buy?

13. What income would be obtained by investing £385 in the purchase of £10 shares in a gas company at £17. 10*s.* per share, the company paying a dividend of 6 per cent. per annum?

14. £10 shares in a certain bank sell for £35, and the bank pays a dividend of 14 per cent. What income would be obtained by investing £945?

15. A company pays a dividend of $4\frac{1}{2}$ per cent., and its £10 shares sell for £15. What percentage does an investor get for his money?

16. A man buys £5 shares in a certain company for £6. 5*s.*, and the company pays a dividend of 5 per cent. What percentage does the investor get on his outlay?

17. A man sells fifty £10 shares in a gas company which pays a dividend of 8 per cent., for £18 each. He invests the proceeds in the purchase of £5 shares in a tramway company at £3. 10*s.* each. Find the change in his income, the tramway company paying a dividend of $3\frac{1}{2}$ per cent.

18. A man buys £10 shares in a company which pays a dividend of 8 per cent., and he buys at such a price that he gets 3 per cent. on his money. At what price does he buy?

19. A bank pays a dividend of 20 per cent., and its £60 shares, of which £20 is paid up, sell for £75. What percentage does an investor get on his money?

20. A man invests £925 in the purchase of £20 shares in a bank which pays a dividend of 16 per cent. per annum. The shares sell for £18. 10*s.*, and £4 is paid up on each share. Find the income derived from the investment.

205. Sometimes the Capital of a Company is not divided into Shares of definite amount; it is then called

Stock. *Any amount of Stock can be bought or sold, whereas only a whole number of shares can be dealt with.*

The price of Stock is given at so much *per cent.*; thus Stock is said to be at 115, when £100 Stock costs £115, and so in proportion for other amounts.

Ex. 1. How much Midland Railway stock at 143 can be bought for £1644. 10*s.*?

Since £143 cash buys £100 stock,
£1 ..... £100÷143 .....
and £1644½ .. .. £100÷143×1644½ .....

Thus the stock required $= £\frac{100 \times 1644\frac{1}{2}}{143} = £1150.$

Ex. 2. How much money will be obtained by the sale of £1150 Midland Railway stock at 143?

Since £100 stock sells for £143,
£1 ..... £143÷100;
∴ £1150 ..... £143÷100×1150 = £1644. 10*s.*

Ex. 3. What income will a man receive who holds £1150 stock in a company which pays a dividend of 4½ per cent.?

Since £100 stock produces £4½ income,
£1 ..... £4½÷100 .....
and £1150 ..... £4½÷100×1150 .....

Thus income $= £\frac{4\frac{1}{2} \times 1150}{100} = £51.\ 15s.$

Ex. 4. What income will be obtained by investing £1644. 10*s.* in a stock which pays a dividend of 4½ per cent. per annum and which sells at 143?

£143 will buy £100 stock which will entitle the holder to £4½ income.

Hence £143 secures £4½ income;
∴ £1 ..... £4½÷143 .....
∴ £1644½ ..... £4½÷143×1644½ .....

Thus income required $= £\frac{4\frac{1}{2} \times 1644\frac{1}{2}}{143} = £51.\ 15s.$

Ex. 5. How much per cent. will a man obtain for his money, if he buys stock at 143 which pays a dividend of 4½ per cent.?

£143 cash buys £100 stock on which the dividend is £4½.

Hence he gets £4½ income for £143;
∴ ..... £4½÷143×100 ..... £100.

Thus percentage required $= \frac{4\frac{1}{2} \times 100}{143} = \frac{450}{143} = 3\frac{21}{143}.$

EXAMPLES XV. B.

1. How much stock at 127 can be bought for £1016?
2. .....164 .....£1066?
3. ..... 93 .....£2092. 10*s.*?
4. ..... 57 .....£413. 5*s.*?
5. How much will £900 stock at 75 sell for?
6. .....£1150 ..... 120 .....
7. .....£825 ..... 128 .....
8. .....£2180 ..... 97 .....

9. What income will be obtained from £900 stock when the dividend is 4 per cent.?

10. A man holds £1375 Railway stock; what will be his yearly income when the two half-yearly dividends on the stock are respectively at the rates of $2\frac{1}{2}$ and 4 per cent. per annum?

11. What income will be obtained by investing £1101. 15*s.* in a stock which pays $3\frac{1}{2}$ per cent. per annum and which can be bought at 113?

12. What income will be obtained by investing £7038 in a $3\frac{1}{2}$ per cent. stock at $97\frac{3}{4}$?

13. How much of a $3\frac{1}{2}$ per cent. stock must a man hold to get an income therefrom of £67. 4*s.*?

14. What per centage on his money will a man get who buys at 125 a stock which pays 4 per cent. per annum?

15. The price of the 4 per cent. debentures of a certain railway is 128. How much per cent. on his money does an investor get?

16. A man gets an income of £66 a year by investing £2145 in 4 per cent. Railway debentures. What was the cost price of the debentures?

17. A man gets an income of £50. 12*s.* 6*d.* a year by investing £1530 in a $4\frac{1}{2}$ per cent. stock. At what price did he buy?

18. A man bought stock at $97\frac{1}{2}$, and on its rising to $103\frac{1}{4}$ he sold out, thereby making a profit of £66. 2*s.* 6*d.* How much stock did he buy, and what did he pay for it?

19. Will a man get a greater or smaller percentage on his outlay by investing in 3 per cent. stock at 95 than by investing in 4 per cent. stock at 127?

20. Which is the better investment, 4 per cent. stock at 129 or $4\frac{1}{2}$ per cent. stock at 145?

206. Governments borrow money to meet exceptional expenditure, and undertake to pay a fixed rate of interest.

Foreign Governments generally give printed papers, called **Bonds,** acknowledging their indebtedness for definite sums. From these Bonds can be cut small certificates, called **Coupons,** which entitle the holder to receive the guaranteed interest for each successive half year until the Bond is redeemed.

Consols. The public debt of England now consists of two portions, on one of which $2\frac{1}{2}$ per cent. per annum is paid, and on the other $2\frac{3}{4}$ per cent. will be paid until April 5th, 1903, after which the interest will be at the rate of $2\frac{1}{2}$ per cent. per annum, the interest being paid in quarterly instalments. This stock is called **Consols.** *Any amount of Consols can be purchased.*

Ex. 1. What amount of $2\frac{3}{4}$ per cent. consols at $97\frac{1}{2}$ can be purchased for £100?

Since £100 consols can be bought for £$97\frac{1}{2}$,
£$100 \div 97\frac{1}{2}$ £1;
$\therefore$ £$100 \div 97\frac{1}{2} \times 100$ ..... £100.

Thus amount required $= £\dfrac{100 \times 100}{97\frac{1}{2}} =$ £102. 11*s.* 3*d.*

N.B. *A fraction of a penny must be disregarded.*

Ex. 2. How much money will be obtained by the sale of £352. 8*s.* 6*d.* consols at 96?

Since £100 consols sell for £96,
£1 ..... £$\frac{96}{100}$;
$\therefore$ £352. 8*s.* 6*d.* ..... £352. 8*s.* 6*d.* $\times \frac{96}{100}$.

And £352. 8*s.* 6*d.* $\times \frac{96}{100}$ will be found to be £338. 6*s.* 6*d.* [disregarding a fraction of a penny].

Ex. 3. A man buys Russian 4 per cent. Bonds of £100 at 96. What percentage does he get on his outlay?

For £96 a Bond for £100 can be bought, on which £4 interest is paid per annum.

Hence £96 cash produces £4 income;
$\therefore$ £100..... £$\dfrac{4 \times 100}{96}$ .....

Thus percentage required $= \frac{400}{96} = 4\frac{1}{6}$.

207. BROKERAGE. Stocks and Shares, except those of small Companies, are bought and sold at a special market called a Stock Exchange. The agent who is employed to buy and sell for the public is called a **Stock Broker,** and the person who deals in Stocks and Shares is called a **Stock Jobber.**

In the lists of prices of stocks and shares quoted in the daily papers there are generally two prices given, the lower price being that at which Stock Jobbers bought and the higher that at which they sold the stock.

Stock Brokers charge a Commission for their services which is called **Brokerage**; this brokerage would be added to the cost price of Stocks or Shares, and subtracted from the selling price.

Brokerage is calculated on the *nominal* value.

Thus if stock be quoted in the newspaper at $112\frac{1}{2}$—113, and brokerage be $\frac{1}{8}$ per cent., a buyer would have to pay $113+\frac{1}{8}$, and a seller would receive $112\frac{1}{2}-\frac{1}{8}$.

The usual brokerage charged by stock brokers is 2*s.* 6*d.* per £100 Stock, that is $\frac{1}{8}$ per cent., for Consols and Foreign Government Stock; and $\frac{1}{4}$ per cent. on Railway Stock; also $\frac{1}{2}$ per cent. on Shares below £100.

We have hitherto supposed that brokerage was already allowed for in the price.

Ex. A man sold out £5000 stock in a company which paid $3\frac{1}{2}$ per cent. at $91\frac{1}{4}$, and invested the proceeds in a stock which paid 4 per cent. and was sold at $101\frac{1}{8}$. What was his change of income? [Brokerage $\frac{1}{8}$ per cent. to be charged on each transaction.]

His original income $= £5000 \times \frac{3\frac{1}{2}}{100} = £175.$

£100 stock realised $£(91\frac{1}{4}-\frac{1}{8}) = £91\frac{1}{8}.$

$\therefore$ £5000 ..... $£91\frac{1}{8} \times 50 = £\frac{36450}{8}.$

£100 of the new stock cost $£(101\frac{1}{8}+\frac{1}{8}) = £\frac{810}{8},$

$\therefore £\frac{800}{810}$ £1;

$\therefore £\frac{800}{810} \times \frac{36450}{8}$ ..... $£\frac{36450}{8}.$

Hence the amount of new stock $= £\frac{800}{810} \times \frac{36450}{8} = £4500.$

Hence new income $= £4500 \times \frac{4}{100} = £180.$

He therefore increased his income by £5 a year.

EXAMPLES XV. C.

Brokerage is allowed for in the price, except when otherwise stated.

1. What amount of consols at $97\frac{1}{2}$ could be bought for £390?
2. ..... 97 ..... £533. 10*s.*?
3. ..... 98 ..... £710. 10*s.*?
4. ..... $96\frac{1}{4}$ ..... £505. 6*s.* 3*d.*?
5. ..... 96 ..... £300?
6. ..... $97\frac{3}{8}$ ..... £500?
7. How much would be obtained by selling £500 Consols at 96?
8. ..... £450 ..... 97?
9. ..... £275 ..... $98\frac{1}{4}$?
10. ..... £325 ..... $97\frac{1}{2}$?
11. ..... £526. 7*s.* 4*d.* ..... 96?
12. ..... £1524. 3*s.* 8*d.* ..... 95?

13. What is the quarterly income obtained from £1250 in the $2\frac{3}{4}$ per cent. consols?

14. What is the quarterly income obtained from £560. 18*s.* 7*d.* in the $2\frac{3}{4}$ per cent. consols?

15. A man holds £780. 12*s.* in $2\frac{1}{2}$ per cent. consols. What quarterly income does he get from it?

16. A man has a yearly income of £49. 10*s.* from $2\frac{3}{4}$ per cent. consols. How much does he hold?

17. A man has a yearly income of £16. 6*s.* 4*d.* from $2\frac{3}{4}$ per cent. consols. How much does he hold?

18. What sum of money must be invested in $2\frac{3}{4}$ consols at $96\frac{3}{4}$ to secure an income of £154 a year? [Brokerage $\frac{1}{8}$ per cent.]

19. How much stock in the $2\frac{3}{4}$ per cent. consols must be sold at $99\frac{1}{8}$, in order to provide for the purchase of £1584 Railway Stock at $126\frac{3}{4}$? [Brokerage $\frac{1}{8}$ per cent. on Consols and $\frac{1}{4}$ per cent. on Railway Stock.]

20. A man invested a certain sum in consols when they were at $96\frac{1}{4}$, and he made a profit of £51 by selling out when they had risen to $98\frac{5}{8}$; and brokerage of $\frac{1}{8}$ per cent. was charged on each transaction. How much money did he invest?

21. A man invested money in consols at $100\frac{5}{8}$, and on their rising to $101\frac{3}{8}$ he sold out, thereby gaining £24. How much stock did he buy and what did he pay for it?

22. A man who has £3465 in Midland Railway Stock sells out at 140 and buys North-Eastern Consols at 165. What is the change in his income, if the Midland Stock pays 5 per cent. and the North-Eastern $6\frac{1}{4}$?

23. A man had £2400 in the $2\frac{3}{4}$ per cent. consols: he sells out at 99, and invests the proceeds in 4 per cent. Railway debentures, thereby increasing his income by £6 a year. At what price did he buy the debentures?

24. A man's income from railway shares paying $4\frac{1}{2}$ per cent. amounts to £378. He sells out at 134; what does he realize? The proceeds being invested in bank shares, £20 paid up, at 84, paying a dividend of 20 per cent., what is the change in his income?

25. A man invested a certain sum of money in the $2\frac{3}{4}$ per cent. consols at $98\frac{1}{4}$ (including brokerage); and his income therefrom, after 6*d.* in the pound had been deducted for income tax, was £214. 10*s.* How much money did he invest?

26. A man purchases £1400 stock in $2\frac{3}{4}$ per cent. consols at $94\frac{1}{2}$ and also invests £3150 in the purchase of Russian Inscribed Five per Cent. Loan at $94\frac{1}{4}$. If he sells the Consols at $95\frac{1}{8}$ and the Russians at $96\frac{1}{2}$, what does he gain on the transactions? [Brokerage on Consols $\frac{1}{8}$ and on Russians $\frac{1}{4}$.]

27. A man having an income of £352 a year in $2\frac{3}{4}$ per cent. Consols, sells out at $96\frac{3}{4}$, and invests the proceeds in 4 per cent. debentures, thereby increasing his income by £48 a year. At what price were the debentures bought? [Brokerage on Consols $\frac{1}{8}$ and on Debentures $\frac{1}{4}$.]

28. A man calls in a sum of £10,000 lent on mortgage at 4 per cent., and invests the money in £3000 London and North Western Railway 4 per cent. debenture stock at 133 (including brokerage) and £3000 Great Western Railway 5 per cent. stock at 167 (including brokerage). The balance of the £10,000 is placed on deposit at $2\frac{1}{2}$ per cent. What is the change in the income derived from the £10,000?

CHAPTER XVI.

SQUARE ROOT AND CUBE ROOT.

208. THE continued product of any number of factors all equal to a given number is called a **power** of that number.

Thus 5×5 is called the *second power* or the **square** of 5; $5 \times 5 \times 5$ is called the *third power* or the **cube** of 5; $5 \times 5 \times 5 \times 5$ is called the *fourth power* of 5; and so on.

Instead of writing a power of a number in full, a small figure called an **index** is put above and to its right to shew how many equal factors are to be taken.

Thus 5^2 is written for 5×5; 5^3 for $5 \times 5 \times 5$; and so on.

A number whose square is equal to a given number is called the **square root** of that number.

Thus 4 is the square root of 16, since $4^2 = 16$.

A number whose cube is equal to a given number is called the **cube root** of that number.

Thus 3 is the cube root of 27, since $3^3 = 27$.

The square root and the cube root of a number are denoted respectively by the symbols $\surd$ and $\sqrt[3]{\ }$ placed before the number.

Thus $\surd 16 = 4$, and $\sqrt[3]{27} = 3$.

209. The square of a number can be at once found by actual multiplication; we shall in the present Chapter shew how to find the square root of a given number.

The squares of the first 12 whole numbers should be known: they are

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144.

It will be seen at once that the square root of an integer is by no means always an integer, in fact the only numbers between 1 and 100 which have an integral square root are 4, 9, 16, 25, 36, 49, 64 and 81.

It will be seen later on that the square root of an integer which is not the square of a whole number can only be found approximately.

An integer (or a fraction) which is the square of another integer (or fraction) is called a **Perfect square.**

Thus 16 and $\frac{4}{9}$ are perfect squares, namely the squares of 4 and $\frac{2}{3}$ respectively.

210. In simple cases the square root of a given number can be found by separating it into factors, as in Art. 83, and making use of the principle that *the square of a product is equal to the product of the squares of its factors.*

For example, to find $\sqrt{324}$.

$$324 = 4 \times 81 = 2^2 \times 9^2 = (2 \times 9)^2;$$

hence $$\sqrt{324} = \sqrt{(2 \times 9)^2} = 2 \times 9 = 18.$$

EXAMPLES XVI. A.

Find the square roots of the following numbers.

1.	196.	**2.**	225.	**3.**	324.	**4.**	400.
5.	441.	**6.**	576.	**7.**	676.	**8.**	1089.
9.	1225.	**10.**	2025.	**11.**	2601.	**12.**	3969.
13.	4225.	**14.**	7056.	**15.**	11025.	**16.**	15876.

Find the least number by which each of the following numbers must be multiplied in order that the result may be a perfect square:

17.	12.	**18.**	20.	**19.**	24.	**20.**	52.
21.	126.	**22.**	140.	**23.**	1176.	**24.**	1344.

211. From the Theorems of Art. 43, we have

$$(60+3)^2 = (60+3) \times (60+3) = (60+3) \times 60 + (60+3) \times 3$$
$$= 60 \times 60 + 3 \times 60 + 60 \times 3 + 3 \times 3 = 60^2 + 2 \times 60 \times 3 + 3^2.$$

The square of the sum of any other pair of numbers can be expressed in a similar form.

Hence *the square of the sum of any two numbers is equal to the sum of their squares plus twice their product.*

We may also write the above result in the following form, which will be used later on,

$$(60+3)^2 = 60^2 + (2\times 60+3)\times 3.$$

212. Since

$$10^2 = 100,\ 100^2 = 10000,\ 1000^2 = 1000000,$$

and so on, it follows that

if a number has 1 digit, its square has either 1 or 2 digits
..... 2 ....., ... 3 or 4 .....
..... 3 ....., ... 5 or 6 .....

and so on.

Hence, if we mark off the digits of a given number, beginning at the units digit, into periods of two, the last of the periods containing either one or two digits; then *the number of these periods will be equal to the number of digits in the square root of the given number.*

For example, by pointing off the numbers 961, 5476, 368449 and 1522756 as follows 9'61, 54'76, 36'84'49 and 1'52'27'56, we see that the square roots of these numbers contain respectively 2, 2, 3 and 4, figures.

213. To find the square root of any whole number. The method will be seen from the following examples:—

Ex. 1. To find the square root of 3969.

By pointing off the digits into periods of two, we see that there are *two* digits in the required root; and, since $60^2 = 3600$ and $70^2 = 4900$, we see that the root lies between 60 and 70. The tens digit must therefore be 6, and we have now to find the units digit.

If we subtract 60^2 from the given number the remainder is 369; and, by Art. 211, this remainder is equal to the product of $(2\times 60 +$ the required units digit) multiplied by the units digit.

Hence, if we use 2×60 as a **trial divisor** we obtain a quotient, namely 3, which is either equal to or *greater* than the required digit. Since $(2 \times 60 + 3) \times 3 = 369$, the trial divisor gives in this case the correct result. The work is written down thus:

39'69 (60+3	39'69 (63
36 00	36
$2 \times 60 + 3 = 123$) 3 69	123) 3 69
3 69	3 69

On the right the process is shortened, as in ordinary division, by the omission of zeros, the periods, of two figures each, being brought down one at a time, one figure of the root corresponding to each period.

Ex. 2. Find the square root of 114244.

11'42'44 (300+30+8	11'42'44 (338
9 00 00	9
600+30) 2 42 44	63) 2 42
1 89 00	1 89
660+8) 53 44	668) 53 44
53 44	53 44

There are here three periods and therefore three digits in the root, the first of which is 3 since 114244 is between 300^2 and 400^2. Using 300×2 as a trial divisor in order to find the second figure in the root, we obtain the quotient 40; this however is too great, for $(600 + 40) \times 40$ is greater than 24200; we therefore try 30, which proves to be correct.

It should be noticed that at the end of every stage we have subtracted from the given number *the square of that part of the root already found.* After subtracting 300^2 the remainder is 24244; after subtracting 330^2 the remainder is 5344; and after subtracting 338^2 the remainder is zero. Hence 338 is the required root.

The process is usually written in the shortened form, the successive 'divisors' being formed by *multiplying the answer already found by 2, and annexing on the right the next figure of the answer.*

Ex. 3. Find the square root of 50126400.

50'12'64'00 (7080
49
1408) 11 26 4
11 26 4
00

Here there are 4 periods and therefore 4 figures in the root. A figure of the root corresponds to each period brought down in the shortened process; and in the present case two figures of the root are noughts.

Ex. 4. To find the square root of 315.

```
        3'15·00'00' ( 17·74.....
        1
      ---
27 ) 2 15
       1 89
      ----
347 ) 2600
        2429
      -----
3544 ) 17100
         14176
        -----
          2924
```

Having found the integral part of the root, namely 17, there is a remainder 26. We therefore place a decimal point after the units figure of the number and of the root; and then bring down periods each containing two noughts. The process can be continued to any extent that may be desired, but *it can never terminate.* [Art. 215.]

Ex. 5. Find to 3 places of decimals the square root of 792·4.

```
        7'92·40'00'00' ( 28·149.....
        4
      ---
48 ) 3 92
       3 84
      ----
561 )   840
          561
        ---
5624 ) 27900
         22496
        -----
56289 ) 540400
          506601
          ------
           33799
```

In finding the square root of a decimal number, **the pointing must be begun at the units figure,** and carried forwards for the integral part, and backwards for the decimal part.

EXAMPLES XVI. B.

Find the square roots of the following :

1. 729.	**2.** 1156.	**3.** 1849.	**4.** 2209.
5. 3481.	**6.** 6084.	**7.** 1681.	**8.** 56169.
9. 4157521.	**10.** 49126081.	**11.** 26625600.	
12. 9345249.	**13.** 1766689024.	**14.** 16642320 25.	
15. 6877219041.			

Find the square roots of

16. 13·69.	**17.** ·2809.	**18.** 4·5796.	**19.** 400·8004.
20. 19650·4324.	**21.** 182·493081.	**22.** ·00022201.	
23. ·0016630084.			

Find to 4 places of decimals the square roots of

24.	5.	**25.**	19.	**26.**	21·5.	**27.**	125·4.
28.	31·046.	**29.**	·4.	**30.**	·081.	**31.**	·01735.

214. Since $\left(\frac{3}{4}\right)^2 = \frac{3\times 3}{4\times 4} = \frac{3^2}{4^2}$,

it follows conversely that

$$\sqrt{\frac{9}{16}} = \frac{3}{4} = \frac{\sqrt{9}}{\sqrt{16}}.$$

Thus *the square root of a vulgar fraction is equal to a fraction whose numerator and denominator are respectively the square roots of the numerator and denominator of the given fraction.*

Ex. 1. Find the square roots of $\frac{144}{169}$, $1\frac{9}{16}$, $\cdot\dot{4}$ and $2\cdot\dot{0}86419753\dot{3}$.

$$\sqrt{\frac{144}{169}} = \frac{\sqrt{144}}{\sqrt{169}} = \frac{12}{13}; \quad \sqrt{1\tfrac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4};$$

$$\sqrt{\cdot\dot{4}} = \sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3};$$

and $\sqrt{2\cdot\dot{0}86419753\dot{3}} = \sqrt{2\frac{86419753}{999999999}} = \sqrt{2\frac{7}{81}}$

$$= \sqrt{\frac{169}{81}} = \frac{\sqrt{169}}{\sqrt{81}} = \frac{13}{9} = 1\cdot\dot{4}.$$

Ex. 2. Find, to four places of decimals,

(i) $\sqrt{\frac{5}{9}}$, (ii) $\sqrt{\frac{4}{5}}$, (iii) $\sqrt{\cdot\dot{3}}$ and (iv) $\frac{5}{\sqrt{3}}$.

(i) $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{1}{3}\sqrt{5}$, which can be found as in Art. 213.

(ii) In examples in which the denominator is not a perfect square, the fraction should be expressed as a decimal. In the present case $\sqrt{\frac{4}{5}} = \sqrt{\cdot 8} = \&c.$

Or thus: $\sqrt{\frac{4}{5}} = \sqrt{\frac{20}{25}} = \frac{\sqrt{20}}{\sqrt{25}} = \frac{1}{5}\sqrt{20} = \&c.$

(iii) $\cdot\dot{3} = \cdot 33'33'33'33'\ldots\ldots$ Then proceed as in Art. 213.

(iv) $\frac{5}{\sqrt{3}} = \frac{5\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{5}{3}\sqrt{3} = \&c.$

The change of form from $\frac{5}{\sqrt{3}}$ to $\frac{5}{3}\sqrt{3}$ will save a great deal of labour.

Ex. 3. What is the length of a side of a square field whose area is one acre, and what is the distance from one corner of the field to the opposite corner?

1 acre = 4840 sq. yds.

Hence, as the field is a square, the length of a side $= \sqrt{4840}$ yards $= 69{\cdot}5701\ldots$ yards $= 69$ yds. 1 ft. $8\frac{1}{2}$ inches very nearly.

To find the distance from one corner to the opposite corner, we must assume Euclid I. 47, from which we have

(diagonal of square)^2 = sum of squares on two sides of the square
= square on one side $\times$ 2.

Hence required length of diagonal $= \sqrt{4840 \times 2}$ yds. $= 98{\cdot}38\ldots$ yards.

Ex. 4. At what rate per cent. at compound interest will £560 amount to £588. 7*s*. in 2 years?

$$\text{Amount} = \text{Principal} \times \left(\frac{100 + \text{rate required}}{100}\right)^2;$$

$$\therefore \left(\frac{100 + \text{rate required}}{100}\right)^2 = \frac{\text{£588. 7}s.}{\text{£560}} = \frac{1681}{1600}.$$

Hence $$\frac{100 + \text{rate required}}{100} = \sqrt{\frac{1681}{1600}} = \frac{41}{40}.$$

Hence rate required $= \frac{1}{40}$ of $100 = 2\frac{1}{2}$.

215. Since the square of a number cannot end with a nought except the number itself ends with a nought, it follows that, if the process of finding a square root does not terminate when the last significant figure is brought down, the process will never terminate.

Expressions such as $\sqrt{3}$, $\sqrt{2{\cdot}5}$ which cannot be found *exactly* are called **Surds.**

Although no definite fractional or decimal number can be found whose square is *exactly* equal to 3, for example, the process of Art. 213, *if continued far enough*, will enable us to find a decimal whose square differs from 3 by as small a quantity as we please.

EXAMPLES XVI. C.

Find the square roots of

1. $\frac{25}{144}$.
2. $1\frac{25}{144}$.
3. $\frac{169}{289}$.
4. $11\frac{14}{25}$.
5. $39\frac{1}{16}$.
6. $\cdot\dot{1}$.
7. $\cdot00\dot{4}$.
8. $\cdot1\dot{3}\dot{4}$.
9. $1{\cdot}36\dot{1}$.
10. $4{\cdot}38204\dot{}$.

Find to four places of decimals the square roots of

11. $\frac{7}{16}$.
12. $\frac{4}{11}$.
13. $3\frac{1}{4}$.
14. $8\frac{2}{7}$.
15. $2{\cdot}\dot{4}$.
16. $\cdot0\dot{4}\dot{1}$.
17. $\cdot0\dot{8}\dot{3}$.
18. $3{\cdot}5\dot{1}6\dot{2}$.

Find, to within one thousandth of their true value

19. $\frac{1}{\sqrt{2}}$. 20. $\frac{10}{\sqrt{9}}$. 21. $\frac{5}{\sqrt{15}}$. 22. $\frac{7}{\sqrt{30}}$.

23. A beam 36 feet long, and whose section is a square contains $182\frac{1}{4}$ cubic feet of timber. What is its width?

24. Find the length of the side of a square field which contains 10 acres.

25. A field of 2 acres is exactly square, find the length of one side correctly to a thousandth part of the whole.

26. Find the cost of putting a fence round a square field of 10 acres at 1*s.* 8*d.* per yard.

27. Find the length of the side of a square field whose area is 4 A. 2 R. 9 P.

28. Find the length of the side of a square field whose area is 12 A. 3 R. 9 P. $18\frac{3}{4}$ Sq. yds.?

29. The length of a field is twice its breadth, and its area is 5 acres. What is its length?

30. The length of a field is thrice its breadth and its area is 1 A. 3 R. 20 P.; what is its breadth?

31. The breadth of a room is twice its height and the length is thrice its height; and it cost £24 to paint the walls at 4*d.* per square foot. What is the height?

32. The length of a room is treble its breadth. The cost of carpeting at 7*s.* 6*d.* per square yard is £28. 2*s.* 6*d.*, and the cost of painting the four walls at $4\frac{1}{2}d.$ per square foot is also £28. 2*s.* 6*d.* What is the height of the room?

33. A rectangular field which is twice as long as it is wide cost 1·0138̇ shillings per square yard to turf, and the whole cost was £191. 17*s.* $0\frac{3}{4}d.$ Find the lengths of the sides of the field.

34. The side of a square is 10 yards; find, to the nearest inch, the length of its diagonal.

35. A rectangular block of timber is 5 ft. long and contains 3 cubic feet. If its section be a square, find its thickness to the nearest tenth of an inch.

36. A square field is bordered by a path one yard wide, the field and path together occupying two acres and a half. Find the cost of covering the path with gravel at 1*s.* 6*d.* per square yard.

37. At what rate per cent. compound interest will £375 amount to £397. 16*s.* 9*d.* in 2 years?

Cube Root.

216. The cubes of the first 10 whole numbers should be known: they are

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

An integer (or a fraction) which is the cube of another integer (or fraction) is called a **Perfect Cube.**

Thus 64 and $\frac{125}{343}$ are perfect cubes, namely the cubes of 4 and $\frac{5}{7}$ respectively.

217. In simple cases the cube root of a number can be found by separating it into factors, as in Art. 83.

For example, to find $\sqrt[3]{9261}$.

$$9261 = 9 \times 1029 = 27 \times 343 = 3^3 \times 7^3 = (3 \times 7)^3;$$

hence $$\sqrt[3]{9261} = \sqrt[3]{(3 \times 7)^3} = 3 \times 7 = 21.$$

EXAMPLES XVI. D.

Find the cube roots of the following numbers:

1. 10648. **2.** 3375. **3.** 35937. **4.** 13824.
5. 19683. **6.** 42875.

Find the least number by which each of the following numbers must be multiplied in order that the result may be a perfect cube.

7. 108. **8.** 392. **9.** 336. **10.** 441.
11. 4032. **12.** 7056.

218. Since

$10^3 = 1000$, $100^3 = 1000000$, $1000^3 = 1000000000$,

and so on, it follows that

if a number has 1 digit, its cube has either 1, 2 or 3 digits
..... 2....., ... 4, 5 or 6 .....
..... 3....., ... 7, 8 or 9 .....

Hence, if we mark off the digits of a given number, beginning at the units digit, in periods of three, the last of the periods containing one, two or three digits; then *the number of these periods will be equal to the number of digits in the cube root of the given number.*

For example, by pointing off the numbers 2744, 32768, 3511808 as follows, namely 2'744, 32'768 and 3'511'808, we see that the cube roots of these numbers contain respectively 2, 2 and 3 figures.

219. $(60+5)^3 = (60+5)^2 \times (60+5)$

$= (60^2 + 2 \times 60 \times 5 + 5^2)(60+5)$ [Art. 211]

$= (60^3 + 2 \times 60^2 \times 5 + 5^2 \times 60)$
$\quad + (60^2 \times 5 + 2 \times 60 \times 5^2 + 5^3)$

$= 60^3 + 3 \times 60^2 \times 5 + 3 \times 60 \times 5^2 + 5^3.$

The cube of the sum of any two other numbers can be expressed in a similar form.

Hence *the cube of the sum of any two numbers is equal to the cube of the first plus three times the square of the first multiplied by the second plus three times the first multiplied by the square of the second plus the cube of the second.*

The above Theorem will enable us to find the Cube Root of any number.

220. To find the cube root of any number. The method will be seen from the following examples:

Ex. 1. To find the cube root of 157464.

By pointing off the figures into periods of three [Art. 218], we see that there are *two* figures in the required root.

The first figure of the root is 5, since 157000 is between 50^3 and 60^3. Subtract 50^3 from the given number, and the remainder will be 32464. Now take *thrice the square of the part of the root already found*, namely 7500, and use this as a '**trial divisor**' in order to obtain the second figure of the root; $32464 \div 7500$ gives a quotient 4, which is either equal or *greater* than the required second figure. If we now take $3 \times 50^2 + 3 \times 50 \times 4 + 4^2$, and multiply the sum by 4, and then subtract the product from 32464, we shall have subtracted altogether $50^3 + (3 \times 50^2 + 3 \times 50 \times 4 + 4^2) \times 4 = (50+4)^3$. [Art. 219.] After this subtraction there is found to be no remainder; and hence the given number is 54^3, so that the required cube root is 54.

$157'464 \,(\, 50+4$

$50^3 = 125\,000$

3×50^2	$= 7500$	$32\,464$
$3 \times 50 \times 4$	$= 600$	
4^2	$= 16$	
	$8116 \times 4 =$	$32\,464$

Ex. 2. Find the cube root of 13312053.

$$
\begin{array}{rl}
 & 13'312'053\ (200+30+7 \\
200^3= & 8\,000\,000 \\
\hline
3\times200^2=120000 & \big|\ 5\,312\,053 \\
3\times200\times30=\ 18000 & \big| \\
30^2=\ \ 900 & \big| \\
\hline
138900\times30= & 4\,167\,000 \\
\hline
 & 1\,145\,053 \\
\hline
3\times230^2=158700 & \big| \\
3\times230\times7=\ 4830 & \big| \\
7^2=\ \ 49 & \big| \\
\hline
163579\times7= & 1\,145\,053
\end{array}
$$

Here there are three periods, and therefore three figures in the root; and, since 13000000 lies between 200^3 and 300^3, the first figure of the root is 2. Subtract 200^3, and the remainder is 5312053. Now take 3×200^2, that is 120000, as a 'trial divisor'; and $5312053\div120000$ will give **40** for quotient. It will, however, be found on trial that 40 is too great, for

$$(3\times200^2+3\times200\times40+40^2)\times40$$

is greater than the remainder 5312053; we therefore try **30**. Take

$$3\times200^2+3\times200\times30+30^2,$$

and multiply this sum by 30 and subtract the product from 5312053; we shall then have subtracted altogether $(200+30)^3$ from the given number, and the remainder will be found to be 1145053.

To find the last figure of the root use 3×230^2, that is 158700, as a 'trial divisor,' and $1145053\div158700$ gives 7 for quotient. Take $3\times230^2+3\times230\times7+7^2$, and multiply this sum by 7, and subtract the product from 1145053. There is now no remainder; and, from Art. 219, we have now subtracted altogether $(230+7)^3$; hence the given number $=237^3$, so that 237 is the required cube root.

Ex. 3. Find the cube root of 252·435968.

The pointing must be begun from the units figure, and carried forwards for the integral part and backwards for the decimal part.

$$
\begin{array}{rl}
 & 252'\cdot435'968'\ (6+\cdot3+\cdot02 \\
6^3= & 216\cdot \\
\hline
3\times6^2=108 & \big|\ 36\cdot435968 \\
3\times6\times\cdot3=\ \ 5\cdot4 & \big| \\
(\cdot3)^2=\ \ \cdot09 & \big| \\
\hline
113\cdot49\times\cdot3= & 34\cdot047 \\
 & \ \ 2\cdot388968 \\
3\times(6\cdot3)^2=119\cdot07 & \big| \\
3\times6\cdot3\times\cdot02=\ \ \cdot378 & \big| \\
(\cdot02)^2=\ \ \cdot0004 & \big| \\
\hline
119\cdot4484\times\cdot02= & 2\cdot388968
\end{array}
$$

The process can be somewhat shortened, as in Square Root; it is, however, very rarely necessary to find a cube root, and it is therefore undesirable to attempt to shorten the above process.

We may remark that the approximate value of a cube root can be found at once by means of Tables of Logarithms, and that if a *very* close approximation be required Horner's process had better be employed. [See Burnside and Panton's *Theory of Equations*, chapter X.]

EXAMPLES XVI. E.

Find the cube roots of

1. 1331.	**2.** 3375.	**3.** 4913.	**4.** 12167.
5. 29791.	**6.** 68921.	**7.** 79507000.	**8.** 148877000.
9. 8869743.	**10.** 733870808.	**11.** 2352637.	**12.** 16974593.
13. 2·197.	**14.** ·004913.	**15.** ·238328.	**16.** 125525·735343.
17. $2\frac{19}{27}$.	**18.** $39\frac{38}{125}$.	**19.** $12568\frac{5}{64}$.	**20.** $240\frac{17712}{19683}$.

Find to three significant figures the cube roots of

21. 10.	**22.** 1·5.	**23.** 3·75.	**24.** ·0675.

25. Find the side of a cube which has the same volume as a beam 40 ft. 6 in. long, 1 ft. 4 in. wide and $\frac{3}{4}$ in. thick.

26. Find the length of one edge of a cube whose volume is 2 yards 14 ft. 145 inches.

27. Find the area of each face of a cube whose volume is 5 c. yds. 2 ft. 1592 in.

28. Find approximately the length of one edge of a cubical vessel which contains a gallon. [A gallon = 277·274 cubic inches.]

29. Find approximately the side of a cube of iron which weighs a ton, assuming that a cubic foot of iron weighs 486 lbs.

30. At what rate per cent. compound interest will £6400 amount to £6892. 2*s.* in three years?

CHAPTER XVII.

MISCELLANEOUS PROBLEMS.

Proportional Parts.

221. When the ratio between the parts of a given quantity are known, the parts themselves can be at once found.

Ex. 1. Divide £100 between A and B so that A may have £3 for every £2 that B has.

For every £3 that A receives, B will receive £2, *and the two together will receive* £3 + £2 = £5.

Hence A receives £3 out of every £5 of the whole;

$\therefore$ A receives $\frac{3}{5}$ of the whole = $\frac{3}{5}$ of £100 = £60.

Also B receives $\frac{2}{5}$ of the whole = $\frac{2}{5}$ of £100 = £40.

Ex. 2. The profits of a business are to be divided between the partners A, B and C, so that A may have 4 parts, B 3 parts, and C 2 parts. How much does each get out of a profit of £4500?

If A has 4 parts to B's 3 parts and C's 2 parts; *A will have 4 parts out of* $(4+3+2)$ *parts divided between them all.*

Hence A will have $\frac{4}{4+3+2}$ of the whole;

$\therefore$ A ..... $\frac{4}{9}$ of £4500 = £2000.

So B ..... $\frac{3}{9}$ of £4500 = £1500,

and C ..... $\frac{2}{9}$ of £4500 = £1000.

Ex. 3. Divide £23. 10*s*. between *A*, *B* and *C*, so that *A*'s share may be to *B*'s as 4 to 5, and *B*'s share to *C*'s as 3 : 4.

Here $\frac{A\text{'s share}}{B\text{'s share}} = \frac{4}{5}$ and $\frac{B\text{'s share}}{C\text{'s share}} = \frac{3}{4}$.

Hence *A*'s share $= \frac{4}{5}$ *B*'s share $= \frac{4}{5} \times \frac{3}{4}$ *C*'s share $= \frac{3}{5}$ *C*'s share.

Hence shares of *A*, *B* and *C* together $= (\frac{3}{5} + \frac{3}{4} + 1)$ of *C*'s share,

that is £$23\frac{1}{2} = \frac{12+15+20}{20}$ of *C*'s share.

Hence *C*'s share $= \frac{20}{12+15+20}$ of £$23\frac{1}{2} = \frac{20}{47}$ of £$\frac{47}{2}$ = £10.

Then *A*'s share $= \frac{3}{5}$ of *C*'s = £6, and *B*'s share $= \frac{3}{4}$ of *C*'s = £7. 10*s*.

Or thus:

A's share : *B*'s $= 4 : 5$
B's : *C*'s $= 3 : 4$.

Now *multiply the terms of the two ratios by such numbers that the numbers corresponding to B's share may be the same in both.* In the present case multiply by 3 and 5 respectively. Then

A's share : *B*'s : *C*'s = 12 : 15 : 20.

Thus *A* gets 12 parts out of (12 + 15 + 20) parts altogether;

hence *A* receives $\frac{12}{12+15+20}$ of £23. 10*s*. = £6, &c.

Ex. 4. Divide £11. 12*s*. between 12 men, 8 women and 20 children, giving to each man twice as much as to each woman and to each woman three times as much as to each child.

A man's share = a woman's share × 2 = a child's share × 6.

Hence 12 men, 8 women and 20 children will have

(12 × 6 + 8 × 3 + 20) shares of a child.

Hence a child's share × (72 + 24 + 20) = £11. 12*s*. = 232*s*.;

∴ a child's share $= \frac{232}{116}$*s*. = 2 shillings.

Whence it follows that each man has 12 shillings and each woman 6 shillings.

Ex. 5. Divide 532 into three parts proportional to $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$.

Since $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$ are respectively $\frac{40}{60}$, $\frac{45}{60}$ and $\frac{48}{60}$, we have merely to divide into parts proportional to 40, 45 and 48.

Hence, as in Ex. 3, the parts are $\frac{40}{40+45+48}$ of the whole, &c.

EXAMPLES XVII. A.

1. Divide £245 into parts in the ratio 3 : 4.

2. Divide £165 into parts in the ratio $2\frac{1}{2}$: 3.

3. Divide £33. 3*s.* into parts in the ratio $\frac{2}{3}$: $\frac{3}{4}$.

4. Divide £54 into three parts proportional to the numbers 1, 2 and 3.

5. Divide £97. 18*s.* $8\frac{1}{2}d.$ into parts proportional to the numbers 7, 9 and 13.

6. Divide £17. 11*s.* into three parts proportional to 5, $5\frac{1}{2}$ and $7\frac{1}{2}$.

7. A sum is divided into parts proportional to the fractions $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{8}$; what fractional part of the whole is the first part?

8. The profits of a business are to be divided between the three partners in proportion to the numbers 5, 3 and 2. How much does each receive out of a total profit of £6237?

9. In a certain business *A* has 7 shares, *B* 5, *C* 3 and *D* 1 share. The profits are £2410. Find each partner's share.

10. *A* provides £5000, *B* £3000 and *C* £1250 to carry on a business. How much should each get out of a profit of £555?

11. *A*, *B* and *C* are partners in a business and have shares in proportion to the numbers 4, 3 and 2 respectively, after 5 per cent. per annum has been paid on the capital. The capital is £20000, of which sum *A* provided £12000 and *B* the remainder. How much does each receive out of a total yearly profit of £3400?

12. *A*, *B* and *C* are partners in a business, *C* receives 10 per cent. of the net profits as manager, the remainder being divided between *A*, *B* and *C* in proportion to the numbers 5, 4 and 3 respectively. In a certain year *A*'s share of the profits amounted to £1520; what were the shares of *B* and *C*?

13. The shares of *A*, *B* and *C* of the capital in a business are as 4 to 3 to 2. After 4 months *A* withdraws half his capital, and the profits at the end of the year are £1518. How should this be divided between *A*, *B* and *C*?

14. Divide £157. 10*s.* between *A*, *B*, *C* and *D*, so that *A* may have as much as *C* and *D* together, *B* as much as *A* and *C* together, and *D* twice as much as *C*.

15. Divide £80 among 22 men, 26 women and 82 boys, so that two men may have as much as three women, and one woman as much as two boys.

16. If 8 men can do as much as 14 women, and 5 women as much as 9 boys, divide £27 among 4 men, 6 women and 9 boys in proportion to the work they do.

17. Divide £759. 11*s.* among three persons *A*, *B* and *C*, so that *A* may get one-fourth of what *B* receives, and *C* may get one-tenth as much as *A* and *B* together.

18. Three partners *A*, *B* and *C* had shares in a business proportional to the numbers 4, 5 and 6 respectively. *C* retired and received as his share of the business £15000. How much of this money should be paid by *A* and *B* respectively in order that after *C*'s retirement their shares might be equal?

19. *A* and *B*, whose capitals were as 3 to 4, joined in business, and at the end of 4 months they withdrew $\frac{2}{5}$ and $\frac{3}{8}$ respectively of their capitals from the business. How should a gain of £624 be divided between them at the end of the year?

20. The volumes of three substances contained in a certain mixture are proportional to the numbers 2, 1 and 4 respectively; also the weights of equal volumes of the substances are as the numbers 1, 32 and 16 respectively. Find the weight of the first substance contained in 3 lb. 1 oz. of the mixture.

MIXTURES.

222. The cost of a mixture of given quantities of two different ingredients is at once found when the price of the separate ingredients is known.

Ex. 8 lbs. of tea which cost 1*s.* 2*d.* a lb. is mixed with 3 lbs. of tea which cost 2*s.* 1*d.* a lb. What was the cost of a lb. of the mixture?

The mixture weighs (8 + 3) lbs. = 11 lbs.

The mixture cost $14d. \times 8 + 25d. \times 3 = 187d.$

Hence each lb. of the mixture cost $187d. \div 11 = 17d.$

223. The ratio in which two different ingredients must be taken in order to make a mixture whose cost is any given sum *intermediate between the costs of the separate ingredients*, will be seen from the following examples.

Ex. 1. In what ratio must two kinds of tea, which cost respectively 1*s.* 2*d.* and 2*s.* 1*d.* per lb., be mixed in order that the mixture may cost 1*s.* 5*d.* per lb.?

The cost of the dearer tea is 8*d.* per lb. *more* than that of the mixture.
..... cheaper..... 3*d.* ..... *less* .....

We must take such quantities of each that the loss on the one just balances the gain on the other.

This will be the case if we take 3 lbs. of the dearer tea and 8 lbs. of the cheaper, for the loss on the one will be $8d. \times 3$ and the gain on the other will be $3d. \times 8$.

Ex. 2. In what ratio must whiskey which cost 18*s.* per gallon be mixed with whiskey which cost 13*s.* 6*d.* per gallon, in order that 25 per cent. profit may be made by selling the mixture at 20*s.* per gallon?

Whiskey which sells at 20*s.* a gallon at a gain of 25 per cent. must have cost $\frac{100}{125}$ of 20*s.* per gallon $= 16s.$ per gallon.

Hence the cost of
the better whiskey is 2*s.* per gallon more than that of the mixture,
and cheaper..... $2\frac{1}{2}s.$ ..... less .....

Hence we must take $2\frac{1}{2}$ gallons of the dearer kind to every 2 gallons of the cheaper.

Thus ratio required $= 2\frac{1}{2} : 2$ or $5 : 4$.

Ex. 3. In what way must 3 kinds of tea worth 1*s.* 4*d.*, 1*s.* 8*d.* and 2*s.* per lb. respectively be mixed that the mixture may be worth 1*s.* 7*d.* per lb.?

When there are 3 (or more) kinds of commodity and only the price of the mixture fixed, there is an indefinite number of ways of satisfying the condition.

In the present case the gain on the cheapest tea, namely 3*d.* per lb., must just balance the loss on the other two kinds at 1*d.* and 5*d.* per lb. respectively.

Possible solutions are 2, 1, 1; 3, 4, 1; 4, 7, 1; 4, 2, 2; &c.

EXAMPLES XVII. B.

1. What would be the cost per lb. of a mixture of 4 lbs. of tea at 1*s.* 4*d.* and 6 lbs. at 1*s.* 9*d.*?

2. What will be the cost of a mixture of 3 gallons of spirit at 12*s.* per gallon and 5 gallons at 16*s.* a gallon?

3. If 180 lbs. of sugar which cost $3\frac{1}{4}d.$ per lb. be mixed with 120 lbs. which cost $2\frac{1}{4}d.$ per lb., at what price must the mixture be sold so as to gain 25 per cent. on the cost?

4. A milkman buys milk at 1*s.* 3*d.* per gallon. He adds 20 per cent. of water to the milk, and sells the mixture at 1*s.* 6*d.* per gallon. What is his gain per cent.?

5. In what ratio must two kinds of tea, which cost respectively 1*s.* 3*d.* and 1*s.* 9*d.* per lb., be mixed in order that the mixture may cost 1*s.* 5*d.* per lb.?

6. In what ratio must biscuits worth respectively $5\frac{1}{2}d.$ per pound and $7\frac{1}{2}d.$ per pound be mixed that the mixture may be worth 6*d.* per pound?

7. How much whiskey worth 15*s.* per gallon must be mixed with 10 gallons worth 18*s.* 6*d.* per gallon that the mixture may be worth 17*s.* 6*d.* per gallon?

8. How much sugar worth $3\frac{1}{16}d.$ per lb. must be mixed with 1 cwt. of sugar worth $2\frac{1}{4}d.$ per lb. in order that the mixture may be worth $3\frac{1}{2}d.$ per lb.?

9. Tea at 2*s.* 9*d.* a lb. is mixed with tea at 3*s.* 3*d.* a lb. In what proportion must they be mixed, so that by selling the mixture at 3*s.* $2\frac{1}{2}d.$ a lb. a profit of 10 per cent. may be made?

10. A grocer buys some tea at 2*s.* a pound, and some at 2*s.* 9*d.* In what proportion must he mix them so that, when he sells the tea at 3*s.* a pound, he may make a profit of 20 per cent.?

11. A grocer buys biscuits at 3*d.* and others at 5*d.* a lb., spending the same money on each kind. He mixes the two sorts and sells the mixture at 4*d.* a lb. What does he gain per cent.?

12. A grocer mixes 120 lbs. of tea at 2*s.* 6*d.* a lb. with a certain quantity at 2*s.* a lb., and sells the whole at 2*s.* 4*d.* a lb., gaining 5 per cent. Find the quantity of 2*s.* tea in the mixture.

WORK AND TIME.

224. We now consider problems with reference to work done in various times. These can all be solved by considering the fractional parts of the whole work which are done in a definite time.

Ex. 1. One man can mow a field in 30 hours and another man can mow the field in 60 hours: how long would it take them working together to do it?

The first man mows $\frac{1}{30}$ of the whole in 1 hour,

... second..... $\frac{1}{60}$ .....;

$\therefore$ ... two together ... $\frac{1}{30}+\frac{1}{60}$.....

And, as the two together would mow $\left(\frac{1}{30}+\frac{1}{60}=\right)\frac{1}{20}$ of the whole in 1 hour, they would mow the whole in 20 hours.

Ex. 2. A cistern could be filled in 20 minutes by its supply pipe and emptied in 35 minutes by its waste pipe. If the cistern be empty and both pipes be opened, how long would it take to fill it?

The supply pipe fills $\frac{1}{20}$ of the cistern in 1 minute,
... waste ... empties $\frac{1}{35}$ .....;
hence together they fill $(\frac{1}{20}-\frac{1}{35}=)\frac{3}{140}$ ....

And, as $\frac{3}{140}$ of the whole is filled in 1 minute, the whole will be filled in 1 min. $\div\frac{3}{140}=46\frac{2}{3}$ minutes.

EXAMPLES XVII. C.

1. *A* can mow a field in 3 days, and *B* can mow the same field in 6 days. In how many days will they do it working together?

2. A bath could be filled by its cold water pipe in 15 minutes and by its hot water pipe in 30 minutes. In what time will it be filled when both are opened?

3. *A* can do a piece of work in 12 days and *B* can do the same in 20 days. *A* works at it for 3 days; how long would it take *B* to finish it?

4. *A* can mow a field in 15 hours and *B* can mow the same field in 25 hours. They work together for $7\frac{1}{2}$ hours, when *A* goes away. How long will it take *B* to finish the work?

5. Two men together can do a piece of work in 20 days, which one of them alone could do in 30 days. How long would it take the other man to do the work alone?

6. When the hot and cold water pipes are both opened a bath is filled in 6 minutes; and when the cold water only is turned on, the bath is filled in 10 minutes. In how long would the bath be filled if the hot water pipe only is opened?

7. *A* and *B* could together finish a piece of work in 25 days. They work together for 15 days, and then *A* finished it by himself in 20 days. How long would it take them to do the whole working separately?

8. *A* and *B* could together do a piece in $22\frac{1}{2}$ days. *A* worked at it alone for 10 days, and then *B* finished it alone in 60 days. How long would it take them separately to do the whole work?

9. *A* can do a piece of work in $2\frac{1}{2}$ days, *B* can do it in 3 days, and *C* can do it in $3\frac{3}{4}$ days; how long would it take them when all working together to do it?

10. A cistern is filled by one pipe in 48 minutes, by another in an hour, and by a third in half an hour. In what time would it be filled if all three pipes were open together?

11. A cistern can be filled by one pipe in 3 hours, by another in 3 hrs. 40 min., and it can be emptied by a third pipe in 2 hrs. 20 min. If it be empty and they are all opened together, in what time will the cistern be filled?

12. A can do a piece of work in 6 days, B in 8 days, and C in 12 days. B and C work together for 2 days, and then C is replaced by A. Find when the work will be finished.

13. C does half as much in a day as A and B can do together, and B does half as much again as A. If all three working together can mow 20 acres of barley in 16 days, how long would each, working by himself, take to mow 5 acres?

14. A and B together can perform a piece of work in 24 hours, A and C in 30, and B and C in 40 hours: in what time would each be able to perform it when working separately?

RACES AND GAMES.

225. The following are examples of questions of this nature.

Ex. 1. In a 100 yards race A can give B 5 yards start and *just* win, also B can give C 5 yards start. How much could A give C?

A runs 100 yards while B runs 95, and B runs 100 yards while C runs 95.

Hence C's distance in any time $= \frac{95}{100}$ of B's $= \frac{95}{100} \times \frac{95}{100}$ of A's.

Hence while A runs 100 yards, C will run $\frac{95}{100} \times \frac{95}{100}$ of $100 = 90\frac{1}{4}$ yards.

Thus A can give $9\frac{3}{4}$ yards to C.

Ex. 2. In a certain game A can give B 1 point in 5, B can give C 1 point in 5, and C can give D 1 point in 8. How many points can A give D in 100?

From the given conditions, we have

$$\begin{array}{cccc} A & : & B & : & C & : & D \\ 5 & : & 4 & & & & \\ & & 5 & : & 4 & & \\ & & & & 8 & : & 7. \end{array}$$

[Numbers in the same horizontal line being proportional to the scores made by the persons below whose names the numbers occur.]

Hence, as in Example 3, Art. 221,

$$25 : 20 : 16 : 14;$$
$$\therefore \quad 100 : 80 : 64 : 56.$$

Thus A can give $(100 - 56 =)$ 44 points in 100 to D.

EXAMPLES XVII. D.

1. A can give B 10 yards start in a race of 100 yards, and B can give C 10 yards start over the same distance. How many yards start can A give C?

2. A can give B 20 yards and C 51 yards start in a quarter of a mile race. How many yards could B give C in a quarter of a mile?

3. A can beat B by 5 yards in a 100 yards race, and B can beat C by 10 yards in a 200 yards race; by how much could A beat C in a 400 yards race, supposing that they always run at the same pace?

4. A wins a race of 100 beating B by 19 yards and C by 10 yards. How many yards start ought C to give B in 200 yards that they may run a dead heat?

5. In a certain game A can give B 1 point in 10, B can give C 1 point in 6. How many can A give C in 100?

6. At a certain game A scores 100 points while B scores 85, and B scores 100 while C scores 80. How many will C score in the time that it takes A to score 500?

7. A can make 9 articles while B makes 14, and B can make 7 while C makes 6: how many can C make in the time that A makes 30?

8. In a certain game A can give B 1 point in 10, B can give C 1 point in 12, and C can give D 1 point in 15. How many can A give D in 1000?

9. In a certain game A can give B 1 point in 5, B can give C 1 point in 8, and C can give D 3 points in 10. How many can A give D in 100?

10. A can give B 20 yards and can give C 41 yards start in a race of a quarter of a mile, and B can give C a start of 3 seconds over the same distance. How long does each take to run a quarter of a mile?

226. The following examples are worth notice.

Ex. 1. A starts at 10 o'clock to walk along a road at the rate of 4 miles an hour; B starts on a tricycle at 45 minutes past 10 and rides after A at the rate of 9 miles an hour. When will B overtake A?

When B starts A has already travelled $\frac{45}{60}$ of 4 miles, that is 3 miles.

B gains on A at the rate of $(9-4=)$ 5 miles an hour.

B will overtake A when he has gained 3 miles, which he will do in $(3\div5)$ hours $=36$ minutes.

Ex. 2. At what time between 4 and 5 o'clock will the hands of the clock be together?

At 4 o'clock the minute-hand is 20 minute-spaces behind the hour-hand. In one hour the minute-hand passes over 60 minute-spaces, and the hour-hand passes over 5 minute-spaces.

Thus the minute-hand gains 55 minute-spaces in an hour.

Now when the two hands are together, the minute-hand must have gained on the hour-hand 20 minute-spaces, and the time required for this $=\frac{20}{55}$ of an hour $=21\frac{9}{11}$ minutes.

Thus the time required is $21\frac{9}{11}$ minutes past 4.

Ex. 3. A train travelling at the rate of 45 miles an hour is observed to completely pass a certain telegraph post in 5 seconds; it also completely passed in 4 seconds a second train which was travelling along a parallel line of rails in the opposite direction at the rate of 30 miles an hour. How long were the trains?

The time the first train takes to completely pass a post is the time it takes to travel a distance equal to the length of the train; and, since the train goes at the rate of 45 miles an hour, it goes in 5 seconds a distance $=\frac{5}{3600}$ of 45 miles $=110$ yards. Thus the first train is 110 yards long.

Again, in the time the trains take to completely pass one another the distance travelled by the two trains together must be the sum of the lengths of the trains; and in 4 seconds the trains will together travel $\frac{4}{3600}$ of 45 miles $+\frac{4}{3600}$ of 30 miles $=146$ yards 2 ft.

Hence the length of the second train

$=146$ yds. 2 ft. -110 yards $=36$ yds. 2 ft.

Ex. 4. Seven fowls are worth 6 ducks, 7 ducks are worth 2 geese, 10 geese are worth 7 turkeys, and a turkey is worth 17*s*. 6*d*. How much is a fowl worth?

One fowl is worth $\frac{6}{7}$ of the worth of a duck,

$\therefore$ ..... $\frac{2}{7}$ of $\frac{6}{7}$ ....goose,

$\therefore$ ..... $\frac{7}{10}$ of $\frac{2}{7}$ of $\frac{6}{7}$ .....turkey,

$\therefore$ ..... $\frac{7}{10}$ of $\frac{2}{7}$ of $\frac{6}{7}$ of 17*s*. 6*d*.

$$=\frac{7}{10}\times\frac{2}{7}\times\frac{6}{7}\times\frac{35}{2}\text{ shillings}=3s.$$

EXAMPLES XVII. E.

1. One boy runs at the rate of 100 yards in 15 seconds, and has a start of 40 yards in front of another boy who runs at the rate of 100 yards in 12 seconds. When will the first boy be caught?

2. One cyclist rides at the rate of 15 miles an hour and starts half-an-hour after another who rides along the same road at the rate of 12 miles an hour. When will the first rider be overtaken?

3. At what time between 5 and 6 o'clock will the hands of the clock be together?

4. At what time between 2 and 3 o'clock will the hands of the clock be at right angles?

5. A train travelling at the rate of 45 miles an hour is observed to completely pass a certain point in 9 seconds. Find the length of the train.

6. A man on the platform of a station observed that a train passed him in 10 seconds, and passed completely through the station, which is 308 yards long, in 24 seconds. How long was the train, and how fast was it going?

7. A passenger train, moving at the rate of 45 miles an hour, overtook a mineral train twice as long as itself which was going along a parallel line of rails in the same direction at the rate of 23 miles an hour; and the passenger train completely passed the mineral train in $22\frac{1}{2}$ seconds. How long was each train?

8. A person lights two candles 12 and 10 inches long respectively at 6 p.m. The former diminishes 5 inches in length in 4 hours, and the latter 1 inch in 2 hours. If kept alight at what time will the former be two inches shorter than the latter?

9. If 3 pears are worth as much as 4 apples, 5 apples as much as 3 plums, 8 plums as much as 3 peaches, and if pears cost 1*s.* 6*d.* a dozen; what is the price of a peach?

10. Twelve fowls are worth as much as 11 ducks, 5 ducks are worth as much as 4 pheasants, 10 pheasants as much as 3 turkeys, and 7 turkeys as much as 10 geese; also a fowl and a pheasant are together worth 6*s.* 6*d.* Find the cost of a goose and a turkey together.

CHAPTER XVIII.

SCALES OF NOTATION.

227. NUMBERS have hitherto been expressed by means of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, on the understanding that every figure placed to the left of another represents *ten* times as many as if it were in the place of that other.

This mode of representing numbers is called the **common scale of notation,** and ten is called the **radix** or **base** of the scale.

228. Any other number might have been used instead of ten as the base of a *system of numeration,* that is of a system by which numbers are named according to some definite plan, and of the corresponding *scale of notation,* that is of a system by which numbers are represented by a few signs according to some definite plan.

Thus 2345 would be written in the scale of six, if 4 stood for 4×6, 3 for 3×6^2, and 2 for 2×6^3, so that every figure placed to the left of another represented *six* times as many as if it were in the place of that other.

229. It would be a good exercise to perform the first four rules of arithmetic with numbers expressed in various scales.

EXAMPLES XVIII. A.

1. Add 351, 462, 325 and 1234. [Scale 7.]
2. Add 3121, 1233, 3012 and 1332. [Scale 4.]
3. Subtract 417 from 1000, and 682 from 710. [Scale 9.]

4. Subtract 1406 from 2132 and 4615 from 11352. [Scale 8.]

5. Multiply 123 by 321. [Scale 4.]

6. Multiply 5432 by 2054. [Scale 6.]

7. Divide 77372 by 325. [Scale 9.]

8. Divide 128796 by 236. [Scale 12.] [Use t for ten and e for eleven.]

230. To shew how to express any number given in the ordinary scale in any other scale of notation it will only be necessary to consider an example.

Ex. 1. Express 853 in the scale of six.

Consider any number, for example 2345, which is already written in the scale of six; this number is equal to

$$5+4\times6+3\times6^2+2\times6^3.$$

If we divide the number by 6, we shall have for remainder 5, the first figure on the right, and the quotient will be

$$4+3\times6+2\times6^2.$$

If we now divide this quotient by 6, the remainder will be 4, the second figure on the right.

Thus *the figures can be found in order from the right by dividing the given number by* 6; *and dividing the quotient so obtained again by* 6; *and so on.* The method is easily seen to be applicable to all numbers and to any scale.

$$\begin{array}{rl} 6\,|\,853 & \\ 6\,|\,142 & \text{remainder } 1 \\ 6\,|\,23 & \ldots\ldots\ldots\ldots\ 4 \\ 3 & \ldots\ldots\ldots\ldots\ 5 \end{array}$$

Thus 853 is expressed in the scale of 6 by 3541.

Ex. 2. Find the number in the ordinary scale which is expressed by 3541 in the scale of 6.

Since $3541=3\times6^3+5\times6^2+4\times6+1,$

the required result can be obtained by ordinary multiplication.

Ex. 3. Change 257 from the scale of 8 to the scale of 4.

We may first change 257 from the scale of 8 to the ordinary scale, as in Ex. 2, and then change to the scale of 4, as in Ex. 1.

The result may, however, be obtained by one process.

$$\begin{array}{rl} 4\,|\,257 & \\ 4\,|\,53 & \text{rem. } 3 \\ 4\,|\,12 & \ldots\ldots\ 3 \\ 2 & \ldots\ldots\ 2 \end{array}$$

Explanation: We first divide 2 *eights* plus five by 4, giving the quotient 5 and remainder 1; we then divide 1 *eight* plus 7 by 4 giving quotient 3 and remainder 3; and so for each line.

The required number is 2233.

Sometimes the scales of two, three, four, ..., ten, eleven, twelve are called the Binary, Ternary, Quatenary, Quinary, Senary, Septenary, Octenary, Nonary, Denary or Decimal, Undenary, and Duodenary or Duodemical, Scales respectively.

EXAMPLES XVIII. B.

1. Express 3159 in the scale of 6.
2. ..... 5624 ..... 8.
3. ..... 1893 ..... 4.
4. ..... 359 ..... 3.
5. ..... 3975 ..... 11.
6. ..... 12345 ..... 12.
7. Transform 3154 from the scale of 6 to the ordinary scale.
8. Transform 15384 from the scale of 9 to the ordinary scale.
9. Change 3561 from the scale of 7 to the scale of 8.
10. Change $1473t$ from the scale of 11 to the scale of 12.

231. Radix Fractions. Radix fractions in any scale correspond to decimal fractions in the ordinary scale; thus ·213 in the scale of 6 stands for

$$\frac{2}{6}+\frac{1}{6^2}+\frac{3}{6^3}.$$

To shew how to express any fraction given in the ordinary scale by a series of radix fractions in any proposed scale, it will only be necessary to consider an example.

Ex. 1. Express the fraction $\frac{23}{108}$ by a series of radix fractions in the scale of 6.

Consider, for example, ·325 which is already written in the scale of six, and which, therefore, is equal to

$$\frac{3}{6}+\frac{2}{6^2}+\frac{5}{6^3}.$$

If we multiply by 6 the integral part of the product will be 3, the first figure after the point, and the fractional part will be $\frac{2}{6}+\frac{5}{6^2}$. If we now again multiply by 6, the integral part of the product will be 2, the second figure after the point.

Thus the figures can be found in order by multiplying the given fraction by 6, and the fractional part of the product so obtained by 6; and so on. The method is easily seen to be applicable to all fractions and to any scale.

We have $\frac{23}{108}\times 6 = 1+\frac{5}{18}$; $\frac{5}{18}\times 6 = 1+\frac{2}{3}$; and $\frac{2}{3}\times 6 = 4$.

Hence $$\frac{23}{108} = \frac{1}{6}+\frac{1}{6^2}+\frac{4}{6^3}$$

$= \cdot 114$, when expressed by radix-fractions in the scale of six.

Ex. 2. Change 135·3 from the scale of 10 to the scale of 9.

The integral and fractional parts of the given number must be changed by separate processes.

9 | 135	·3
9 | 15 rem. 0	9
1 rem. 6	2·7
	9
	6·3

Thus the required result is $160\cdot\dot{2}\dot{6}$.

Ex. 3. Express $\cdot 5\dot{3}\dot{1}$ as a vulgar fraction, all the numbers being in the scale of 6.

Since $$\cdot 5\dot{3}\dot{1} = \cdot 5313131\ldots$$

$$\cdot 5\dot{3}\dot{1}\times 10 = 5\cdot 3131\ldots$$

and $$\cdot 5\dot{3}\dot{1}\times 1000 = 531\cdot 3131\ldots$$

Hence, by subtraction,

$$\cdot 5\dot{3}\dot{1}\times(1000-10) = 531-5;$$

$$\therefore\ \cdot 5\dot{3}\dot{1} = \frac{531-5}{1000-10} = \frac{522}{550}.$$

EXAMPLES XVIII. C.

1. Express 416·25 in the scale of 8.
2. ..... 328·125 ..... 4.
3. ..... 413·24 ..... 5.
4. ..... 175·0625 ..... 6.
5. ..... 317·125 ..... 12.
6. ..... 517·56 ..... 7.
7. ..... 149·23 ..... 11.
8. Change 31·45 from the scale of 6 to the ordinary scale.
9. Change 147·24 from the scale of 8 to the ordinary scale.
10. Change 123·45 from the scale of 6 to the scale of 9.
11. Change 719·24 from the scale of 12 to the scale of 7.

CHAPTER XIX.

FOREIGN EXCHANGES.

232. If a person in a foreign country, say France, imports goods from England, he might pay for the goods by sending to England the proper amount of gold in the form either of coined money, called **specie**, or of uncoined gold, called **bullion**. The trouble and expense of sending specie or bullion from one country to another is, however, almost entirely done away with by means of Bills of Exchange.

233. A Bill of Exchange is a written order for the payment of a specified sum of money. It must state the name of the **Drawer**, that is of the person giving the order; the name of the **Drawee**, that is of the person on whom the order is given; and the name of the **Payee**, that is of the person in whose favour the bill is drawn. When the bill has received the Drawee's acceptance, or written agreement to pay it, the Drawee is called the **Acceptor**.

A Bill payable in any particular place, Paris for example, is called a bill **on** Paris.

The amount of a Foreign Bill is usually expressed in the money of the country in which it is to be paid.

234. The comparative value of the coins of two countries, estimated by the actual value of the metal contained in them, is called the '**Par of Exchange**.'

For example, the amount of gold in 1000 sovereigns is equal to that in 2517 ten-franc gold pieces; hence the 'par of exchange' is £1 = 25·17 francs.

235. The actual amount paid for Bills of Exchange, for example paid in London for bills on Paris, varies from time to time; the current price paid for Bills, called the '**Rate of Exchange**,' cannot however ordinarily be much above or below par; for if it would cost more to discharge a debt by means of a bill than by the actual transmission of bullion, the latter method would naturally be adopted.

It should be noticed that if all countries had exactly the same coinage there would still be fluctuations in the rate of exchange between two countries, as the balance of indebtedness between those two countries varied.

For instance, 'the course of exchange between London and Australia stands occasionally *at par*, but there is generally a premium on London ranging between 1 and 5 per cent.; £100 in London = £101 to £105 in Australia.' Tate's *Modern Cambist*.

236. The following Table, taken from Tate's *Modern Cambist*, gives the average prices of Bills of Exchange in London. The Table also shews the way in which the prices are usually given.

France, Belgium, Switzerland	[1 Franc = 100 Centimes]	25·25 fr. for £1.
Italy	[1 Lira = 100 Centesimi]	25·25 fr. .....
Germany	[1 Mark = 100 Pfennige]	20·55 M. .....
Austria	[1 Florin or Gulden = 100 Kreuzer]	11·75 Fl. .....
Holland	[1 Florin or Gulden = 100 Cents = 20 Stivers]	12 Fl. 3 St. .....
Norway, Sweden, Denmark	[1 Krone = 100 ore]	18·40 Kr. .....
Russia	[1 Rouble = 100 Copecks]	25d. for 1 Rouble.
Portugal	[1 Milreis = 1000 Reis]	52d. for 1 Milreis.
Spain		4*s*. 8*d*. for 1 Dollar.
Turkey		112 Piastres for £1.
United States	[1 Dollar = 100 Cents]	$4·80 for £1.
Calcutta &c.		1*s*. 8*d*. for 1 Rupee.
Hong Kong		3*s*. 9*d*. for 1 Dollar.
Shanghai		5*s*. 2*d*. for 1 Tael.

237. The approximate average values of the different foreign coins can be found from the Table in Art. 236. The most important are

1 Franc &c.	$=9\frac{1}{2}d.$	1 Rouble	$=2s.\ 1d.$
1 Mark	$=11\frac{3}{4}d.$	1 Milreis	$=52d.$
1 Austrian Florin	$=1s.\ 8d.$	1 Piastre	$=2\frac{1}{4}d.$
1 Dutch Florin	$=1s.\ 7\frac{3}{4}d.$	1 Rupee	$=1s.\ 8d.$
1 Scandinavian Crown	$=1s.\ 1\frac{1}{2}d.$	1 Tael	$=5s.\ 2d.$
1 U.S. Dollar	$=4s.\ 2d.$		

238. Suppose a debtor in Paris has to pay money to a creditor in London. The debtor can (1) either buy a Bill of Exchange on London for the amount of his debt and send this to his creditor in London; or (2) the debtor can direct his creditor to '**draw**' on him in Paris, in which case the creditor would make out a bill of exchange payable by the debtor in Paris, and for this he would obtain money in London.

The debtor might, however, adopt a more circuitous course. He might, for example, send to London a bill for the proper amount on Hamburg, and direct his agent at Hamburg to draw on him in Paris.

Operations of exchange between two countries through the medium of one or more other countries are called **indirect exchanges**, and the rates resulting therefrom are called **arbitrated rates of exchange**.

The object of an operation in indirect exchange is to obtain a better rate than is afforded by the direct exchange.

239. The following extract from the *Daily News* of Nov. 18, 1891, will shew the way in which Exchanges are quoted.

Exchanges on London (telegraphed from)—Paris, 25.19; Berlin, eight days, 20.31½; Frankfort, short, 20.32; Vienna, 118.10; Hamburg, three months, 20.18; Amsterdam, 12.05½; Calcutta, 1*s.* 4⅝*d.*; Hong Kong, 3*s.* 1½*d.*; Shanghai, 4*s.* 3¾*d.*; St Petersburg, three months, 106.

Rates in London for bills on Foreign Countries:—

Amsterdam, 12.1 to 12.2; ditto, three months, 12.3½ to 12.4; Antwerp and Brussels, three months, 25.46¼ to 25.51¼; Berlin and German Bank Places, 20.55 to 20.59; Paris, cheques, 25.17½ to 25.22½; Paris, three months, 25.38¾ to 25.43¾; Austria, three

months, 11.97½ to 12.02½; Petersburg, three months, 22 to 22⅛; Genoa, Leghorn, Naples, &c., three months, 26.18¾ to 26.23¾; Spain, three months, 40¾ to 41; Lisbon and Oporto, three months, 41 5/16 to 41 5/16; Copenhagen, three months, 18·42 to 18.46.

On reference to the Table given in Art. 236 nearly all the above quotations will be obvious. The Vienna quotation on London gives the number of florins for £10, the St Petersburg quotation on London gives the number of Roubles for £10. Amsterdam on London is given in florins and *cents*.

Cheques or Short is for Bills payable at sight.

240. The following are examples of different cases which have to be considered. In actual transactions brokerage [usually $\frac{1}{10}$ per cent.] would have to be taken into account.

Ex. 1. When the course of Exchange is £1 in London for 25·24 francs in Paris, how much would a bill on Paris for 10000 francs cost in London?

£1 cash in London buys 25·24 francs to be paid in Paris;

$\therefore$ £$\frac{10000}{25{\cdot}24}$ ..... 10000 francs .....

Thus, sum required = £$\frac{10000}{25{\cdot}24}$ = £396·1965 = £396. 3*s*. 11*d*.

Ex. 2. When the course of Exchange is 25·30 francs in Paris for £1 in London, for how much should a bill on London be drawn so as to be worth 10000 francs in Paris?

£1 to be paid in London sells for 25·30 fr. in Paris;

$\therefore$ £$\frac{10000}{25{\cdot}30}$ ..... 10000 fr. .....

Thus, sum required = £$\frac{10000}{25{\cdot}30}$ = £395·2569 = £395. 5*s*. 1*d*.

Ex. 3. A merchant has to pay 10000 francs in Paris when the rates of exchange are: London on Paris 25·24,
Paris on London 25·30.

Which will be the better way (1) for him to buy direct bills on Paris or (2) for his creditor in Paris to draw upon him in London?

The two cases are considered separately in the preceding questions, from which it appears that the second course is the most advantageous, the difference amounting to 18*s*. 10*d*.

Ex. 4. Find the arbitrated rate of exchange at London on Paris through Hamburg, when

London on Hamburg £1 = 20·4 marks,
Hamburg on Paris 1 mark = 1·24 francs.

Here £1 in London buys 20·4 marks payable in Hamburg, and 1 mark in Hamburg buys 1·24 francs payable in Paris.

Hence £1 in London will buy $20{\cdot}4 \times 1{\cdot}24$ francs in Paris.

Ex. 5. The exchanges are as under:—

London on Berlin, at 3 months, £1 for 20·48 marks,
Berlin on London, at 3 months, £1 for 20·12 marks.

What should be the exchanges for bills payable *at sight*, interest being allowed at 4 per cent.?

£1 cash paid in London buys 20·48 marks to be paid in 3 months at Berlin.

Now the discount on a three months bill for 20·48 marks at 4 per cent. per annum is ·2048 [see Art. 195].

Hence 20·48 marks at 3 months is worth (20·48 − ·20) marks.

Thus London on Berlin at sight should be £1 for 20·28 marks.

Again 20·12 marks cash paid in Berlin buys £1 to be paid in London in 3 months, or $£1 \times \frac{99}{100}$ to be paid in London at sight.

Hence £1 *to be paid in* London at sight should cost

$$20{\cdot}12 \text{ marks} \times \frac{100}{99}, \text{ which is } \textit{very nearly } 20{\cdot}12 \text{ marks} \times \frac{101}{100}$$

$$= (20{\cdot}12 + {\cdot}20) \text{ marks} = 20{\cdot}32 \text{ marks}.$$

Thus Berlin on London at sight should be £1 for 20·32 marks.

EXAMPLES. XIX.

Find the cost in London of a Bill of Exchange for

1. 50000 francs on Paris at 25·30 [francs for £1].
2. 1250 marks on Berlin at 20·55 [marks for £1].
3. 1500 florins on Amsterdam at 12 $1\frac{1}{2}$ [12 florins $1\frac{1}{2}$ stivers for £1].
4. 1560 milreis on Lisbon at $42\frac{1}{2}$ [pence for 1 milreis].
5. 1000 gulden on Vienna at 11·69 [gulden for £1].
6. 2560 crowns on Stockholm at 18·30 [Kr. for £1].
7. 1765 roubles on St Petersburg at $24\frac{1}{2}$ [pence for 1 rouble].
8. 1725 lira on Genoa at 25·35 [lira for £3].
9. 10000 dollars on New York at 4·80 [$ for £1].

10. 1575 rupees on Calcutta at 1*s.* 5*d.* [for 1 rupee].

11. What should be the quotations for 3 months bills, having given the following quotations, and assuming that bills are discounted in all cases at 4 per cent.

(i) London on Paris cheques 25·27,
(ii) London on Amsterdam at sight 12 $1\frac{1}{2}$,
(iii) London on Lisbon " " $42\frac{1}{2}$,
(iv) London on St Petersburg " " $24\frac{3}{4}$?

12. When the course of Exchange is

London on Antwerp at 3 months 25·55 [francs for £1],
London on Vienna at 3 months 11·90 [gulden for £1],
St Petersburg on London at 3 months 94·75 [roubles for £10],
New York on London at 60 days 4·80 [$ for £1],

find the cost of the following bills, assuming that the rate of discount is in all cases 4 per cent. per annum :—

(i) London on Antwerp 1560 francs at sight,
(ii) London on Vienna 7640 gulden " "
(iii) St Petersburg on London £1500 " "
(iv) New York on London £1750. 12*s.* 6*d.* " "

13. Find the arbitrated rate of exchange at London on St Petersburg through Berlin, having given

London on Berlin at 3 mos. 20·55,
Berlin on St Petersburg at 3 mos. 188 [Marks for 100 Roubles].

MISCELLANEOUS EXAMPLES.

1. Express in words 5006017, and in figures thirteen millions twenty-five thousand and eleven.

2. Find the least multiple of 3157 which is greater than a million.

3. How many articles worth £2. 17*s.* 9*d.* each should be given in exchange for 154 articles worth £3. 16*s.* 6*d.* each?

4. Reduce 5 tons 7 cwt. 1 qr. 5 lbs. 11 oz. to ounces.

5. Find the G. C. M. and also the L. C. M. of 3432 and 3575.

6. Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{11}{12}$ and $\frac{13}{24}$.

7. Divide $43\frac{1}{43}$ by $28\frac{28}{129}$, and express the result as a fraction of 12.

8. Divide ·221312 by 5·32.

9. Add together ·375 of 13*s.* 4*d.* and ·07 of £2. 10*s.*, and subtract the result from £·45.

10. Find the rent of 134 A. 3 R. 25 P. at £3. 13*s.* 4*d.* an acre.

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11. Multiply 905741 by 518963, and express the result in words.

12. A certain number was divided by 77 by short divisions; the quotient was 137, the first remainder was 9 and the second remainder was 6. What was the dividend?

13. Reduce 15 m. 2 fur. 15 p. 3 yds. to inches.

14. A grocer mixed 48 lbs. of tea which cost him 2*s.* 8*d.* a lb. with a certain quantity which cost 2*s.* 6*d.* a lb. He then sold the whole for £16. 1*s.* 6*d.*, and gained £2. 3*s.* 6*d.* by the transaction. How much tea did he sell?

15. Express 756, 1155 and 1176 as the product of prime factors.

16. Simplify $4\frac{1}{2}+\frac{1}{3}-3\frac{3}{4}+5\frac{7}{8}-6\frac{11}{16}$.

17. Simplify $\frac{3}{4}$ of $\frac{5}{12}$ of $4\frac{8}{13}\div 1\frac{2}{3}$ of $\frac{5}{13}$.

18. Simplify $2{\cdot}9015\times{\cdot}01702\div{\cdot}005803$.

19. Find $\frac{3}{7}$ of £2. 11*s.* 11*d.* − ·115625 of £1 + ·75*d.*

20. A farm of 359 acres 2 roods 32 poles is let at £1. 3*s.* 4*d.* per acre. What is the whole rent?

21. Find the difference between seventy-six millions and eight, and four hundred and ninety-nine thousand four hundred and forty; and divide the result by ninety-nine.

22. What is the greatest number which will divide 2000 with remainder 11, and will divide 2708 with remainder 17?

23. Multiply 4 fur. 30 p. 9 in. by 144.

24. Taking the average length of a lunar month from full moon to full moon to be 29·5306 days and the length of a year to be 365·2422 days; shew that 4131 lunar months are very nearly equal to 334 years.

25. What is the least number of grains which is an exact number both of lbs. Troy and of lbs. Avoir.? If the number of lbs. Troy in a certain weight exceed the number of lbs. Avoir. by 496, what is that weight in grains?

26. Simplify $\dfrac{3\frac{1}{2}\times 3\frac{1}{2}\times 3\frac{1}{2}-3}{3\frac{1}{2}\times 3\frac{1}{2}-3}\div 8\frac{23}{37}$.

27. Find the value of a property if the owner of $\frac{3}{7}$ of it can sell $\frac{4}{5}$ of his share for £492.

28. Divide ·00625 by 2500, and 6·25 by ·0025.

29. Express 20 lbs. 8 oz. 9 dwts. 6 grs. as a decimal of 254 lbs. 10 oz.

30. Find the difference between the value of 13 cwt. 2 qr. 24 lbs. of sugar at a guinea per cwt., and that of 1 qr. 27 lbs. 12 oz. of tobacco at £28 per cwt.

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31. Write MDCCCXCIX in Arabic figures, and express 1489 by means of Roman numerals.

32. A man takes 100 steps a minute, and the average length of his step is 30 inches. How far will he walk in 4 hours?

33. How much coal is required to supply 12 fires for 27 weeks, each fire consuming 1 cwt. 1 qr. 14 lbs. of coal weekly?

34. Find the greatest number by which when 4344 and 5943 are divided the remainders will be 31 and 41 respectively.

35. Simplify $267\frac{3}{4}$ of $\frac{16}{19}\times(\frac{5}{9}-\frac{3}{7}-\frac{1}{15})$.

36. Simplify $\dfrac{4\frac{1}{5}-3\frac{1}{4}}{14\frac{1}{5}+4\frac{1}{4}}\div\dfrac{4\frac{1}{5}\text{ of }4\frac{3}{4}}{13\frac{13}{14}\div 1\frac{21}{41}}$.

37. If $\frac{18}{59}$ of $1\frac{1}{3}$ of $29\frac{1}{2}$ of a certain sum is £1938. 6*s.* 9*d.*, what is the sum?

38. Reduce ·63, ·48324 and ·01654 to vulgar fractions in their lowest terms.

39. What decimal is £2. 17*s.* 10½*d.* of £5? Find the value of ·78125 of £1 − ·0625 of 5*s.* − 2·625 of 2*d.*

40. Find by Practice the cost of a silver cup weighing 2 lbs. 5 oz. 17 dwts. 12 grs. at 7*s.* 4*d.* an ounce.

41. The sum of £602. 3*s.* 11*d.* is to be divided in equal portions among 226 people. How much will each receive?

42. The heights of 5 boys are respectively 5 ft. $4\frac{1}{2}$ in., 5 ft. 2 in., 5 ft. $1\frac{1}{2}$ in., 4 ft. 10 in., and 4 ft. $8\frac{1}{2}$ in. What is the *average* height?

43. Reduce 726314 inches to miles, fur., poles, &c.

44. A circular running path is 902 yards round. Two men start back to back to run round, and one runs at the rate of 10 miles and the other at the rate of $10\frac{1}{2}$ miles an hour. When and where will they meet for the first time?

45. Find the greatest length of which both 42 yds. 9 in. and 55 yds. 9 in. are multiples.

46. Find the least fraction which added to the sum of $\frac{11}{56}$, $\frac{8}{49}$, and $\frac{13}{80}$ will make the result an integer.

47. What fraction of 27*s.* is $\frac{9}{11}$ of 1*s.* $10\frac{1}{2}d.$?

48. Simplify $\frac{5{\cdot}76}{{\cdot}018} \times \frac{{\cdot}00196}{{\cdot}64}$; and divide $\dot{\cdot}7\dot{2}$ by $\dot{\cdot}i17936\dot{}$, expressing the result as a recurring decimal.

49. Find the value of $\dot{\cdot}05\dot{}$ of $\dot{\cdot}i0i\dot{}$ of £74. 18*s.* 6*d.*

50. A person buys 4 cwt. 3 qrs. 14 lbs. of sugar at £1. 8*s.* 4*d.* per cwt., and sells it at $3\frac{1}{2}d.$ per lb. How much does he gain?

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51. Find the sum of all the numbers between 100 and 200 which are divisible by 13.

52. If a person's income be 325 guineas a year, find what he will save in 4 years after spending on an average £5. 6*s.* $10\frac{3}{4}d.$ a week, taking 52 weeks to a year.

53. Divide 69 miles 7 fur. 39 po. 2 yds. 1 ft. 10 in. by 136.

54. The L. C. M. of two numbers is 11160, the G. C. M. is 15, and one of the numbers is 465. What is the other number?

55. Simplify $\frac{1}{9}$ of $1\frac{1}{4}$ of $4\frac{1}{2} \div \frac{5}{36}$ of $1\frac{1}{3}$ of $3\frac{1}{2}$.

56. Express $\cfrac{1}{3+\cfrac{3}{5+\cfrac{5}{7+\frac{7}{9}}}}$ as a simple fraction.

57. Subtract $1\frac{3}{14}$ of $\frac{1}{6}$ of a guinea from $\frac{2}{5}$ of $\frac{7}{16}$ of £1; and express the difference as a fraction of half a crown.

58. Divide ·37592 by ·0125 and 3759·2 by ·000125.

59. Express $\frac{3}{8}$ of £2·625 – $2\frac{4}{5}$ of 1·375 shillings as a fraction of a guinea.

60. Find the value of 7 A. 1 R. 31 P. 22 yds. at £36. 13*s.* 4*d.* an acre.

61. Find the difference, in pounds, between one million eight hundred and three thousand eight hundred and forty farthings, and one hundred and fifty thousand four hundred threepenny pieces.

62. How many miles, furlongs, poles, &c. are there in a hundred million inches?

63. If butter be bought at $5\frac{1}{2}$ guineas per cwt. and sold at 1*s.* $4\frac{1}{2}$*d.* per lb., how much will be gained on every cwt.?

64. Find the G. C. M. of 1035, 391 and 598.

65. Simplify $4\frac{1}{4}$ of $3\frac{1}{3} - 2\frac{1}{2} \div 5\frac{5}{7} + 6\frac{6}{11} \div 3\frac{3}{11}$.

66. Simplify $\dfrac{2}{3 - \dfrac{4}{5 - \frac{3}{2}}}$ and $\dfrac{1551{\cdot}55}{65{\cdot}1} \times \dfrac{21}{20{\cdot}02}$.

67. Multiply $31{\cdot}5\dot{6}$ by 123, and $\cdot00\dot{7}\dot{2}$ by $\cdot\dot{4}\dot{5}$.

68. Add ·00125 of an acre, ·0625 of a rood, and ·00375 of a pole; and subtract the result from 91·6634375 yards.

69. If a bankrupt pays 9*s.* 7*d.* in the pound, how much will a creditor receive to whom he owes £48. 9*s.*?

70. If 8 cwt. 3 qrs. 20 lbs. cost £5. 4*s.* 2*d.*, what would a ton cost at the same rate?

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71. Multiply £4. 17*s.* 9*d.* by 237, and find the least sum which must be added to the product to make it divisible by 169.

72. The distance between two stations is 234 miles 4 fur. 38 yards 2 ft. An engine wheel revolves 142878 times in travelling from one station to the other. How many inches does it travel in one revolution of the wheel?

73. What is the greatest weight of which both 1 ton 19 cwt. 1 qr. 22 lbs. and 4 tons 13 cwt. 18 lbs. are multiples?

74. Subtract $\dfrac{5\frac{4}{9}}{8\frac{10}{11}}$ of $2\frac{5}{11}$ from $5\frac{2}{7}$ of $3\frac{9}{25}$ of $\frac{40}{111}$.

75. Add together $\frac{11}{35}$ of $4\frac{1}{2}$ guineas, ·615 of a pound, and $3\frac{3}{11}$ of $4\frac{5}{7}$ of $1\frac{5}{9}$ of 16*s.* 7*d.*

76. The sum of the ages of a father and of his son is now 88 years, and 12 years ago the father was three times as old as the son. How old are they?

77. Obtain the cost of 2 tons 17 cwt. 2 qr. 7 lbs. at £1. 8*s.* 4*d.* per cwt.

78. If 4 A. 3 R. 27 P. of land cost £17707. 10*s.*, how much would 5 A. 2 R. 13 P. cost at the same rate?

79. What is the acreage of a rectangular field whose sides are respectively 201 yds. 2 ft. and 60 yards?

80. Find two numbers, one of which is double the other, and whose product is 8192.

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81. A number is divided by 210 in three steps, the factors being 5, 6, 7 in order; and the remainders are 2, 3, 4 in order. What would have been the remainders if the number had been divided by 7, 6, 5 in order?

82. What is the smallest number of farthings that must be added to £59. 17*s.* 6½*d.* to make the sum exactly divisible by 157?

83. Reduce 216875 inches to miles, fur., &c.; also 57637 square yards to acres, roods, poles and yards.

84. Simplify $\frac{3\frac{7}{16} \text{ of } 2\frac{10}{11}}{\frac{4}{13} \text{ of } 8\frac{2}{3}} + 1\frac{3}{13} \text{ of } \frac{5}{12} + \frac{63}{104} + \frac{1}{6}\left(\frac{3}{4} + \frac{1}{26}\right)$.

85. Divide 8·0262 by ·000504, and $\cdot 1\dot{3}\dot{8}$ by $\cdot 4\dot{1}\dot{5}$.

86. After paying an income tax of 8*d.* in the pound, a man had £968. 9*s.* 7*d.* left. What was his income?

87. If the carriage of 36 cwt. 2 qrs. 7 lbs. for 49 miles cost £20. 9*s.* 6*d.*, what weight should be carried 35 miles for £3. 17*s.* 6*d.*?

88. What length of carpet 27 inches wide would be required to cover the floor of a room 24 feet long and 21 feet wide?

89. A rectangular field contains 2 A. 3 R. 14 P., and its length is 6·25 chains. What is its breadth?

90. Find, to within a thousandth of the whole, the square roots of 15, $\frac{3}{80}$ and ·081.

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91. Knowing that July 4, 1891 was on a Saturday, find what day of the week May 11, 1844 was on.

92. How many yards worth 4*s.* 2¼*d.* a yard would be a fair exchange for 402 yds. worth 3*s.* 5½*d.* a yard?

93. Find the least length which is a multiple of 1 ft. 6 in., 4 ft. 6 in., 7 ft. 6 in. and of 15 ft. 9 in.

94. Simplify $4\frac{3}{7} \div \left(\frac{5}{28} \times \frac{4}{15} \times 10\frac{1}{3}\right)$ and $4\frac{3}{7} \div \frac{5}{28} \times \frac{4}{15} \times 10\frac{1}{3}$.

95. Which is the greater $\cdot\dot{3}\dot{4}$ of a ton, or $6\cdot\dot{9}\dot{6}$ of a cwt.? Express their difference as a decimal of a lb.

96. A creditor received only 5*s.* 7*d.* in the pound of a debt due to him, and thereby lost £908. 12*s.* 2½*d.* What was the debt?

97. If 5 men working 10 hours a day mow 30 acres of grass in 6 days, in how many days of 11 hours each could 3 men mow 33 acres?

98. How much will it cost to paint the ceiling of a room 15 ft. 6 in. long and 12 ft. 6 in. wide at 8*d.* per square foot?

99. How many loads (cubic yards) of gravel would be required to cover to a depth of 2 inches a path 90 yards long and 5 ft. wide?

100. One side of a square field of 22½ acres abuts on a road. This side is divided into building plots 100 feet deep and having a frontage along the road of 30 ft. each. The building plots are let at £12 each, and the rest of the field at £5. 10*s.* an acre. What is the total rental of the property?

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101. Divide £219979. 6*s.* $4\frac{1}{2}d.$ by 841.

102. How many square inches are there in 5 acres 2 roods 1 pole; and how many acres, roods, poles, &c. are there in one hundred million square inches?

103. Find the smallest number of pounds which is a multiple of a guinea and also of a dollar (4*s.* 2*d.*).

104. Multiply together $\frac{16\frac{9}{13}}{1\frac{11}{38}}$, $\frac{1\frac{8}{31}}{2\frac{13}{22}}$, $\frac{2\frac{16}{21}}{1\frac{2}{33}}$, $\frac{4\frac{1}{11}}{1\frac{9}{49}}$ and $\frac{7}{22}$.

105. Find the value of ·01625 of a ton, and shew that when divided by ·0175 it becomes 18 cwt. 2 qrs. 8 lbs.

106. Find by practice, the cost of 94 lbs. 10 oz. 18 dwts. 6 grs. at £3. 6*s.* 8*d.* per oz.

107. If 12 men can dig a trench 36 ft. long, 5 ft. wide and 4 ft. deep in 1 day; how long a trench 7 ft. wide and 3 ft. deep could 14 men dig in 5 days?

108. What is the acreage of a rectangular field whose length is 2 fur. 37 poles and whose breadth is 1 fur. 15 poles?

109. The number of sheep on a farm increased for 4 years at the rate of 20 per cent. each year, and there were originally 625 sheep. How many were there at the end of the 4 years?

110. A man makes a profit of 20 per cent. by selling an article for 1*s.*; how much per cent. would he make by selling it for 1*s.* 1*d.*?

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111. How many persons can each receive £1. 2*s.* $6\frac{1}{2}d.$ out of £545. 10*s.* 2*d.*?

112. Find the least number which when divided by 17 leaves a remainder 12, and when divided by 29 leaves a remainder 24.

113. The age of a father is three times the sum of the ages of his three sons, and two years ago the father's age exceeded the sum of the ages of the three sons by 36 years. How old is the father?

114. Reduce to their simplest forms:

(i) $\frac{3}{11}(\frac{2}{3}+\frac{5}{6}+\frac{7}{12}-\frac{1}{4})-1\frac{1}{4}$ of $\frac{1}{3\frac{1}{3}}$.

(ii) $(\frac{1}{2}+\frac{2}{5}+\frac{3}{4}+\frac{4}{5})\div(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5})$.

115. Find the value of $\dot{3}\dot{8}\dot{4}$ of £1. 7*s.* 9*d.* + $\dot{3}\dot{8}\dot{4}$ of £1. 7*s.* 6*d.* + $\dot{3}\dot{8}\dot{4}$ of £1. 17*s.* 6*d.*, and express the result as a decimal of £10.

116. Find the rent of a farm containing 480 acres 3 roods 12 poles at £3. 8*s.* 4*d.* an acre.

117. If 8 men can mow 35 acres of grass in 5 days working 10 hours a day, how many men would be required to mow 63 acres in 12 days, if they worked at the same rate for 12 hours a day?

118. How many turves each 3 ft. by 1 ft. would be required to turf a lawn 96 feet by 75 feet, and how much would they cost at 7*s.* a hundred?

119. A man bought eggs at 1*s.* 6*d.* a score, and sold them at 11 for a shilling. What profit did he make per cent.?

120. Find the Simple Interest on £672. 12*s.* 6*d.* for 15 months at 4 per cent. per annum.

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121. Divide 570326 by 63 by 'short' divisions, explaining clearly the formation of the remainder.

122. Two men have £11. 13*s.* between them, and one has 12*s.* 6*d.* more than the other. How much has each?

123. Reduce to its simplest form

$$(\tfrac{1}{2}+\tfrac{2}{3}) \text{ of } (\tfrac{3}{4}+\tfrac{4}{5})+\tfrac{5}{6} \text{ of } (\tfrac{1}{8}+\tfrac{1}{10})+\frac{1\frac{1}{2}}{22\frac{1}{2}} \div \frac{21\frac{1}{3}}{1\frac{1}{3}}.$$

124. Multiply 36·2 by ·057, and divide 5752·8 by ·00376, and ·0025 by 3·1.

125. Find the value of a bar of gold weighing 5 lbs. 10 oz. 17 dwts. 22 grs. at £4. 5*s.* per oz.

126. Knowing that 10000000 acres are equivalent to 4046711 hectares, find the number of hectares in a square mile.

127. In a room 22 ft. by 18 ft. there is a Turkey carpet with a border 2 ft. wide all round it. The carpet cost 20 guineas; how much was that a square yard?

128. How many gallons will a cistern 6 ft. by 4 ft. by 3 ft. hold, taking a gallon to be 277·274 cubic inches?

129. A man has a landed estate whose gross rental is £2550 a year, but he makes an allowance of 15 per cent. to all his tenants, and pays 8 per cent. of the rent received to his agent; also repairs and other outgoings come to £350 a year. How much does the landlord receive?

130. A grocer mixes 9 lbs. of coffee at 2*s.* 3*d.* a lb. with 6 lbs. of chicory at 7½*d.* a lb. At what price per lb. must he sell the mixture in order to get a profit of 25 per cent.?

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131. The total number of votes given for two candidates at an election was 127345, and the successful candidate had a majority over the other of 17377. How many votes did each get?

132. Divide £875 between three persons so that the first may have £50 more than the second and the second £75 less than the third.

133. A certain number less than 1000, when divided by 56 or by 72 leaves 13 as remainder. What is the number?

134. Subtract $\frac{3}{14}$ of 17*s.* $9\frac{1}{2}d.$ from the sum of $\frac{5}{11}$ of £2. 8*s.* $9\frac{3}{4}d.$ and $\frac{9}{16}$ of £3. 19*s.* 4*d.*

135. A wire ·2346 yards long is cut up into pieces each ·007 yards long. How many pieces will there be, and what length will be left over?

136. If the carriage of 8 tons 10 cwt. for 120 miles cost £1. 1*s.* 3*d.*, how much would be charged for the carriage of 19 tons 3 cwt. 3 qrs. for 160 miles?

137. A room is 21 ft. long, 17 ft. wide and 12 ft. high. How many pieces of paper 21 inches wide and 12 yards long must be bought to paper the room supposing 150 square feet of the walls are left uncovered?

138. A class contains 19 boys; and in an examination 6 boys got 56 per cent. of the full marks each, one got 90 per cent., and the rest got 39 per cent. each, except one boy who got no marks at all. What was the average per centage got by the boys in the class?

139. By selling 60 lbs. for a sovereign a tradesman lost $12\frac{1}{2}$ per cent. At what price per cwt. must he sell so as to gain $12\frac{1}{2}$ per cent.?

140. What sum will amount to £2528. 2*s.* 6*d.* in three months at $4\frac{1}{2}$ per cent.?

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141. A sum of £5 was distributed amongst a certain number of boys, each of whom received either threepence or fourpence. Two hundred and forty received fourpence each, how many received threepence each?

142. Reduce 563147 inches to miles, &c.

143. Find the prime factors of 58212. What is the greatest square number of which 58212 is a multiple?

144. Simplify $\dfrac{2\frac{109}{180}+\frac{1}{4}\left(\frac{2}{3}+\frac{3}{5}\right)-\frac{2}{3}\left(\frac{1}{4}+\frac{3}{5}\right)}{\frac{3}{4}\text{ of }\frac{5}{6}-\frac{2}{3}\text{ of }\frac{4}{7}}$.

145. Find the value of ·0078125 of £2. 13*s.* 4*d.*

146. Find the cost of 15 cwt. 3 qrs. 9 lbs. of cheese at £5. 9*s.* 8*d.* per cwt.

147. Find the acreage of a rectangular field whose length is 25 chains 80 links and whose breadth is 8 chains 75 links; find also the rent at 50 shillings an acre.

148. How many gallons are there in a kilolitre, knowing that 1 gallon = 277·274 cubic inches and that 1 litre = 61·027 cubic inches?

149. A man, after deducting £250 from his income, pays £46. 17*s.* 6*d.* income tax on the remainder. Without making the deduction the tax would have been £52. 1*s.* 8*d.* Find the amount of the income tax per pound, and the man's whole income.

150. On what sum would the total amount of simple interest payable in 4 years at $4\frac{1}{2}$ per cent. be £76. 10*s.*?

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151. If 16 cwt. 1 qr. 4 lbs. cost £19, what was the price per lb.?

152. A train 110 yards long was observed to pass a certain point in 10 seconds. How many miles an hour was it then going?

153. Find the G.C.M. of 464321 and 683111, and hence find all the common measures of those numbers.

154. Simplify $2\frac{5}{6}$ of $1\frac{1}{11} - 2\frac{8}{11} + \dfrac{4\frac{9}{11} \text{ of } 2\frac{19}{17}}{1\frac{29}{51} \text{ of } 5\frac{3}{10}} - 1\frac{9}{17}\left(3\frac{3}{13} - 2\frac{1}{4}\right)$.

155. Express as recurring decimals the products

$$3{\cdot}6\dot{2}8\dot{7} \times 1{\cdot}875 \text{ and } 1{\cdot}\dot{6}\dot{3} \times {\cdot}0\dot{6}.$$

156. If bread for 15 men for 7 days cost a certain sum when wheat is 36 shillings a quarter; what must be the price per quarter if it cost the same sum to provide bread for 10 men for 8 days?

157. The wages of A and B together for 45 days amount to the same sum as the wages of A alone for 72 days. For how many days will this sum pay the wages of B alone?

158. Ten loads (cubic yards) of gravel are spread uniformly over a path 180 feet long and 4 feet wide; what is the depth of the gravel?

159. Find the weight in kilogrammes of the air in a room 60 ft. long 37 ft. wide and 21 ft. high, assuming that a cubic metre is 61027 cubic inches and that a litre of air weighs 1·29 grammes.

160. Find to the nearest penny the interest on £568. 12*s.* 6*d.* from March 12th to July 5th 1891 at 4 per cent.

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161. Determine the number which when divided by 231 by the method of 'short' division, gives a quotient 583 and 2, 6 and 10 as successive remainders.

162. Find, by the shortest method you can, the value of 8573 articles at £3. 18*s.* $11\frac{3}{4}$*d.* each.

163. Find the least length which is a multiple of 5 yds. 1 ft. 3 in. and also of 7 yds. 2 ft. 9 inches.

164. Simplify $(4\frac{1}{3} - 2\frac{5}{8} \text{ of } \frac{3}{7} + 2\frac{1}{5}) \div \{(4\frac{1}{3} - 2\frac{5}{8}) \text{ of } \frac{3}{7} + 2\frac{1}{5}\}$.

165. Shew that $\frac{111139}{2111109} - \frac{1}{19} = \cdot\dot{0}00009\dot{0}$.

166. If 6 bricklayers can build a wall 150 feet long and 6 feet high in 12 days, how many men would be required to build a wall of the same thickness 80 ft. long and 10 ft. high in 8 days?

167. A room is 20 ft. 7 in. long, 15 ft. 5 in. wide and 11 feet high. Find the number of pieces of paper, each 12 yards long and 21 inches wide, which would have to be bought to paper the walls, supposing that windows &c. which are not papered make up one-sixth of the whole surface of the walls.

168. A person's net income, after deducting income tax at 4*d.* in the pound, was £1062; what would his net income have been if the income tax had been 6*d.* in the pound?

169. A grocer buys sugar at £1. 11*s.* 6*d.* per cwt., and sells it at 4*d.* per lb. Find his gain per cent. allowing 4 lbs. in each cwt. for waste.

170. In what time will £450 amount to £463. 10*s.*, it being known that £1250 will amount to £1278. 2*s.* 6*d.* in 6 months at the same rate of interest?

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171. If rabbits are worth 9*s.* per dozen, and a man pays 10*s.* for a gun license, and 10*s.* per hundred for cartridges, how many shots must he fire in order that the value of the rabbits killed may be equal to the cost of the license and the cartridges used, assuming that he takes 5 shots to kill 2 rabbits?

172. Sound travels at the rate of 1140 ft. a second; how far off is a thunder-cloud when the sound follows the flash after $5\frac{1}{2}$ seconds?

173. A father, who had three children, left his second son £500 more than he left the third son, and his eldest son twice as much as the third. They had £8500 between them: how much had each?

174. A bill of £23. 11*s.* 8*d.* was paid with an equal number of sovereigns, half-crowns, shillings and pence: how many were there of each?

175. Simplify $\dfrac{2\frac{3}{7} \text{ of } \frac{5}{9} - 8\frac{1}{3} \text{ of } \frac{26}{171}}{8\frac{1}{3} \text{ of } (\frac{6}{19} - \frac{2}{7}) \text{ of } \frac{59}{180}}$.

176. Find the value of $\cdot\dot{5}\dot{4}$ of 8*s.* 3*d.* + $\cdot 0\dot{2}\dot{7}$ of £3. 14*s.* + ·3125 of £2. 2*s.*

177. How far should 12 cwt. 2 qrs. 4 lbs. be carried for £4. 12*s.* 1*d.* if 18 cwt. 3 qrs. 6 lbs. are carried 70 miles for £2. 9*s.* 7*d.*?

178. What will it cost to carpet a room 22 ft. 6 in. long and 15 ft. 9 in. wide at 4*s.* 8*d.* per square yard?

179. In a certain examination every candidate took either Latin or Mathematics, also 79·4 per cent. of the candidates took Latin and 89·6 per cent. took Mathematics. If there were 1500 candidates altogether, how many took both Latin and Mathematics?

180. For what sum must goods worth £637 be insured at 2 per cent. premium so that in case of loss the owner may recover the value both of the goods and the premium?

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181. Amongst how many persons must £59. 10*s.* 3*d.* be equally divided that each may get £2. 4*s.* 1*d.*?

182. On what day of the week was January 1st, 1799, knowing that July 6th, 1891, was on Monday?

183. Fifteen apples can be bought for 8*d.*, 4 pears for 3*d.*, 20 oranges for 9*d.*, and 30 plums for 7*d.* If I buy the same number of each of the four fruits, and spend an exact number of pence on each kind, what is the least sum I can spend?

184. Simplify $1\frac{2}{3}$ of $\dfrac{6\frac{3}{4}-2\frac{7}{9}}{3\frac{1}{2}-1\frac{1}{18}} \div \left\{2\frac{5}{6}-\frac{3}{5} \text{ of } \frac{1}{4\frac{4}{5}}\right\}$.

185. *A* and *B* respectively received $\frac{3}{19}$ and $\frac{4}{17}$ of a certain sum of money, and *C* the remainder. *A* received £107. 14*s.* 9*d.*; how much did *C* get?

186. Reduce £·2$\dot{9}$ + ·045 of £2. 10*s.* − ·$\dot{1}$ of 13*s.* 6*d.* to the fraction of $4\frac{1}{2}$ guineas.

187. A man pays £1. 5*s.* 6*d.* per acre for his farm, and his total rent is £319. 7*s.* 9*d.* What is the acreage of the farm?

188. If a number of men in a certain time earn £86. 5*s.*, how much will four times that number of men with double wages earn in one-third of the time?

189. What is the cost of a plot of building-land 242 feet long and 21 feet wide at £2000 an acre?

190. At the beginning of a year the population of a town was 16400. The deaths during the year were 3 per cent. of the population at the beginning of the year and 80 per cent. of the births. What was the population at the end of the year, neglecting changes caused by travelling?

191. Express 1887 by means of Roman numerals.

192. If 5 horses are worth as much as 14 cows, and 4 cows are worth as much as 25 sheep, and 5 cows cost £62. 10*s.* Find how much 6 horses and 420 sheep are worth.

193. Express in tons and fractions of a ton the weight of lead required to cover a flat roof, 147 square yards in extent, with sheet lead one-eighth of an inch thick, supposing that a cubic foot of lead weighs 820 lbs.

194. Simplify $\frac{11}{221}\left\{\left(\frac{4\frac{1}{11}}{20}+\frac{70}{8\frac{3}{4}}\right)-1\frac{11}{25}\left(\frac{3\frac{2}{3}+6\frac{1}{10}}{4\frac{1}{9}-3\frac{2}{5}}\right)\div 4\frac{79}{200}\right\}$.

195. If the first of two numbers be divided by the second the quotient is $4\cdot\dot{5}71428\dot{5}$. Express as a vulgar fraction in its lowest terms the quotient obtained when the second number is divided by the first.

196. Simplify $\dfrac{5\cdot58\dot{3}\times 3\cdot2\dot{3}\dot{7}}{3\cdot7\dot{2}\times\cdot809\dot{3}\dot{4}}$.

197. Find the value of a silver cup weighing 2 lbs. 7 oz. 7 dwts. 12 grs. at 7*s.* 4*d.* an ounce.

198. If an acre of land cost £51. 6*s.* 8*d.*, how much could be bought for £218. 3*s.* 4*d.* at the same rate?

199. Air contains 23·01 per cent. of its weight of oxygen. If a cubic foot of air weigh 1·23 ounces Avoir., what will be the weight of oxygen in a room 48 feet by 27 feet by 12 feet?

200. Find the cost of painting the sides and bottom of a cistern 3 yards long, 5 feet wide and $3\frac{1}{2}$ ft. deep at 3*s.* 9*d.* per square yard.

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201. What number is the same multiple of 354 that 86445 is of 765?

202. Subtract $\frac{3}{5}$ of $\frac{5}{6}$ from $1\frac{1}{2}$ of $\frac{4}{9}$; and divide the result by $(\frac{2}{3}-\frac{4}{7})\times(\frac{4}{5}-\frac{5}{8})$.

203. Subtract $3\cdot95\dot{3}$ from the sum of $7\cdot0\dot{1}\dot{5}$ and $5\cdot13\dot{2}$.

204. Find the cost of 25 cwt. 3 qrs. 25 lbs. 12 oz. at £2. 6*s.* 8*d.* per cwt.

205. A lidless cistern 10 ft. 6 in. long, 7 ft. 4 in. broad, and 5 ft. 4 in. high is to be painted outside. Find the cost at $2\frac{1}{4}d.$ per square foot.

206. Find, to the nearest farthing, the price per lb., in English money, of tea which costs 3·50 francs per kilogramme. [Take £1 = 25·17 francs, and 1 kilo. = 2·204 lbs.]

207. At what rate per cent. will the simple interest payable in 5 years on £780. 18*s.* 9*d.* amount to £156. 3*s.* 9*d.*?

208. Find the sum of money which in 3 years at 5 per cent. compound interest will amount to £4630. 10*s.*

209. Find the theoretical discount on a bill for £2670. 14*s.* 1*d.* drawn on June 24 at 9 months and discounted at 5 per cent. on August 16.

210. A man bought Russian bonds of £100 at 89 and sold them when they had risen to 95. He paid a brokerage of $\frac{1}{8}$ per cent. on each transaction, and made a profit of £86. 5*s.* How many bonds did he buy?

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211. The nearest of the fixed stars is roughly twenty billions of miles distant. Shew that it would take light about $3\frac{1}{3}$ years to traverse this distance at the rate of 190000 miles a second.

212. Simplify $17\frac{1}{2} \times \{6 - 3 \div (\frac{1}{2} + \frac{1}{3})\} - 17\frac{1}{2} \div \{6 - 3 \times (\frac{1}{2} + \frac{1}{3})\}$.

213. Express 1 d. 4 hrs. 31 min. $52\frac{1}{2}$ secs. as a decimal of 3 d. 4 hrs. 5 min.

214. Find the value of 11 oz. 13 dwts. 8 grs. of gold at £3. 17*s.* $10\frac{1}{2}$*d.* per oz.

215. What will it cost to carpet a room 18 ft. long and 15 ft. wide, the carpet being 27 in. wide and costing 3*s.* 9*d.* a yard?

216. A bar of gold is worth £31. 2*s.*; find the value of an equal bulk of silver, having given that the weights of equal volumes of gold and silver are as 19 to 10, and that gold and silver are worth £3. 17*s.* 9*d.* and 57*d.* per ounce respectively.

217. A ship is worth £9475. For what sum must it be insured at £5. 5*s.* per £100 in order that the owner in case of loss may receive both the value of the ship and the amount of premium paid?

218. At what rate per cent. simple interest will £1354. 10*s.* amount to £1444. 16*s.* in two and a half years?

219. Find, to the nearest shilling, the present value of £1000 due 4 years hence, compound interest being reckoned at 4 per cent.

220. What is the price of a 4 per cent. stock, if a man who invests £4301 gets an income of £136 a year on his investment?

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221. A train is travelling at the rate of 35 miles an hour: how many feet does it go in a second?

222. Find $\frac{3}{4}$ of £2. 13*s.* $9\frac{1}{2}d. + \frac{3}{4}$ of £10. 2*s.* $10\frac{1}{2}d. - \frac{8}{9}$ of £9. 3*s.* 9*d.*; and express the result as a fraction of a guinea.

223. Simplify $2{\cdot}42 \div {\cdot}0025 \times {\cdot}02 \div {\cdot}055$.

224. Find the rent of 375·4875 acres at £2. 13*s.* 4*d.* an acre.

225. How many loads (cubic yards) of gravel will be required to cover a court yard 60 feet long and 45 ft. wide to a depth of 2 inches, and how much will the gravel cost at 3*s.* 6*d.* a load?

226. If it cost as much to feed 3 men as to feed 4 boys, and if the cost of feeding 3 boys is found to be 19*s.* $1\frac{1}{2}d.$ per week, how much per week will it cost to feed 51 men?

227. 6 horses and 8 cows cost £375, and they were sold for £446. 5*s.* altogether, a profit of 25 per cent. being made on the horses and a profit of 10 per cent. on the cows. What was the average cost of each horse and of each cow?

228. Find, to the nearest penny, the compound interest on £5612. 13*s.* 4*d.* in 2 years at $3\frac{1}{2}$ per cent. per annum, the interest being payable half yearly.

229. Find the theoretical discount on £1368. 8*s.* due 8 months hence at $5\frac{1}{2}$ per cent.

230. A man has an income of £525. 5*s.* from $2\frac{3}{4}$ per cent. consols. He sells out at $96\frac{1}{2}$, and buys 4 per cent. Russian bonds of £100 at $95\frac{1}{2}$. What will be the change in his income?

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231. Find the G.C.M. of 70851 and 194997, and thence find all the common measures of the two numbers.

232. Simplify $\dfrac{15\frac{1}{4} - 4\frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 23\frac{1}{3} + 2\frac{19}{36}}$ and $\cfrac{5}{11 + \cfrac{1\frac{1}{2}}{3 + \cfrac{2}{5 - 3\frac{1}{4}}}}$.

233. Find the value of £·16875 + ·375 of 3*s.* 4*d.* $- \frac{3}{16}$ of 5*s.*

234. Find the value of 404 acres 1 rood 24 poles 20 sq. yds. at £226. 17*s.* 6*d.* an acre.

235. A gallon of water weighs 10 lbs. and contains 277·274 cubic inches. How many gallons will fall on a square mile in a rainfall of $\frac{1}{10}$ of an inch, and how many tons will the water weigh?

236. What is the price per lb., to the nearest farthing in English money, of Normandy butter which costs 3 francs per kilogramme. [Take £1 = 25·17 francs and 1 kilo. = 2·204 lbs.]

237. A spirit merchant buys whiskey at 12*s.* 6*d.* a gallon, and after adding water retails it at 15*s.* a gallon, making thereby a profit of 35 per cent. on his outlay. How much water was added to each gallon of whiskey?

238. A man embarks his whole property in four successive ventures. In the first he gained 60 per cent., and in each of the others he lost 20 per cent. What was his total loss per cent.?

239. The difference between the commercial discount and the theoretically true discount at $4\frac{1}{2}$ per cent. on a bill due in 8 months is 9*s.*: what is the amount of the bill?

240. A man invested £3840 in the $2\frac{3}{4}$ per cent. consols at 96 (including brokerage); how much stock at the same price could be bought with the first quarterly instalment of interest?

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241. Find the least number by which 3088 must be multiplied that the product may be a perfect square.

242. A man spent one-third of his income on lodgings, one-fourth the remainder on food, one-fifth what was left on clothes, one-sixth of the remainder on books, and then had £120 left. What was his income?

243. Add $42{\cdot}3\dot{6}$, $9{\cdot}4\dot{6}\dot{7}$, $\cdot0\dot{5}\dot{3}$ and $3{\cdot}2\dot{9}1\dot{6}$.

244. Find the quantity of water which will flow through a pipe in 7 days 11 hrs. 33 min. 45 secs. at the rate of 50 gallons an hour.

245. The four walls of a room 23 ft. long, 19 ft. wide and 13 ft. 6 in. high were painted, and the total cost was 9 guineas. How much was this a square yard?

246. The total yield of the wheat crop in the United Kingdom in 1888 was 74493133 bushels, with an average yield of 27·97 bushels per acre. On what area (to the nearest acre) was wheat grown, and what was the value (to the nearest pound) of the wheat at 31*s.* 10*d.* per quarter?

247. A dealer bought 80 sheep, and then sold 35 at a profit of 25 per cent., 25 at a profit of 20 per cent. and the remainder at a profit of 30 per cent., thereby making a total profit of £49. 7*s.* 6*d.* What did he pay for each sheep?

248. In what time will £1455 amount to £1564. 2*s.* 6*d.* at simple interest; having given that, at the same rate per cent. of simple interest, £430 amount to £494. 10*s.* in 3 years?

249. A bill for £1500 drawn on May 1 at 3 months was discounted at 4 per cent. on May 13. How much discount did the banker charge?

250. A man invested in $2\frac{3}{4}$ per cent. consols at 96 and sold out when they had risen to 98, paying $\frac{1}{8}$ per cent. brokerage on each transaction, and his total profit was £98. How much money did he invest?

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251. What is the least sum which must be added to £10000 that the result may be a multiple of £27. 13*s.* $6\frac{1}{4}d.$

252. Simplify $(3\frac{3}{4}-1\frac{2}{3})$ of $2\frac{5}{9}\div 1\frac{1}{4}$ of $(1\frac{1}{2}+2\frac{1}{3})$.

253. Find $\cdot\dot{1}\dot{1}42857\dot{}$ of £1. 15*s.*, and $2\cdot\dot{2}30769\dot{}$ of £2. 12*s.*

254. Find the value of 47 A. 2 R. 36 P. $2\frac{3}{4}$ sq. yds. at £40. 6*s.* 8*d.* an acre.

255. In the centre of a room 23 ft. square there is a carpet 18 ft. square and the rest of the floor is covered with oil-cloth which also goes everywhere 6 inches under the carpet. The carpet cost 16*s.* 6*d.* and the oil-cloth 3*s.* 6*d.* a square yard. What was the whole cost?

256. If 5 men do as much as 8 boys in a day, how many days will it take 32 boys to finish a piece of work a quarter of which was done by 15 men in 16 days?

257. For what sum must a ship worth £5663. 15*s.* be insured at a premium of £1. 10*s.* for £100, in order that in case of loss the owner may recover the value of the ship and the money paid for its insurance?

258. In what time will £385. 6*s.* 8*d.* amount to £421. 9*s.* 2*d.* at $3\frac{3}{4}$ per cent. simple interest?

259. Find, to the nearest penny, the present value of £500 due at the end of 3 years, reckoning compound interest at 4 per cent.

260. If London and North Western Railway stock pays a dividend of 7 per cent. per annum, at what price must the stock be bought so that an investor may get 4 per cent. on his money?

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261. Divide 168 acres 3 roods 38 poles 13 sq. yards by 51.

262. A person who owned $\frac{1}{3}$ of an estate sold $\frac{2\frac{7}{11}}{8\frac{7}{10}}$ of $\frac{1}{2}\frac{1}{5}$ of his share for £96; what would be the value of $\frac{2\frac{1}{2}}{3\frac{1}{3}}$ of $\frac{5}{1\frac{1}{2}}$ of the whole estate at the same rate?

263. Express 4 min. 12 secs. as a decimal of a week.

264. Find the amount of the wages of a man for 7 weeks 2 days 3 hours at 30 shillings a week, reckoning 6 days to the week and 9 hours to a day.

265. A rectangular field whose area is 1 acre 1 rood 25 poles is 137 yds. 1 ft. 6 in. long. What is its breadth?

266. Assuming that a Russian verst is ·6629 of a mile, and that a kilometre is ·6214 of a mile; find, to the nearest thousandth of the whole, the number of versts in 1000 kilometres.

267. A man sold 25 articles for the same price as he paid for 35. What was his profit per cent.?

268. The interest on a certain sum for one year is £5. 8*s.* 4*d.*, and the compound interest at the same rate per cent. on the same sum for 2 years is £11. 1*s.* Find the rate per cent. and the sum.

269. Find the theoretically true discount on £1250. 0*s.* $2\frac{1}{2}d.$ due 7 months hence at $3\frac{1}{2}$ per cent.

270. A man sold $2\frac{3}{4}$ per cent. consols at $97\frac{1}{2}$, and invested the proceeds in a stock which paid 5 per cent. interest and which cost 156, thereby increasing his income by £30 a year. What was his original income?

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271. How many acres, roods &c. are there in a hundred million square inches?

272. What fraction of a ton added to $\frac{11}{28}$ of 2 cwts. will make the result equal to 1 cwt. 2 qrs. 11 lbs.?

273. What decimal of a guinea is equal to $\cdot\dot{3}\dot{6}$ of 8*s.* $0\frac{1}{4}d.$?

274. If 4 cwt. 1 qr. 26 lbs. of tobacco can be bought for £56. 9*s.* 6*d.*, what quantity can be bought for £4. 19*s.*?

275. A cistern 9 ft. long, 8 ft. broad and 6 ft. deep is supplied with water by a pump which will send in 27 gallons a minute. Having given that a gallon of water contains 277·274 cubic inches, find how long it will take to fill the cistern.

276. Find, to the nearest farthing, the value of

$$£18{\cdot}63125 \times 14{\cdot}917 \div 18{\cdot}6142.$$

277. A draper sells three-fourths of a length of carpet at a profit of 20 per cent., and the rest at a loss of 5 per cent. What was his average profit per cent.?

278. Find, to the nearest penny, the interest on £2396. 15*s.* 4*d.* at $3\frac{3}{4}$ per cent. for 50 days.

279. The difference between the interest and the true discount on a certain sum for a year at 5 per cent. is £5. What is the sum?

280. A man has an income of £126. 13*s.* 4*d.* per annum from $2\frac{1}{2}$ per cent. consols. He sells out at $94\frac{1}{2}$, and buys 4 per cent. Railway debentures at 114 with the proceeds. Find the change in his income?

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281. The number of ounces Avoir. in a certain weight exceeds the number of ounces Troy by 17: what is the weight Avoir.?

282. Subtract $\frac{4\frac{7}{8}}{3\frac{1}{4}}$ of $1\frac{5}{12}$ from $2\frac{5}{11}$ of $2\frac{4}{9}$ of $1\frac{3}{5}$.

283. Simplify $4{\cdot}3\dot{5} \div {\cdot}10\dot{8}$ and $\dfrac{7{\cdot}375 - 2{\cdot}6\dot{8}\dot{1}}{2{\cdot}\dot{8} - 2{\cdot}4\dot{3}}$.

284. A room is 20 ft. long, 17 ft. wide and 12 ft. high, and there are in it two doors each 7 ft. by $4\frac{1}{2}$ ft., two windows each 6 ft. by $4\frac{1}{2}$ ft., and a fire-place $5\frac{1}{2}$ ft. by 6 ft. How many pieces (each 12 yards long) of paper 21 inches wide must be bought to paper the room, supposing that one yard out of every 14 is wasted?

285. Having given that 1 metre = 39·3708 inches, express an inch in centimetres and a square inch in square centimetres.

286. A man bought 12 sheep for £30 and 10 oxen for £105. He sold the sheep at a gain of 10 per cent., and the oxen at a gain of 8 per cent. What was his total profit?

287. If 12 men could dig 12 roods of ground in 12 days working 12 hours a day, how many roods of ground could 8 men dig in 8 days working 9 hours a day at the same rate?

288. If in 5 years at simple interest £6520 amount to £7824, in what time will £450 amount to £531 at the same rate per cent. of simple interest?

289. A banker discounted on May 19 at 3 per cent. a bill for £500 drawn on February 1 at 6 months. What was given for the bill?

290. A man sold £5462 Railway stock, part at $127\frac{1}{2}$ and the rest at 129, and the total sum realised was £6969. 6*s.* How much stock was sold at 129?

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291. If a number when divided by 391 leave a remainder 300, what will be the remainder when the number is divided by 17?

292. In one shop a certain fractional part of a sum of money was spent, $\frac{3}{4}$ of the remainder was spent in a second shop, and then $\frac{5}{72}$ of the original sum was left and this amounted to 4*s.* 2*d.* How much was spent in the first shop?

293. Which is the greater, $\cdot\dot{3}\dot{4}$ of a ton or $6\cdot\dot{9}\dot{6}$ of a cwt.? Express their difference as the decimal of a pound.

294. Find the annual cost of repairing a road 47 miles 3 fur. 5 yards long at £35. 4*s.* per mile.

295. How much will it cost to carpet a room 18 ft. long and 16 ft. 6 in. wide, a border 3 ft. wide all round being left uncovered by the carpet, the carpet being 27 inches wide and costing 2*s.* 3*d.* a yard?

296. A man leaves by will £3600 to his wife, and the remainder of his property to be equally divided between his four children; and it was found that the share of each child was one-seventh of the whole property. How much did the man leave?

297. A grocer bought tea at 1*s.* 4*d.* per lb. and sold it so as to gain 25 per cent. The duty on tea was reduced, and he then bought similar tea at 2*d.* per lb. less, and sold it at 2*d.* per lb. cheaper. What was his gain per cent.?

298. Find, to the nearest penny, the compound interest on £1723. 6*s.* 8*d.* for $1\frac{1}{4}$ years at 4 per cent. per annum payable quarterly.

299. How much would a banker pay for a bill for £847. 15*s.* 9*d.*, drawn on February 5 at 9 months and discounted at 3 per cent. on April 30?

300. A farm containing 517 acres 2 roods was let at a yearly rent of 15 shillings an acre. The owner sold the farm for 17 times the annual rent, and invested the proceeds in $2\frac{3}{4}$ per cent. consols at $97\frac{3}{4}$ (including brokerage). What annual income did he lose?

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301. If a wood-pigeon eats two pints of wheat in 3 days, find the value of the wheat eaten by 300 wood-pigeons from July 29th to September 30th (both days inclusive), when wheat is 32*s.* per quarter?

302. Simplify $\dfrac{423 \div \frac{3}{5} + 1\frac{4}{5} \div 3 + 4\frac{1}{2} \div 1\frac{2}{3}}{\frac{3}{7} \times 41 + 32 \times \frac{2}{3} - 1 \div \frac{7}{10}}$.

303. Find, to the nearest penny, the rent of
89 a. 3 r. 14 p. $2\frac{3}{4}$ sq. yds. at £2. 15*s.* 6*d.* an acre.

304. If 5 men working 6 hours a day do a piece of work in 112 days; how long would it take 8 men working 7 hours a day?

305. The expense of painting the four walls of a room which is 14 feet square at 3*s.* 9*d.* per square yard was £11. 13*s.* 4*d.* What is the height of the room?

306. A man buys eggs at 1*s.* 3*d.* per dozen and sells them at 11*s.* 8*d.* per hundred. What is his gain per cent.?

307. A man lent a certain sum of money at 5 per cent., and the yearly interest, after deducting income tax at 4*d.* in the pound, was £36. 17*s.* 6*d.* How much did he lend?

308. A man invests £7480 partly in $2\frac{3}{4}$ per cent. Consols at $96\frac{1}{4}$ and the remainder in 4 per cent. Railway debentures at 132. Find how much he must invest in each stock so that his income may be the same from the two sources.

309. The product of two numbers is 183·63177 and their quotient is 3·857. What are the numbers?

310. An express train runs a distance of $96\frac{1}{2}$ miles in 2 hours; part of the journey is run at the average speed of 40 miles per hour and the remainder at the average speed of 50 miles an hour. Find the lengths of these two portions.

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311. If a man buys 560 tons of coal at 15*s.* 9*d.* per ton and sells the whole at 10*d.* per cwt., how much does he gain by the transaction?

312. What fractional part is 3*s.* $7\frac{1}{2}d.$ of £4. 3*s.* $10\frac{1}{4}d.$?

313. Express $\dfrac{\cdot\dot{2}\dot{3}+\cdot\dot{1}\dot{4}}{\cdot\dot{2}\dot{3}+\cdot\dot{1}\dot{4}}$
as a vulgar fraction in its lowest terms.

314. Find the rental of 500 acres 3 roods 31 poles at £1. 6*s.* 8*d.* per acre.

315. In the centre of a square court is a square of grass covering $\frac{9}{16}$ of the whole area of the court, and the side of the square of grass is 60 feet. Find the cost of gravelling the remainder of the court to a depth of 3 inches, the gravel and labour costing 4*s.* 6*d.* a cubic yard.

316. If the theoretically true discount on £457. 7*s.* 6*d.* due 27 days hence be £1. 2*s.* 6*d.*, what is the rate per cent.?

317. A man invests £463. 2*s.* 6*d.* in 4 per cent. Railway Stock, and obtains an income of £14. 5*s.* from his investment. At what price did he buy the stock?

318. A grocer pays 1*s.*, 1*s.* 3*d.* and 1*s.* 8*d.* per lb. respectively for three different kinds of tea. If he mixes together weights of these teas proportional to the numbers 6, 4 and 3 respectively, and sells the mixture at 1*s.* 6*d.* per pound, what profit does he make per cent.?

319. What is the square root of a number which exceeds the square root of 130400 by 13564304?

320. *A* can do a piece of work in 12 days, and *B* can do it in 15 days. They work together for 5 days, after which *C* joins them, and the work is finished by the three in one day more. How long will *B* and *C* take to do the whole?

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321. What is the greatest number which will divide 9640 and 10250, and leave the remainders 4 and 30 respectively?

322. Simplify

$$\frac{3}{2}-\left[\frac{1}{3}+\left\{1-\left(\frac{1}{3}-\frac{1}{4}\right)\right\}\right]\div\frac{1}{2}\text{ of }\frac{\frac{3}{4}-\frac{1}{2}}{\frac{2}{3}-\frac{1}{4}}+\frac{19}{20}.$$

323. If ·00744 cwt. cost £·028644, what was the price per lb. in pence?

324. Six chickens are worth 17*s.* 6*d.*, 3 ducks are worth 4 chickens, 2 geese are worth 7 ducks and 5 turkeys are worth 9 geese; find the value of a turkey.

325. The population of a town increases every year at the rate of 2 per cent. of the population at the beginning of the year. What was the population in 1888 if the population in 1891 was 132651?

326. A dealer bought a certain number of articles at the rate of 40 for a pound, and twice the same number at the rate of 50 for a pound. He sold the whole at the rate of 36 for a pound. How much per cent. did he gain?

327. The compound interest on a sum of money for 3 years at 5 per cent. is £331. 0*s.* 3*d.*; what is the simple interest on the same sum for the same time and rate per cent.?

328. A square field contains 3 a. 1 r. 13 p. $5\frac{3}{4}$ yds.; find the length of each side.

329. A person has a certain capital half of which he invests in Midland 3 per cent. debentures at $100\frac{1}{4}$ and the other half in Great Western 4 per cent. debentures at 132. His total income from both sources is £265. 13*s.* 4*d.* How much did he invest?

330. Two partners A and B commence a business with £11996. 13*s.* 4*d.* and £10797 respectively, and after 8 months they are joined by C who puts £7198 into the business. At the end of a year the net profit amounts to £2898: find their respective shares.

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331. Sound travels at the rate of 1140 feet per second, and light may be considered to travel instantaneously; how far off is a thunder cloud when the sound follows the flash after 11 seconds?

332. Simplify $\dfrac{4\frac{3}{7}-\frac{5}{9} \text{ of } 4\frac{2}{13}+6\div 4\frac{2}{5}}{18\frac{20}{91}\div 8-\frac{2}{3} \text{ of } 1\frac{6}{7}}$.

333. Add together £3·4175 + £2·834 + £5·186.

334. If 10 men or 25 boys can do a piece of work in 14 days, in what time will 8 men and 15 boys do it?

335. In a school in which 120 scholars were examined, the average percentage of passes in reading, writing and arithmetic was 85, and the numbers who passed in reading and writing were 114 and 100 respectively. How many passed in arithmetic?

336. Find, to the nearest penny, the compound interest on £350.12*s.* 6*d.* in 5 years at 5 per cent.

337. What rate of interest per annum does a man get for his money when, in discounting a bill due in 10 months, he deducts as discount 4 per cent. of the total amount of the bill?

338. There are in a manufactory a certain number of workmen who receive 50*s.* a week, twice as many who receive 31*s.* 6*d.* each a week, and eleven times as many who receive 14*s.* each a week; also the total amount of the workmen's wages for one week is £93. 9*s.* How many workmen are there?

339. Find the square root of $154751\frac{77}{169}$, and the cube root of 1194389981.

340. A man who held £24628 in the $2\frac{1}{2}$ per cent. Consols, sold out at 93 and bought 4 per cent. Railway debentures at 131 with the proceeds. The debentures rose to 139 when he sold out, and re-invested in $2\frac{1}{2}$ per cent. Consols at 94. Find the change in his quarterly income.

341. The whole taxation for a certain district is at the rate of 2*s.* $7\frac{1}{2}$*d.* in the pound; how much is paid for a property whose annual rateable value is £3567. 6*s.* 8*d.*?

342. Simplify

$$\frac{65}{569}\left\{\left(\frac{3\frac{11}{13}}{25}+\frac{40}{4\frac{1}{6}}\right)-11\frac{1}{7}\left(\frac{4\frac{1}{9}+6\frac{3}{8}}{7\frac{5}{12}-3\frac{6}{7}}\right)\div 32\frac{19}{23}\right\}.$$

343. Find the difference between 235 months of 29·5306 days each, and 19 years of 365·24224 days each; and express the result in hours, minutes and seconds.

344. Four fowls are worth 3 ducks, 7 ducks are worth 3 geese, 9 geese are worth 7 turkeys, and a turkey is worth 18 shillings. How much is a fowl worth?

345. The length of a floor is to its breadth as 9 is to 7, and the cost of covering it with carpet 3 ft. 6 in. wide at 7*s.* 9*d.* per yard is £12. 13*s.* 2*d.* What are the dimensions of the floor?

346. Find, to two places of decimals, the cube root of 1037.

Give a rule for determining by inspection the cube root of every perfect cube less than a million.

347. A man would get £8. 1*s.* 4*d.* a year more by investing a certain sum in 4 per cent. Railway stock at 129 than he would get by investing the same sum in $2\frac{3}{4}$ per cent. Consols at $96\frac{1}{4}$. What is the sum?

348. *A* and *B* can reap a field in 10 days, *A* and *C* in 12 days, *B* and *C* in 15 days. They all work together for 4 days when *A* leaves: *B* and *C* work together for 6 days more, when *B* leaves. In how many days can *C* finish the work?

349. At what time between 5 and 6 o'clock do the hands of a clock coincide in direction.

350. Given that a ten franc piece is worth 7*s.* $11\frac{1}{4}$*d.*, a ten dollar piece is worth £2. 1*s.* $1\frac{1}{4}$*d.*, and a ten mark piece is worth 9*s.* $9\frac{1}{2}$*d.*, express £10000 (1) in francs, (2) in dollars, (3) in marks.

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351. The 19th of June, 1889, was on Wednesday, on what day of the week will June 19th, 1909, fall?

352. Multiply together

$$\frac{6\frac{9}{11}}{1\frac{1}{14}},\quad \frac{7\frac{19}{19}}{\frac{25}{57}},\quad \frac{3\frac{8}{13}}{\frac{17}{23}} \text{ and } \frac{1\frac{5}{46}}{9\frac{2}{5}}.$$

353. A piece of work can be done in 48 days by 15 men, but after 9 days two of the men go away. In how many more days will the men who remain finish the work?

354. Multiply £3462. 3*s.* 6*d.* by 294·3487254, so as to find the result correct to the nearest penny.

355. By selling goods for £9. 10*s.* a man lost 5 per cent.; what would he have gained if he had sold for £11. 17*s.* 6*d.*?

356. Divide £857. 5*s*. $8\frac{1}{4}d$. among 4 persons, *A*, *B*, *C* and *D*, so that *A* may have twice as much as *B*, *B* three times as much as *C*, and *C* four times as much as *D*.

357. Find the square root of $514\frac{185}{256}$, and the cube root of 12812904.

358. At what rate per cent. compound interest will £1166. 13*s*. 4*d*. amount to £1274. 0*s*. 7*d*. in 2 years?

359. A man invested £1864. 10*s*. in $2\frac{3}{4}$ per cent. Consols at 96. How much stock did he buy, and what is his quarterly dividend? [Brokerage $\frac{1}{8}$ per cent.]

360. In a race of 100 yards *A* can beat *B* by 5 yards, *B* can beat *C* by 5 yards, and *C* can beat *D* by 5 yards. How should they be handicapped for a 100 yards race, putting *A* at scratch and giving him the advantage of any odd fraction of a foot?

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361. A clock is set right at noon, but when it strikes 12 that night it is 80 seconds fast. Find how many minutes it will gain in a week.

362. Simplify

$$1\tfrac{1}{2} \text{ of } 2\tfrac{4}{5} + 6\tfrac{7}{8} \div 2\tfrac{3}{4} - \left(5\tfrac{1}{2} + \frac{\cdot 12 + \cdot 5\dot{3}}{2{\cdot}2 - \cdot 6\dot{4}}\right).$$

363. What is the least number which when divided by 15 leaves a remainder 9, when divided by 35 leaves a remainder 29, and when divided by 42 leaves a remainder 36?

364. If 24 men in 6 days of 8 hours can dig a trench 140 yards long, 4 yards wide, and 2 yards deep, in how many days of 9 hours will 42 men dig a trench 3 yards wide, 4 yards deep, and 490 yards long?

365. The discount and interest on a certain sum for the same time are £22 and £24 respectively; find the sum.

366. A man invests £1800 partly in a 3 per cent. stock at 105 and the remainder in a 5 per cent. stock at $152\frac{1}{2}$, and he obtained just 3 per cent. on his whole outlay. How much did he invest in each stock?

367. Find the square roots of $\cdot 0\dot{1}2345679$ and $19{\cdot}95\dot{1}$.

A cube contains $18{\cdot}96\dot{2}$ cubic yards: what is the length of an edge?

368. A bag contains sixpences, shillings and half-crowns, and the three sums of money expressed by the three different sets of coins are the same; if there are 102 coins in the bag, how many are there of each kind?

369. A grocer has two sorts of tea, which cost him 2*s.* 8*d.* and 2*s.* 1*d.* per pound respectively. In what ratio must he mix them so that he may gain 25 per cent. by selling the mixture at 2*s.* 11*d.* per pound?

370. Three persons A, B and C working together complete a piece of work which it would have taken them respectively 9, 10 and 12 days to complete if working separately. They receive in payment £2. 13*s.* which they are to divide in proportion to the quantity of work done by each. Find their shares.

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371. What is the smallest integral number of pounds which can be made up of sovereigns, half-sovereigns, crowns, half-crowns, florins, shillings, sixpences and threepences; an equal number of each of the coins being taken?

372. Reduce ·891 of £5. 4*s.* 2*d.* to the fraction of £7. 16*s.* 9*d.*

373. Divide $\cdot 05\dot{2}6\dot{7}$ by $\cdot 026\dot{5}\dot{7}$ and multiply $\cdot 3\dot{1}42857\dot{}$ by $1\cdot 6\dot{9}$.

374. A gallon of water at its maximum density weighs 10·016 lbs. Avoir., and a cubic centimetre is ·0002201 gallons. Express a gramme in grains.

375. A room is $18\frac{1}{4}$ feet long, $15\frac{1}{2}$ feet broad and 12 feet high. Find the cost of papering the walls with paper 21 inches wide at 2*s.* 4*d.* per piece of 12 yards, deducting 90 square feet for doors and windows.

376. A man bought a certain number of eggs at the rate of two a penny, three times the number at the rate of three a penny, six times the number at $5\frac{1}{2}d.$ per dozen, and ten times the number at the rate of 8*d.* per score, and sells them at the rate of 3*s.* 9*d.* per hundred, gaining by the transaction 15*s.* How many eggs did he purchase, and what did he gain per cent.?

377. A square field contains 22 acres 2 roods. How long will it take a boy to run round the boundary of the field at the rate of 12 miles an hour?

378. At what rate per cent. compound interest will £4166. 13*s.* 4*d.* amount to £4553. 0*s.* 7*d.* in 3 years?

379. Two pipes, P and Q, can fill a cistern in 20 minutes and 30 minutes respectively; in what time will the cistern be filled when both P and Q are running? Also, if when a third pipe R is also running, the cistern is filled in two-thirds of this time, in what time would R fill the cistern alone?

380. A man had 4 per cent. Railway Preference Stock which brought him in £664 a year. He sold out at $119\frac{1}{4}$, and invested in Ordinary Stock at 145, paying $\frac{1}{4}$ per cent. brokerage on each transaction. If the Ordinary Stock paid a dividend of 6 per cent., what was his gain in income?

381. A party of 31 people pay a bill of £97. 9*s*. 5½*d*. Fourteen pay £3. 18*s*. 7*d*. each. How much will each of the rest have to pay?

382. A person standing on a railway platform noticed that a train took 21 seconds to pass completely through the station, which was 88 yards long, and that it was 9 seconds in passing him. How long was the train, and at what rate per hour was it travelling?

383. *A* had at first £1. 8*s*., *B* paid him $\left(2+\frac{3\frac{1}{7}}{1\frac{2}{5}}\right)$ of £1. 11*s*. 6*d*., and found that he had remaining $\frac{1}{4\frac{1}{3}}$ of what *A* then had. How much had *B* at first?

384. Find by Practice the value of 147 a. 3 r. 24 p. 15 yds. at £196. 12*s*. 6*d*. an acre.

385. Find the present value of £17230. 5*s*. due three years hence at 2½ per cent. compound interest.

386. A room three times as long as it is broad is carpeted at 4*s*. 6*d*. per square yard, and the walls are coloured at 9*d*. per square yard, the respective costs being £8. 5*s*. 4½*d*. and 4 guineas. Find the dimensions of the room.

387. A grocer has two kinds of tea. By selling the first at 3*s*. 9*d*. per lb. he gains 25 per cent., and by selling the second at 3*s*. 6*d*. per lb. he gains 40 per cent. If he mix them in equal quantities and sell the mixture at 3*s*. 8*d*. per lb., what will be his gain per cent.?

388. The rental of a farm is £500 a year, but the owner has to return 20 per cent. to his tenants, and also to pay certain fixed outgoings amounting to £75 a year. For what sum must the farm be sold in order that the same net income may be obtained by investing the proceeds in 4 per cent. Railway debentures at 130?

389. Express $\frac{1}{7}$ by radix fractions in each of the scales 2, 3, 4, 5, 6, 7, 8, 9 and 10.

390. When the course of Exchange is

London on Paris 25 f. 27 c. at sight,

what should be paid for 3 months' bills on Paris, the rate of discount at Paris being 4 per cent. per annum?

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391. Find the value of 128 tons 12 cwt. 3 qrs. 24 lbs. at £8. 3*s*. 4*d*. per ton.

392. Express ·016 of 13*s*. 4*d*. as a decimal of 16*s*. 8*d*.

393. Two trains start at the same time from A and B and proceed towards each other at the rate of 35 and 45 miles per hour respectively. When they meet, one train has gone $17\frac{1}{2}$ miles further than the other. What is the distance from A to B?

394. A man's net income after deducting income-tax at 6*d.* in the pound, and also poor rate, is £654. The poor rate exceeded the income-tax by £30. Find the gross income.

395. A grocer has two kinds of tea one of which cost 1*s.* 8*d.*, and the other 1*s.* 10*d.* per lb. In what proportion must he mix them that, by selling the mixture at 2*s.* 1*d.* per lb., he may get a profit of 20 per cent.?

396. In the centre of a room 21 feet square there is a square of carpet, and the rest of the floor is covered with oil-cloth. The carpet and oil-cloth cost respectively 16*s.* 6*d.* and 8*s.* 6*d.* a square yard, and the whole cost was £35. 4*s.* 6*d.* Find the width of the oil-cloth border.

397. A sum of £1700 was borrowed, and is to be paid back in two years by two equal annual payments, allowing 4 per cent. interest. Find the amount of the annual payments.

398. In a certain game A can give B 1 point in 10, B can give C 1 point in 10, and C can give D 1 point in 15. How many can A give D in 3000?

399. The sum of £5998. 10*s.* was invested partly in $2\frac{3}{4}$ per cent. consols at 96, and partly in $2\frac{1}{2}$ per cent. consols at $94\frac{1}{2}$ (brokerage included in both prices), and the income from the two portions is the same. What is the total income?

400. Find the arbitrated rate of Exchange at London for bills on Paris payable at sight, having given

London on Hamburg at 3 months £1 = 20 m. 55 pf.

Hamburg on Paris at 3 months 20 marks = 25·05 fr.

Bills being discounted at 4 per cent. per annum.

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401. The length of a room is double the breadth, and three times the height is twice the breadth; also, it costs £11. 5*s.* to carpet the floor at 4*s.* 6*d.* per sq. yard. How high is the room?

402. By the conversion of Consols from 3 per cent. to $2\frac{3}{4}$ per cent., a person's income was diminished by £27 per annum. What cash would he have obtained by selling out at $97\frac{1}{8}$, brokerage being neglected?

403. A man bought a house and sold it so as to gain 5 per cent. Had he given 10 per cent. more for the house, and sold it for £27 more than he did sell it, he would have lost $2\frac{1}{2}$ per cent. Find what he gave for the house.

404. A carpet which covered the central portion of a room cost £10. 15*s.* 10*d.* The room was $25\frac{1}{2}$ feet long, and $19\frac{1}{2}$ feet wide, and between the edge of the carpet and the walls was a distance all round of $2\frac{1}{4}$ feet; what was the cost of the carpet per square yard?

405. Divide the square of 51 into three parts proportional to the quantities 1·6, ·16 and 1·13.

406. The breadth of a rectangular field is $\frac{3}{7}$ of its length, and its area is 405741 square yards. Find the sides of the field.

407. A watch which is set correctly at noon indicates 10 minutes past 9 the same evening when the true time is 10. What is the true time when the watch indicates 11 on the same evening?

408. A man buys a farm of 120 acres for £3225: he repairs the buildings and lets the farm at 28*s.* an acre, thereby getting a return of $3\frac{1}{2}$ per cent. on his money. How much did he spend on repairs?

409. If from any number, *e.g.* 8176249, any other number with the same digits in different order, *e.g.* 1294678, be subtracted, the remainder will be exactly divisible by 9. Explain this.

410. How can I divide 5 half-crowns, 6 florins and 29 sixpences between two persons so that each may receive half the money and half the actual coins?

411. What is the side of a square field which contains 3 acres 2 roods 16 poles?

412. If an income tax of 6*d.* in the pound be deducted before payment of the dividend on $2\frac{1}{2}$ per cent. Stock at 91, how much money must be invested in that Stock to yield a net income of £208 per annum?

413. The capital of a railway company is five millions. The average quarterly income is £222892. 13*s.* 4*d.*, and the average weekly expenditure is £12818. 13*s.* 4*d.* What dividend can be paid to the shareholders?

414. How many persons are there in a procession which, moving along uniformly at the rate of $3\frac{3}{4}$ miles per hour and containing 8 persons in each yard and a half of its length, takes 75 minutes to pass a place on the route?

415. If 172800 blocks of wood, 6 in. high, $2\frac{1}{2}$ in. wide, and 8 in. long, be packed close together in a rectangular pile, 40 ft. long and 10 yds. wide, how high will it be?

416. A man expects to let a house for £60 per annum, and calculates that, after deducting an annual sum for repairs, etc., he will get $7\frac{1}{2}$ per cent. on the purchase-money. He lets the house for £56 per annum, spends yearly £5 more than he intended on repairs, and gets 6 per cent. on the purchase-money. What did he give for the house?

417. A thin rectangular lamina of metal, 3 feet 2 inches long and 2 feet 9 inches wide, has cut from its four corners four squares whose sides are 3 inches long. The four projecting portions are turned up at right angles to the rest of the lamina, and thus form a lidless box. Find its cubical content.

418. What fractional part of any sum, due in $2\frac{1}{2}$ years, at 4 per cent. per annum, is the difference between the present values, according as Simple or Compound Interest is reckoned?

419. Which is the better investment, the $2\frac{3}{4}$ per cent. Consols at $96\frac{1}{2}$, or 4 per cent. Stock at 141?

Divide £7737. 10*s.* between *A* and *B* so that *A*, who invests his share in the former, may have the same income as *B*, who invests his share in the latter Stock (no brokerage to be reckoned).

420. If articles bought at any price per dozen are sold at twice that price per score, what is the gain per cent.?

421. If when *A* gives *B* 50 yards start in a mile race *A* is beaten by 20 yards, by how much will *A* beat *B* in a half-mile race, supposing they both always run at the same rate?

422. The minute-hand of a clock overtakes the hour-hand at intervals of 64 minutes of true time. How much a day does the clock gain?

423. The gross receipts for 1888 of a railway company were £3735935, the working expenses 55·8 per cent. on the gross receipts, and £600 less than the net receipts paid a dividend of $4\frac{3}{8}$ per cent. for the year. Find, to the nearest thousand pounds, the amount of Stock.

424. Spirits at 25*s.* 4*d.*, 22*s.* 10*d.*, and 21*s.* 8*d.* per gallon are mixed in the proportion of the numbers 1, 2, 3, respectively, and sold at 4*s.* 6*d.* per $1\frac{1}{4}$ pint bottle. Other expenses amounting to $2\frac{1}{2}d.$ per bottle, what is the gain per cent.?

425. On the floor of a room 20 feet long and 16 feet wide there is a border one yard wide which is stained, and the rest of the floor is carpeted. The carpet is 20 inches wide and costs 3*s.* 6*d.* a yard; and the staining costs 1*s.* a square yard. Find the whole cost of the carpet and staining.

426. What must be the gross income of an estate if, after paying an income tax of $2\frac{1}{2}$ per cent. of the gross income and rates to the amount of 3*s.* $6\frac{1}{2}$*d.* in the pound, the net income is £8809?

427. The outside dimensions of a metal cistern without a lid are 12 feet 6 inches long, 8 feet 6 inches wide, and 5 feet deep; the sides, ends, and bottom are 2 inches thick. Find (i) the area of an end outside, (ii) of the bottom inside, and (iii) the number of cubic inches of metal it contains.

428. A person borrowed a sum of money, for which 5 per cent. interest was charged. He paid off at the end of each year one-third of the amount then owing. Shew that, just after the third payment, he owed ·343 of the original sum. If he then owed £900. 7*s.* 6*d.*, what sum did he borrow?

429. A model of a machine is made of wood, and is found to weigh 24·2 pounds. The machine itself is to be made of iron, and every line in it is to be eight times as long as the corresponding line in the model. If a cubic foot of wood weigh 44 lbs. and a cubic foot of iron weigh 490 lbs., what will be the weight of the machine in tons, &c.?

430. *A* with £2500 capital joins in partnership with *B* who has £4000 capital: at the end of a year's trading they lost £520, and their capitals are diminished accordingly. *C* now joins them with £4500 capital. At the year's end they had gained £2358. How should this be divided between them?

431. Show that ·0067 of the amount of $2\frac{3}{4}$ per cent. Consols will give the quarterly dividend, with sixpence in the pound income tax deducted, with an error not greater than one penny for every £1000 Stock.

432. The inside dimensions of a packing case being 4 feet 7 inches long, 4 feet 1 inch wide, and 3 feet deep: how many boxes, 11 inches long, 7 inches wide, and 4 inches deep, can be put in the case, and how should they be packed?

433. A London merchant having to pay a Berlin merchant for goods received, finds the rates of exchange as follows:—

London on Berlin, at 3 months, 20·53 marks for £1;

Berlin on London, at 3 months, 20·27 marks for £1.

Which will be the better way—for him to remit direct bills to Berlin, or for his correspondent in Berlin to draw upon him in London? and how much per cent. better, interest being allowed at 4 per cent. per annum for the time the bills have to run?

434. A man buys £4575 Stock at 75, and pays for it with borrowed money for which he gives 8 per cent. At the end of a year he sells out, the Stock having paid a dividend of 3 per cent. for the year, and after repaying his loan with interest the man has £30. 10*s.* left. At what price was the Stock sold?

435. A man lends money at 20 per cent., but in consequence of bad debts he only makes a profit of 8 per cent. on his money. What is the proportion of his bad debts to the whole sum of money he lends?

436. The length of a room is to its breadth as 7 : 6, and the cost of covering the floor with carpet 27 inches wide at 3*s.* 9*d.* a yard is 10 guineas. Find the dimensions of the room.

437. Find the cost of putting a fence round a square field of 2 A. 2 R. 29 P. $18\frac{3}{4}$ yds., at 1*s.* 4*d.* per yard of fence.

438. The excess of the true present value of a sum due in one year, reckoning interest at 4 per cent., over the present value when interest is reckoned at 5 per cent. is 10 shillings. Find the sum.

439. A market woman sold 4 fowls and 6 ducks to one customer for 28 shillings, and 10 fowls and 1 duck to another customer for the same money. Find what she charged for each fowl and for each duck.

440. A man invests £13125 partly in $2\frac{3}{4}$ per cent. Consols at 96 and the rest in 4 per cent. Railway debentures at 126, and gets exactly 3 per cent. on his money. How much does he invest in each Stock?

441. Show that, if the rats in a parish increase in number by 50 per cent. per annum, they will, after an interval of 8 years, be more than 25 times as numerous as they were.

442. Having two liquids *A* and *B*, mixed in parts proportional to the numbers 2 and 3, a chemist took some of the mixture, added thereto a quantity of the liquid *B*, and thus obtained a gallon in which the liquids were mixed in parts proportional to the numbers 3 and 5. How much of the liquid *B* did he add?

443. A bicyclist travelling with uniform velocity along a straight level road on a foggy day, when objects were just visible a furlong off, saw a milestone in front of him for half a minute. Afterwards he overtook a carriage, the back of which he saw for a minute and a half; with what velocity (supposed uniform) was the carriage travelling?

444. With an income of £5 per week, a man intends to spend 7 guineas per week during part of the year and 4 guineas per week during the remainder of the year. What is the greatest exact number of weeks that he may spend 7 guineas per week and not exceed his income, and how much will remain at the end of the year?

445. A farmer possessed a certain number of ewes; forty per cent. of them had two lambs each, the rest one lamb each. Two per cent. of the ewes and 6 per cent. of the lambs died, and then he had 84 more lambs than ewes. How many ewes did he own at first?

446. Nine men engaged on a work would finish it in 180 days, but at the end of every 12 days 9 additional men are engaged: in how many days will the work be finished?

447. A tradesman's charges are 25 per cent. above cost price; if he allows his customers 12 per cent. discount for cash, what profit does he make?

448. A sum of £1000 is borrowed at 5 per cent., and is to be repaid in three years by three equal annual payments. Find, to the nearest penny, what the annual payment should be.

449. A bag contains sixpences, shillings and half-crowns, and the three sums of money in the different coins are the same. There are 612 coins in all. How many half-crowns are there?

450. In the Persian calendar there are 8 years of 366 days and 25 years of 365 days in every 33 years. Given that the true length of the year is 365·242216 days, find in how many years the Persian Calendar will be in error by one day.

451. A man bought 15 A. 1 R. 20 P. of land for £1860. 7*s.* 6*d.*, and sold a plot 150 yards square for 50 per cent. more than it cost him: for how much did he sell it?

452. A London merchant has to pay a debt of 10000 francs due in Paris when the course of exchange is quoted

London on Paris £1 for 25·35 francs,
Paris on London 25·40 francs for £1.

Should the London merchant buy bills on Paris or direct his creditor in Paris to draw on him in London; and how much difference would it make?

453. The present value, reckoning compound interest, of a certain debt due three years hence is £112. 10*s.* In two years time its value will be £120. 2*s.* 6*d.* What is the debt?

454. A company makes in the year a profit of £75118. 17*s.* 6*d.* on a capital of £872325; find, to the nearest quarter per cent., the highest dividend that can be paid, and how much will be carried forward.

455. *A* can do in 5 days as much work as *B* can do in 7 days, and *B* can do in 8 days as much work as *C* can do in 7 days; how long will *A*, *B* and *C* working together take to do a piece of work which would occupy *A* alone for 31 days?

456. If the hands of a clock coincide every sixty-six minutes, how much does the clock gain or lose in a day?

457. A certain quantity of work could be done in $4\frac{1}{2}$ days by 6 men or 9 women or 12 boys. How long would it take 4 men, 6 women and eleven boys working together to do it?

458. A number of men spent at an inn as many farthings each as there were men, and the whole sum spent was £1. 10*s.* 1*d.* How many men were there?

459. A cubical block of metal of 7·84 inches side weighs ·25 lbs. per cub. inch. A hole of square sectional area is to be cut completely through the metal, perpendicular to a face of the cube, in order that the weight of metal left may be 100 lbs. Find to three places of decimals the side of the square section.

460. A sum of £237. 14*s.* 3*d.* was made up of sovereigns, half-crowns, shillings and pence; and for every 7 pennies there were 3 shillings, 5 half-crowns, and 8 sovereigns. Find the number of coins of each kind.

461. An ordinary train going at the rate of 25 miles an hour starts on a journey an hour before an express train which goes at the rate of 40 miles an hour. The ordinary train arrives 15 minutes before the express. Find the distance.

462. *A* gave *B* half a minute start in a race over a measured mile, and had run 11 yards past the winning post when *B* reached it. But if, when *B* started to run the mile *A* had started simultaneously 176 yards behind him, they would have reached the winning post exactly in the same time. Find the time each takes to run a mile.

463. *A* and *B* ride a race of 31 miles on bicycles. The driving wheel of *A*'s machine makes 3,410 revolutions per hour and has a circumference of 168 inches; that of *B* makes 3,520 revolutions per hour, and has a circumference of 162 inches: which will win, and by how much?

464. A horse and two cows cost as much as 50 sheep, and a score of sheep are worth £50; also a horse costs as much as three cows. Find the cost of 4 horses.

465. Divide £523. 10*s.* among three persons, *A*, *B*, and *C*, so that *A* may get one-third of what *B* receives, and *C* may receive as much as *A* and *B* together.

466. The price per ton of coals in the winter half of the year is (on the average) thirty per cent. more than in the summer half of the year, and five tons are used by a person in the winter for every two tons in the summer. What would be the cost of coals for the summer half if the whole cost for the year were £16. 3*s.*?

467. If 20 men can finish in 12 days a piece of work which 40 boys can do in 15 days, how many boys will be required to help 18 men to finish the same piece of work in 10 days?

468. A bag contains 150 coins, which are sovereigns, half-crowns or shillings. The value of the half-crowns is twice that of the shillings and half that of the sovereigns. How many coins of each kind are there in the bag?

469. A clock which was right at 1 o'clock was found to be 2 minutes fast at 5 o'clock the same afternoon, What will be the true time when the clock strikes 12 that night?

470. A watch is set correctly at noon, but gains at a uniform rate. It is noticed that the hands are together between the figures V and VI when it is really five o'clock. How much does the watch gain in a day, and at what time will the hands be together again?

471. Two bicyclists ride round a circular path in the same direction; one of them travels at the rate of 20 miles an hour, and the other at the rate of 15 miles an hour, and the slower is passed by the quicker every 10 minutes. How often would they meet one another if they went round in opposite directions, and how far is it round the path?

472. A bookseller is allowed by a publisher 20 % discount on the nominal price, and 5 % on this last amount for cash; he also is charged for 13 copies as 12. The bookseller allows his cash customer 3*d.* in the shilling off the nominal price. What is the bookseller's profit on cash sales?

473. A dealer bought 56 sheep, and then sold 21 at a profit of 15 %, 16 at a profit of 10 %, and the remainder at a loss of 5 %, thereby making a total profit of £5. 14*s.* What did he pay for each sheep?

474. A man buys land at £30 an acre and sells a portion of it in allotments at £90 an acre. By so doing he clears £150 besides keeping 25 acres for himself. How many acres did he buy?

475. *A* buys an estate and some time afterwards sells it to *B* for £5400. *B* gets a net return of $2\frac{3}{4}$ per cent. on his money; also *A* had previously obtained a net return of 3 per cent., although his net income from the estate was £36 less than *B*'s. What did *A* originally pay for the estate?

476. If 5 men do $\frac{3}{5}$ of a piece of work in $2\frac{1}{4}$ hours, how long will it take 4 boys to finish it, knowing that 6 men and 9 boys did a similar piece of work in 2 hours?

477. The annual average depth of rainfall for the three years 1879, 1880, 1881 at a certain place was 24·98 inches; the succeeding three years it was 29·62. The year 1883 was the rainiest when there fell 4·8 inches more than in 1882, 6·36 inches more than in 1884, and 7·47 more than in 1880. The year 1881 was short of the preceding year by only ·17 inches. Find the depth of rain that fell in each of these six years.

478. A man starts from Cambridge at 3 P.M. to meet a carriage which left London at 1.30 P.M. The man walks $4\frac{1}{2}$ miles an hour and the carriage travels at the rate of $8\frac{1}{2}$ miles an hour. When and where will the man meet the carriage, the distance from Cambridge to London being $51\frac{3}{4}$ miles.

479. If 3 men can do as much work an hour as 4 women, and a man twice as much as a boy, and if the aggregate wages of 6 men, 15 women and 10 boys be £89, how ought the money to be divided?

480. If 7 men and 17 boys can do a piece of work in 9 days, and 5 men and 9 boys can do it in 15 days; how many boys must help 5 men that the work may be done in 11 days?

481. A man leaves £12498. 15*s.*, and the money is to be divided between his wife, his two sons and his brother. The wife is to have half the share of the elder son who is to have twice as much as his younger brother and four times as much as his uncle. A wife pays no legacy duty, a son pays 1 per cent. and a brother pays 3 per cent. How much legacy duty was paid altogether?

482. Two passengers have between them 5 cwt. of luggage, and are charged for the excess above the weight allowed free 5*s.* 2*d.* and 9*s.* 10*d.* respectively. If one passenger had travelled with the same amount of luggage he would have been charged 19*s.* 2*d.* How much luggage is allowed to each passenger free of charge?

483. A bankrupt paid a certain amount in the pound; twelve years afterwards he paid off the remainder, adding thereto simple interest at the rate of $4\frac{1}{6}$ per cent. per annum for the 12 years. The second payment made to each creditor was the same as the first payment; how much in the pound was paid at the time of the bankruptcy?

484. Find the cost of making a road half-a-mile long, 36 ft. wide; the soil being first excavated to a depth of one foot at a cost of 1*s.* per cubic yard, rubble being then laid in, 9 inches deep, at a cost of 1*s.* 6*d.* per cubic yard, and 3 inches of gravel, at 3*s.* 3*d.* per cubic yard, being laid on the top, and the whole consolidated by a steam roller at a cost of 2*d.* per square yard.

485. If a population is now ten millions and the births every year are 1 in 20 and the deaths 3 per cent. of the population at the beginning of the year, find what the population will be at the end of 5 years.

486. The rates of two bicyclists are as 11 : 8. They start together from the winning post and race round and round a circuit of 480 yards. It is noticed that the best man passes the other every four minutes, and that at the moment when the race ends they are passing the winning post together for the first time. How long did the race last, and how many minutes' start can the best man afford to give in a nine mile race without losing?

487. A multiplication sum was worked correctly, and then all the figures except those given below were rubbed out, and the lines shew the places where the figures rubbed out were: find all the missing figures.

$$\begin{array}{r} 4-- \\ 3- \\ 36-- \\ --7- \\ \hline --3-- \end{array}$$

488. On a map on the scale of 6 inches to a mile a rectangular field is represented by a space 1 inch long and $\frac{1}{4}$ inch broad; find its area in acres. Also find how many yards less of paling would be required to enclose a square field of the same area.

489. *A* and *B* run a race twice round a circular course of 450 yards circumference. They start together, and run opposite ways. The faster runner *A* after going once round first passes *B* at 9 yards from the starting post. *B* then quickens his pace by $\frac{1}{49}$th of his previous pace; while *A* slackens down to $\frac{59}{51}$ths of his previous pace. Which will win the race, and by how many yards?

490. When a body is falling freely to the earth, the square of its velocity at any point of its fall is always proportional to the distance through which it has fallen. In falling through 16·1 feet a body acquires a velocity of 32·2 feet per second; and in falling through 2·503745 metres it acquires a velocity of 23 feet per second. Find the number of feet in a metre correct to four places of decimals.

491. Three kinds of tea are mixed in the proportion of 1 : 2 : 3. The value of the mixture is 2*s.* 9*d.* per lb. If only half of the second kind had been used the mixture would have been worth 2*s.* 6*d.* per lb. But if all the second kind had been used and only half of the third kind, the mixture would have been worth 3*s.* per lb. Find the worth per lb. of each kind of tea.

492. *A* and *B* began with the same number of counters, and played five games. At the end of each game, the winner received from the loser one-third of the counters then possessed by the loser. *A* lost the first, second, and third games, won the fourth and fifth games, and then had 118 more counters than *B* had. How many counters had *A* at first?

493. Three riders pass a given point on a circular track at the same instant, with velocities 10, 12, and 15 miles per hour respectively. How long, from that instant, must they continue to move round the track with those velocities, before they next pass the same point abreast (the distance round the track for each rider being 704 yards)?

494. Divide £383 among four persons, so that the first may have thrice as much as the second, the third twice as much as the fourth, and the first and second together £95 more than the third and fourth together.

495. When 52 lbs. of coffee are worth as much as 12 lbs. of tea, 22 lbs. of tea are worth as much as 572 lbs. of sugar, a cask of sugar costs 2 guineas, and 1 cwt. of coffee costs 8 guineas, what is the weight of a cask of sugar?

496. In a hundred yards race *A* can beat *B* by 4 yards; in a quarter of a mile race *C* can beat *A* by 11 yards; by how much can *C* beat *B* in a mile race, supposing that the average speeds of each man when running a hundred yards, a quarter of a mile, and a mile are proportional to 9 : 8 : 7?

497. One train started 20 min. later than another to run 45 miles and arrived 25 min. before it. The slower train was 10 miles from the terminus when the fast train arrived. Where did the fast train pass it, and at what speed per hour did it seem to the passengers in the slow train to be going?

498. An ounce of pure gold is worth £3. 17*s.* 10½*d.* In making sovereigns 2 parts out of 23 in the metal used consist of an alloy worth 1½*d.* an ounce, and the remaining parts are pure gold. How much gold and how much alloy are contained in 2617 sovereigns, on the supposition that the metal in a sovereign is just worth £1.

499. A man went to market and spent £200. He spent his money in buying sheep at £5 each, pigs at £1 each and young geese at 1*s.* each; and he bought 200 animals altogether. How many of each did he buy?

500. A man leaves £162625 to his two sons, aged just 19 and 17 years respectively, each son to receive the same sum when he comes of age. Assuming the money to accumulate at the rate of 4 per cent. per annum, Compound Interest, determine the amount which should be paid the elder when he comes of age.

EXAMINATION PAPERS.

I. CAMBRIDGE LOCAL EXAMINATIONS.

JUNIOR STUDENTS. *Dec.* 17, 1890. 9 to 11.

A 1. WHEN a certain number is divided by 1890, the quotient is 365 and the remainder 52. Find the number.

A 2. Multiply £3. 4*s.* 7½*d.* by 416; and divide 1 mile 2 furlongs 35 p. 4 yds. 2 ft. 11 in. by 17.

A 3. How many tons, cwts., &c. are there in 1367941 drams?

A 4. Find by Practice the rent of 7 a. 3 r. 15 p. at £1. 10*s.* 8*d.* an acre.

B 1. Find the sum of $£7\frac{3}{8}\frac{5}{8}$ and $(3\frac{1}{4} - 2_{1}\frac{7}{2})$ of 3*s.* 6*d.*

Multiply $\frac{3}{4\frac{1}{2} \text{ of } 1\frac{1}{3} + 4\frac{1}{4}}$ by $\left(\frac{5}{12} + \frac{4}{15}\right)$.

B 2. Divide ·04130756 by 1·3678.

Reduce the sum of ·625 of £3 and ·175 of £3. 2*s*. 6*d*. to the decimal of £6.

B 3. A bankrupt has assets to the amount of £10252. 4*s*. 8*d*. and liabilities to the amount of £13901. 6*s*. 8*d*. How much can he pay in the pound?

B 4. Find the simple interest on £3695. 15*s*. for one year and eight months at $4\frac{1}{4}$ per cent. per annum.

C 1. A merchant buys whisky at 17*s*. 6*d*. a gallon, and after adding water to it sells the mixture at 18*s*. the gallon, and by doing so he gains 20 per cent. on his outlay. How much water did he add to each gallon of the whisky?

C 2. Find the amount of £520. 16*s*. 8*d*. at 4 per cent. per annum compound interest for a year and a half, interest being due half-yearly.

C 3. A man invests £1405. 16*s*. in $2\frac{3}{4}$ per cents. at 99; how much must he invest in $3\frac{1}{2}$ per cents. at 105 that the rate of interest on his whole investment may be 3 per cent.?

C 4. A train whose length is 176 feet is moving at the rate of 45 miles an hour, and overtakes a second train going at the rate of 30 miles an hour. How long does the first train take to pass a passenger in the second train? And if the first train takes a quarter of a minute to pass the second train completely, find the length of the second train.

II. CAMBRIDGE LOCAL EXAMINATIONS.

Senior Students. Dec. 17, 1890. 9 to 11.

A 1. Find the cost of 2 furlongs 7 poles $1\frac{1}{2}$ yds. of fencing at £29. 6*s*. 8*d*. per mile.

A 2. Simplify $\dfrac{2\cdot46\dot{}-2\cdot3\dot{0}}{\cdot\dot{3}+\cdot1\dot{2}\dot{7}}+\dfrac{4\frac{1}{3}}{19}$.

A 3. Find in tons what weight of water falls upon $85\frac{1}{3}$ acres of ground during a rainfall of $1\frac{3}{4}$ inches: a cubic foot of water being taken to weigh 1000 ounces avoirdupois.

A 4. A rate of 2*s*. $7\frac{1}{2}$*d*. in the £ on a parish produces £745. 1*s*. 3*d*. What is the rateable value of the parish?

A 5. Find to the nearest penny the interest on £7000 for 85 days at 4 per cent. per annum.

B 1. Three partners *A*, *B*, and *C* invest £4730, £3680, and £2840 respectively in their business; what is the share of each in a profit of £595. 6*s*. 3*d*.?

B 2. The cost of levelling and turfing a square cricket-field at £175. 9*s*. 4*d*. per acre is £987; find the cost of surrounding it with a railing costing 3*s*. 2*d*. per yard.

B 3. If 4 men and 6 boys working 9 hours per day mow 69·3 acres of corn in 10 days, in how many days will 5 men and 4 boys working 10 hours and 40 minutes per day mow 67·76 acres, it being assumed that 2 men mow as much as 5 boys?

B 4. If a merchant mixes 29 gallons of whisky costing him 15*s*. 6*d*. per gallon with 17 gallons costing him 11*s*. 6*d*. per gallon, how much water must he add in order that by selling the mixture at 14*s*. 3*d*. per gallon, he may gain $18\frac{3}{4}$ per cent.?

B 5. A passenger train moving at the rate of 45 miles per hour overtakes a mineral train $1\frac{1}{2}$ times as long moving at the rate of 27 miles per hour and passes it completely in 25 seconds. How long would the passenger train take to pass completely through a station 165 yards in length?

B 6. Which is the better Stock to invest in, the 5 per cents. at $142\frac{1}{2}$ or the 4 per cents. at $113\frac{3}{4}$?

What sum of money must a man have, so that by investing one-third of it in the former and two-thirds in the latter Stock, he may obtain an income of £455. 13*s*. 4*d*.?

III. CAMBRIDGE HIGHER LOCAL EXAMINATIONS.

Dec. 17, 1889. 9 to 11.

Part I.

1. A cook buys 10 lbs. of sugar at $2\frac{1}{2}d$. per lb.; 18 eggs at 8 for a shilling; $1\frac{1}{2}$ lbs. of sausages at $9\frac{1}{2}d$. per lb.; 3 lbs. 6 oz. of bacon at 10*d*.; $1\frac{3}{4}$ lbs. of cheese at 7*d*.; and a joint of mutton weighing 10 lbs. 4 oz. She receives £4. 1*s*. 3*d*. change out of a five pound note. What price per lb. was she charged for the mutton?

2. If the area of Bedfordshire is 294,983 acres, and the population 149,173, find to the nearest integer the number of persons per square mile.

3. Prove that a fraction is not altered in value when its numerator and denominator are multiplied by the same number.

Simplify (1) $\dfrac{46875}{265625}$.

(2) $\dfrac{4\frac{2}{13}\times 2\frac{1}{19}\times 3\frac{1}{6}}{19\frac{2}{7}-5\frac{2}{5}\times 3\frac{3}{14}}$.

Express as decimals

(3) $\dfrac{3}{2\cdot 01\dot{2}6\dot{5}+2\cdot\dot{8}\dot{7}+3\cdot 10\dot{8}55946\dot{}}$.

(4) $\cdot 39\dot{2}85714\dot{}\times\cdot 15\dot{9}0\dot{}$.

4. ·378125 of an acre is let in allotments so as to produce a rental of £·75625. How much is the rent per perch?

5. A square room is carpeted with carpet 27 inches wide costing 3*s.* 4½*d.* per yard. If the cost of the carpet is £15. 12*s.* 6*d.*, find the length of a side of the room.

6. Two pipes, *A* and *B*, can fill a cistern separately in 12 and 20 minutes respectively, and a waste-pipe, *C*, can carry off 15 gallons per minute. If all the pipes are opened the cistern fills in two hours. How many gallons does it hold?

7. Find by Practice the value of 27 tons 13 cwt. 3 qrs. 26 lbs. 4 oz. at £9. 6*s.* 8*d.* per ton.

8. Find (*a*) the simple interest on £468. 11*s.* 10½*d.* for four years at 4 per cent.; (*b*) the present worth of £86. 16*s.* 5¼*d.* due three years hence, *compound* interest being reckoned at 5 per cent.

9. A man has £4750 of a 4 per cent. stock. He sells out at 88 and invests in a 5 per cent. stock at 104½. Find the change in his income.

Part II.

10. 100 cubic feet of air contain 20·9 cubic feet of oxygen, the rest being nitrogen. If 15·96 cubic feet of nitrogen weigh as much as 14·01 cubic feet of oxygen, find to two places of decimals how many ounces of oxygen are contained in 100 ounces of air.

11. If in 1871 when 702,201 tons of sugar were consumed in the United Kingdom, the consumption was 49·93 lbs. per head, and the corresponding numbers for 1881 were 989,208 tons and 62·77 lbs. per head; find to the nearest whole number the increase per cent. in the population during the interval.

12. Two trains leaving London at 9.45 and 10 a.m. reach Grantham 105 miles distant at 11.47 and 12.2 respectively, each running uniformly but losing a minute in starting and a minute in stopping. An up-train meets the first express at 10h. 42m. $31\frac{1}{2}$s., and the second at 10h. 50m. 24s. If it proceeds uniformly, when will it reach London?

13. A man invests £16,213 partly in shares at 105 bearing a dividend (free from income-tax) of 8 per cent., and partly in a mortgage at 5 per cent. interest on which he pays income-tax at 8*d.* in the pound. His net income from each investment is the same. What is his whole income?

IV. OXFORD LOCAL EXAMINATIONS, JUNIOR CANDIDATES. July 14, 1891. 9 to 11.

1. (1) Simplify $2\frac{3}{4}+1\frac{13}{8}+2\frac{3}{10}-3\frac{5}{24}+1\frac{8}{15}$.
   (2) Multiply ·00125 by 176.
   (3) Divide 2·86 by ·013.

2. (1) Reduce to seconds 9 wks. 1 day 7 hrs. 12 min. 35 sec.
   (2) Express $2\frac{1}{2}$ guineas as the fraction of £$1\frac{3}{4}$.

3. How much would it cost to fill a trench 18 feet long, 15 feet broad, and 12 feet deep, with material at £1. 13*s.* 4*d.* per cubic yard?

And what would be the weight of the contents at 1 oz. per cubic inch?

4. Find by Practice the cost of 3 oz. 17 dwt. 12 gr. at 18*s.* 6*d.* per ounce.

5. If 16 horses eat 12 pecks in 10 days, how long will 63 bushels last 56 horses?

6. Find the simple interest on £320. 15*s.* 0*d.* at $2\frac{1}{2}$ per cent. per annum for 32 years.

7. Two partners make a profit of £182; one put £300, the other £400, into the business: how should they divide the profit?

8. Find the square root of 112487236.

9. What is the income arising from an investment of £250 in a 3 per cent. stock at $66\frac{2}{3}$?

10. If oranges are bought at 11 for 10*d.* and sold at 10 for 11*d.*, what is the gain per cent.?

V. OXFORD LOCAL EXAMINATIONS, SENIOR CANDIDATES. JULY 14, 1891. 9 to 11.

1. (1) Simplify $\dfrac{8\frac{7}{12}}{5\frac{11}{18}+5\frac{5}{6}} - \dfrac{11\frac{3}{8}}{16\frac{11}{12}+5\frac{5}{6}}$.

(2) Multiply $\cdot\dot{0}76923$ by 169.

2. Find the L. C. M. of 64, 72, 80, 84, 96.

3. Find by Practice the value of 3 tons 3 cwt. 3 qrs. 14 lbs. of coal at £1. 3*s.* 4*d.* per ton.

4. Divide £847. 19*s.* 3*d.* among 4 people in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

5. Find the compound interest on £50,000 in three years at $2\frac{3}{4}$ per cent. per annum.

6. Find the cube root of 17053975224.

6*. Find the square root of 22581504.

7. A merchant buys two kinds of wine, the price of one being 5 per cent. cheaper than that of the other; by selling the dearer wine at 39*s.* per dozen he makes a profit of 17 per cent.; at what price per dozen must he sell the other so as to make on it a profit of 20 per cent.?

7*. A merchant buys 54 dozen of wine for £97. 4*s.*; at what price per bottle must he sell it so as to gain $5\frac{5}{9}$ per cent.?

8. A person in Paris owes another in Vienna 6670 Austrian florins; instead of paying direct, he pays through a broker in London, and thereby gains in English money at current rates £1. 5*s.* $9\frac{3}{25}d.$; the exchange between London and Paris being 25 francs for £1 and between London and Vienna 11·60 florins for £1, find the rate of exchange between Paris and Vienna.

8*. A person buys £500 Stock at 66, and afterwards £500 of the same Stock at 69; he sells out the whole when the price has risen to 89; find the increase in his capital.

9. Within a rectangular court of length 200 feet a gravelled path 10 feet wide is laid down along the four sides, the cost of the gravel being 3*d.* per square foot; if the path had been twice as wide, the gravel would have cost £81 more: find the breadth of the court.

9*. Find, in tons, cwts., &c., the weight of water contained in a tank 6 yards long, 2 yards high, and 8 feet broad, the tank being full, and the weight of a cubic foot of water being 1000 oz.

10. One man, three women, and four boys can do a piece of work in 96 hours; two men and eight boys can do it in 80 hours; and two men and three women can do it in 120 hours; in how many hours can it be done by five men and twelve boys?

10*. If 17 men can do a piece of work in 24 days of 10 hours each, in how many days of 8 hours can 18 men do the same piece?

VI. OXFORD AND CAMBRIDGE SCHOOL EXAMINATIONS.

For Lower Certificates. 1891.

1. How many rupees worth 1*s.* $6\frac{1}{2}d.$ each are there in 29748 guineas?

2. Multiply £246. 7*s.* $10\frac{1}{2}d.$ by $4\frac{2}{9}$, and 2 cwt. 3 qrs. 6 lbs. by 3·75.

3. Find the L.C.M. of 777, 4329, and 819, and explain shortly the reason of the method you employ.

4. Simplify

(i) $10\frac{1}{2}+6\frac{1}{4}+9\frac{9}{16}-23\frac{3}{8}$;

(ii) $\dfrac{3\frac{1}{2}+2\frac{1}{3}+2\frac{4}{5}}{\frac{1}{7}\text{ of }\frac{11}{12}\text{ of }\frac{14}{55}}$.

5. Express $\frac{31}{82}$ as a decimal fraction; and find the value of $\dot{1}\dot{6}$ of £3. 4*s.* $10\frac{1}{2}d.$

6. Find by Practice the cost of 1020 things at £5. 3*s.* $7\frac{1}{2}d.$ each.

7. Find the simple interest on £976. 10*s.* for a year and four months at $2\frac{3}{4}$ per cent. per annum.

8. Fifteen hundred men in a fortress have provisions for 48 days, but five hundred men join them: how long will the provisions now last?

9. A tradesman who is selling off makes a reduction of 10 per cent. from the marked price: at what price should he mark goods for which he wishes to receive 15*s.* 9*d.*?

10. A watch which gains uniformly is 150 seconds slow on Sunday at noon: on the following Friday at 12 minutes past 7 a.m. it is 210 seconds fast; when was it right?

11. What will it cost to cover with zinc the sides and bottom of a cistern whose length, breadth, and depth are respectively 7 ft. 10 in., 5 ft. 4 in., and 1 ft. 9 in. at 6*s.* 9*d.* per square yard?

VII. OXFORD AND CAMBRIDGE SCHOOL EXAMINATIONS. For Higher Certificates. 1891.

1. A train 160 ft. long is travelling at the rate of 25 miles an hour. Find (1) how long it will be in passing completely over a bridge 240 yds. long; (2) how long it will be in passing completely a train 170 ft. long travelling in the opposite direction at the rate of 20 miles an hour.

2. Add together $2\frac{2}{9}$, $3\frac{11}{12}$, $4\frac{3}{8}$, $2\frac{5}{6}$ and divide the result by the difference between $3\frac{7}{18}$ and $2\frac{5}{16}$.

3. Find the value of $\frac{2}{7}$ of 3 miles $-\frac{8}{21}$ of a furlong $+\frac{24}{35}$ of a pole $+2\frac{2}{15}$ yds.

4. Find the first 5 decimal places in the quotient of $3{\cdot}065 \div {\cdot}0291$.

5. Express an ounce Avoirdupois as a decimal of an ounce Troy, having given that a pound Avoirdupois contains 7000 grains Troy.

6. Find the present value of £1616. 2*s.* 8*d.* due 3 years 9 months hence, reckoning 4 per cent. per annum simple interest.

7. If 8 per cent. is gained by selling a piece of ground for £773. 11*s.*, what would have been the gain per cent., and what would have been the actual amount gained, if it had been sold for £787. 17*s.* 6*d.*?

8. Three men together fill a pit in an hour and a half, throwing in respectively 8, 10, and 12 equal shovelfuls of earth per minute. In what time could each man have filled up the pit by himself?

9. The external length, breadth, and thickness of a closed box are 4 ft., 2 ft. and 16 inches respectively, and the wood of which it is made is an inch thick. Find (1) the number of cubic inches of wood in the box; and (2) the cost of lining it with metal at $4\frac{1}{2}d.$ per square foot.

10. The rent of a house is £200 per annum, due by half-yearly payments on Jan. 1st and July 1st, and no payment has been made since the beginning of last year. What sum will be due at the beginning of next year, interest being added to arrears, at the beginning of each half year, at the rate of 5 per cent. per annum?

11. A man sells out $2\frac{3}{4}$ per cent. consols at $96\frac{1}{4}$, and by investing the proceeds in shares which pay an annual dividend of £4 per share, raises his income 5 per cent. What was the price of each share?

12. Find (correct to the tenth of an ounce) the weight of a hollow ball of copper 3·9 ft. in diameter and $1\frac{4}{5}$ in. thick, assuming that the volume of a sphere equals the cube of the diameter multiplied by $\frac{11}{21}$, and that a ton of copper contains 3 cubic feet and 1693·782$\dot{1}$ cubic inches.

A. Find by Practice the rent of 113 acres, 3 roods, 35 perches at £1. 16*s.* 8*d.* per acre.

B. Find the compound interest on £1800 in 3 years at 5 per cent.

C. A room is 14 ft. long, 13 ft. 6 in. wide, and 7 ft. 8 in. high. What will be the cost of covering the walls with paper half a yard wide at 13*s.* 6*d.* per dozen yards?

D. Find the first five decimal places in the square root of ·0565.

E. What sum must be invested in $2\frac{3}{4}$ per cent. consols at $96\frac{1}{4}$ in order to derive an income of £100 a year?

What will be the reduction in this income when the dividends are reduced from $2\frac{3}{4}$ to $2\frac{1}{2}$ per cent.?

F. Three men enter into partnership, putting into the business £300, £400 and £500 respectively. How much should each receive out of every £100 of profits?

VIII. COLLEGE OF PRECEPTORS.

Certificate Examination. Third Class. 1891.

1. Divide eight hundred and seventy-one millions nine hundred and nineteen thousand seven hundred and fifty-two by one hundred and seventy-five thousand three hundred and sixty-six; and shew that your answer is the correct one.

2. A guardsman, in a march of $4\frac{3}{4}$ miles, takes 10032 steps; find the length of his step.

3. How many times is £237. 6*s*. 8*d*. contained in £9256?

4. Reduce 19 ac. 3 rd. 36 po. 20 yd. 6 ft. to feet: and find how many miles, yards, and feet there are in 253617 inches.

5. A tradesman sold 318 cwt. of sugar for £775. 2*s*. 6*d*., and realised a profit of £39. 15*s*. by the sale. What did the sugar cost him per cwt.?

6. Name the different kinds of Vulgar Fractions with which you are acquainted, and give an example of each kind. Reduce to their Least Common Denominator $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$.

7. Find the values of—

(i) $\frac{1}{2}+\frac{2}{3}+\frac{1}{6}+\frac{3}{5}+\frac{4}{15}$;

(ii) $\frac{6}{7}-\frac{3}{13}$ of $1\frac{6}{7}$;

(iii) $(\frac{1}{4}+\frac{3}{5})\times(\frac{5}{6}-\frac{5}{7})\div(\frac{7}{8}-\frac{7}{9})$;

(iv) $\dfrac{\frac{1}{2}+\frac{1}{4}+\frac{1}{5}}{2\frac{1}{4}+4\frac{1}{5}+5\frac{1}{4}}$.

8. What must be given for 2874 sacks of potatoes at 8*s*. $4\frac{3}{4}$*d*. the sack?

9. When 54 yds. 2 ft. 6 in. of carpet exactly cover a floor which is 23 ft. 6 in. long and 15 ft. 9 in. broad, what must be the width of the carpet?

10. Two watches are together at 10 o'clock this morning (June 17th), one loses $8\frac{1}{2}$ seconds, and the other gains 9, in 24 hours. When will one be half-an-hour before the other, and what time will each then shew?

IX. COLLEGE OF PRECEPTORS.

CERTIFICATE EXAMINATION. SECOND CLASS. 1891.

1. REDUCE 3 tons 11 cwt. 15 lbs. to oz. In 4578038 inches, how many miles, &c. are there?

2. Find, by two different methods, the cost of labour for 335 working days, at the rate of £2. 2*s*. 6*d*. per week of 6 days.

3. If 72 horses consume 16 quarters of corn in 12 days, how many will 24 quarters feed for 16 days?

4. Add together the fractions $\frac{4}{5}$, $\frac{5}{7}$, $\frac{3}{11}$, and $\frac{17}{385}$; and reduce $(17 \text{ of } 4\frac{5}{11}) \times (\frac{1}{7} \text{ of } \frac{2}{51}) \times (\frac{3}{28} \div \frac{17}{33})$ to a simple fraction in its lowest terms.

5. Multiply ·056277 by 5·07; and divide ·3728 by ·0283 correct to three places of decimals.

Obtain the recurring decimals corresponding to the fractions $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{11}$, and $\frac{1}{13}$, respectively.

6. Subtract ·046 of 2 cwt. 1 qr. from 1·3 of 9 lbs., and give the answer as a decimal of 5 ounces.

7. In what time will £827. 10*s.* amount to £1072. 13*s.* 7½*d.*, when put out at Simple Interest at the rate of 3½ per cent.?

8. *A*, *B*, and *C* are partners. The sums deposited by *A* and *B* amount together to £900; and those by *B* and *C* amount together to £1600. If the entire capital is £2200, what is the Interest each one is entitled to at 4 per cent.?

9. A square plot of ground contains 6773948416 square inches. What is the length of each side in yards?

10. If a French are is a square with a base of 10 metres, and a metre is taken as equal to 39·36 English inches, shew that there are 40½ French ares in an English acre, within a small fraction of an are, *or* within a very few square feet.

X. COLLEGE OF PRECEPTORS. Certificate Examination. *First Class.* June 17, 1891.

1. Taking the prices per quarter of wheat for the 5 years commencing 1837 to be £2. 15*s.* 10*d.*, £3. 4*s.* 6*d.*, £3. 10*s.* 8*d.*, £3. 6*s.* 4*d.*, and £3. 4*s.* 4*d.*; also for the 5 years ending 1889 to be £1. 12*s.* 10*d.*, £1. 11*s.*, £1. 12*s.* 6*d.*, £1. 11*s.* 10*d.*, and £1. 9*s.* 9*d.*, determine how much less the average price per quarter for the latter 5 years is than half the average price per quarter for the former 5 years.

2. Divide the sum of $\frac{7}{22}$ and $\frac{1894}{5011}$ by their difference, and express one-millionth part of the quotient as a decimal.

A and *B* respectively received $\frac{3}{13}$ and $\frac{1}{17}$ of a certain sum of money, and *C* the remainder. *A* received £107. 14*s.* 9*d.*; (i) what sum did *B* get, and (ii) what fractional part was *A*'s share of *C*'s share?

3. Obtain 37 times the sum of $\cdot 41\dot{6}$, $\cdot 2\dot{2}3\dot{4}$, and $\cdot 00\dot{4}\dot{5}$; also find (to the nearest pound) the cost of 5122·7 kilogrammes at 3·43 francs per kilogramme, reckoning 25·42 francs to the pound.

4. A square field contains 57 acres 2 roods 16 poles; what is (i) its perimeter, and (ii) the rent at 25*s.* per acre?

5. In what time at $4\frac{1}{2}$ per cent. per annum simple interest will £583. 18*s.* 4*d.* amount to £656. 18*s.* $1\frac{1}{2}d.$?

6. If 30 men, working 8 hours per day at a uniform rate, earn £72 in 9 days, how much will 63 men earn, who work for 13 hours per day, at $\frac{2}{3}$ of the same uniform rate, in 10 days, the wages being proportional to the amount of work done.

7. What weight of tea worth 3*s.* 6*d.* per lb. should be mixed with 60 lbs. of tea worth 2*s.* 3*d.* per lb., so that, when the mixture is sold at 2*s.* 9*d.* per lb., there may be (i) neither gain nor loss, (ii) a gain of 10 per cent.?

8. The present worth of a certain sum due in 3 years at 5 per cent. per annum compound interest is £966. 13*s.* 4*d.* What is the discount?

The interest on a certain sum for 10 years is greater than the discount on the same sum due in 10 years, by one quarter of the discount. Find the rate per cent. per annum, simple interest, which is the same in both cases.

9. Find the quarterly income (to the nearest penny) derived from £3,500 $2\frac{3}{4}$ per cent. Consols, after a deduction of income tax at the rate of 4*d.* in the pound.

A person bought Consols at 94 and sold the same at $95\frac{5}{8}$ thereby gaining £55. What cash did he receive for the Stock, reckoning $\frac{1}{8}$ per cent. brokerage on each transaction?

10. A train passed through a station with a velocity which continued uniform until it passed a station 40 miles away, and at the latter station was 2 minutes late. Had the velocity been 50 miles per hour, it would have been 10 minutes late. At what rate did the train travel?

11. On one day as many hundreds of oranges could be bought for half a sovereign as dozens could be bought for a shilling on the previous day. A woman bought 75 on each day, and paid altogether for them 5*s.* 6*d.* How many oranges could be bought for sixpence on the first day?

XI. CAMBRIDGE PREVIOUS EXAMINATION.

JUNE 12, 1891. 9—11½.

1. MULTIPLY two millions seven thousand and sixty-two by two hundred and seven thousand one hundred and sixty-two and divide the result by two thousand and thirty-one.

2. Find the greatest common measure and the least common multiple of 255, 289, 340.

3. Simplify $\dfrac{2\frac{1}{11}+6\frac{2}{7}}{2\frac{3}{4}-\frac{3}{5}}-\dfrac{2\frac{1}{2}-\frac{1}{5}}{2\frac{2}{5}+\frac{1}{6}}$.

4. Divide 1·37852 by 28·6. Express $\cdot\dot{5}\dot{4}\dot{3}$ and $\cdot 3\dot{4}\dot{5}$ as vulgar fractions reduced to their lowest terms and their difference as a circulating decimal.

5. The area of a square field is 25 acres 2 roods 16 poles, find the length of its side in yards.

6. Find the cost of 71 cwt. 2 qrs. 14 lbs. at £2. 11*s.* 6*d.* per cwt.

7. If forty-two men working eight hours a day do a piece of work in six days, how many men working nine hours a day would be required in order to do it in seven days?

8. *A* can run 100 yards in $10\frac{1}{5}$ secs., and *B* in $10\frac{2}{5}$ secs.; what handicap must be given to *B* in order to make the race a dead heat?

9. Find the true discount on a bill for £360. 16*s.* 10*d.* paid 73 days before due, the rate of interest being $3\frac{1}{2}$ per cent. per annum.

10. A man sells 120 articles for the cost price of 130, find his rate per cent. of profit.

11. A man invests £7000, part in 3 per cent. stock at 97 and the remainder in 4 per cent. stock at 104. Find how much he must invest in each in order that he may have equal returns from the two sources.

XII. UNIVERSITY OF CAMBRIDGE EXAMINATION FOR COMMERCIAL CERTIFICATES.

DEC. 17, 1889. 9 to 11.

1. Multiply 3415·6782998456 by 42176·37124 so as to obtain the product correct within 100.

2. Add together $\frac{2}{3}$ of $\frac{1}{2}$, $\frac{7}{8} \div \frac{5}{12}$, and $\frac{\frac{3}{4}+\frac{7}{8}}{2\frac{1}{2}}$.

3. Find the number of inches in 3 miles 2 fur. 4 po. 5 yds. 2 ft. 5 in., and reduce 345241 sq. ft. to acres, roods, poles, &c.

4. Find the total cost of 320 tons 16 cwt. 32 lbs. at £3. 17*s.* $10\frac{1}{2}d.$ per cwt., and 27 tons 2 cwt. 15 lbs. at 3*s.* $10\frac{1}{2}d.$ per lb.

5. Find, correct within a penny, the compound interest on £1 in 20 years at 5 per cent. per annum.

6. Give the relative values of a shilling (English), a franc (French), and a mark (German); and the relative magnitudes of a yard (English), and a metre (French).

7. What is a Bill of Exchange?

If the cheque exchange in London on Paris is 25·30 and the rate of discount for 3 months' bills in London is at 5 per cent. per annum, what debt in Paris can be discharged by a person in London who holds a three months' bill on London for £1000?

8. A man calls in a sum of £10,000 lent on mortgage at 4 per cent. and invests the money in £3000 L. and N. W. Ry. 4 per cent. deb. stock at 133 (including brokerage) and £3000 G. W. Ry. 5 per cent. stock at 167 (including brokerage). The balance of the £10,000 is placed on deposit at $2\frac{1}{2}$ per cent. What is the change in the income derived from the £10,000?

9. A wine merchant buys 10,000 litres of wine at 2 francs a litre, and immediately sells it for cash at 2*s.* 6*d.* a quart. Find the amount of his gross profit assuming that 8 litres are equal to 7 quarts and that a franc is equal to $9\frac{1}{2}d.$

Again, supposing that he works with borrowed capital for which he pays 5 per cent. per annum interest, that his working expenses are at the rate of 10 per cent. upon the price he pays for the wine, and that he gives a year's credit and makes bad debts to the extent of 10 per cent., find the amount of net profit at the end of the year.

10. A discount house takes money on deposit and uses the money in discounting bills. It pays 4 per cent. per annum interest on the deposits and makes $4\frac{1}{4}$ upon them. If the deposits average £10,000,000, what annual profit is made?

By how much would this profit be increased if the interest on the deposits were paid after deduction of income tax upon this interest at the rate of 6*d.* in the £1?

11. The following entries occur in a metal merchant's register of transactions :

1889

Jan. 1. Sold R. Sachs 20 tons copper at £50 payable in 3 months and took his bill in payment.

Discounted R. Sachs' bill at 5 per cent. per annum.

Jan. 5. Bought 1000 tons pig iron at 60*s.* payable in 3 months and gave bill in payment.

Give the usual Ledger entries which should be made on account of these transactions.

XIII. OXFORD AND CAMBRIDGE SCHOOL EXAMINATIONS. For Commercial Certificates.

1. See separate paper. [Addition.]

2. Find the value of 6·14 of £1 + $\frac{2}{7}$ of 10*s.* + 6·8 of 6*d.* and reduce the result to the decimal of £10. 13*s.* 4*d.* correct to 6 decimal places.

3. If a cwt. of lead cost £1. 9*s.* 2*d.*, find by Practice the cost of 15 tons 17 cwt. 2 qrs. 10 lbs.

4. Find the rent of a farm of 899 acres 2 roods 30 poles at £1. 16*s.* 4*d.* an acre.

5. Simplify (i) $\dfrac{(2\frac{3}{4}+\frac{1}{4}) \text{ of } 2\frac{4}{9}}{1\frac{1}{4} \text{ of } (\frac{3}{2}+2\frac{1}{3})}$;

(ii) $\dfrac{3\frac{1}{3}+8\frac{4}{9}}{6\frac{1}{12}-3\frac{7}{8}} \div \dfrac{3\frac{1}{7} \text{ of } 9\frac{7}{9}}{3\frac{13}{14} \text{ of } 4\frac{2}{5}}$.

6. Divide ·00742 by 2·56; and express as a vulgar fraction

$$\frac{\cdot 13+(\cdot 1 \text{ of } 3\cdot 7)}{\cdot 24-(\cdot 007 \text{ of } 20)}.$$

7. A man allows his agent 5 per cent. on his gross income of £16292. 10*s.* for collecting it. He spends one-seventh of what he has then left in insuring his life; and this portion is exempt from income tax. The income tax being 8*d.* in the £ find its amount.

8. Find the expense of paving a pathway 6 feet wide round and immediately outside a tennis court 21 yards long and 10 yards broad at 9¼*d.* per square yard.

9. By selling goods for £240 a tradesman gains 25 per cent.; how much would he have gained per cent. by selling them for £204?

10. A man invests £1980 in the $3\frac{1}{2}$ per cents. at 99 and £3220 in the $4\frac{1}{2}$ per cents. at 105; find the average rate of interest on his whole investment.

11. Find the banker's discount on a bill for £2240 drawn June 8 at 3 months, discounted June 30 at 4 per cent.

12. At what rate per cent. will £150 amount to £164. 1*s.* 3*d.* in $2\frac{1}{2}$ years, simple interest; and what would be the amount at compound interest for the same time at the same rate paid yearly?

13. Give as nearly as possible the present English value of the following foreign monies and measures, and say in what country or countries they are used:

(*a*) Franc, Rupee, Thaler, Centesimo, Mexican Dollar.

(*b*) Pood, Kilometre, Maund, Quintal.

14. Explain the following quotations of Exchange:

'India demand rates were cabled 1*s.* $6\frac{1}{2}$ for Bombay and 1*s.* $6\frac{9}{16}$ for Calcutta.'

'Berlin: foreign Bills weaker, both long and short on London being half a pfennig lower.'

'Hamburg: 3 mos. 20·54 to 20·58.'

S. Petersburg: ditto $27\frac{3}{4}$ to $27\frac{7}{8}$.

New York: 60 days. $48\frac{5}{8}$ to $48\frac{3}{4}$.

15. James Wilson and Thomas Jackson enter into partnership on 1st January 1890, to trade as Wilson, Jackson & Co. with a capital of £8000. Wilson contributing £6000, and his partner the remainder. Interest on capital is to be reckoned at 5 % per annum, and the former is to take $\frac{2}{3}$rds of profits or losses, the latter $\frac{1}{3}$. At the close of the year the balance sheet shews a profit of £1125. 15*s.* 0*d.* Render the account of each partner with the firm, not reckoning Interest on the amount drawn out for personal expenses which have been £480 by Mr Wilson and £330 by Mr Jackson.

XIV. COLLEGE OF PRECEPTORS. MERCANTILE EXAMINATION. JUNE 15, 1891. 11.30 to 1.30.

1. Find the Simple Interest on £425 for 1 year 233 days, at $3\frac{1}{2}$ per cent. per annum.

2. By the shortest method multiply ·83210753 by 5·8621537, and divide ·03784563 by 2·498637, in both cases correct to 7 decimal places.

3. *A* sold goods to *B* at a profit of 5 per cent. *B* became bankrupt, and paid 13*s.* 7*d.* in the £. What was *A*'s loss per cent.?

4. Exchange 15,000 kroner into sterling, at 18·19 kroner, and £576. 18*s.* 3*d.* into florins at 12 fl. $7\frac{1}{2}$ cents.

5. A tradesman buys articles at £4. 7*s.* 6*d.* per gross. He reckons that he will sell 75 per cent. of the articles at a certain price per article, and the remainder at half that price. What must that price be if he wishes to gain 20 per cent.?

6. A bill for £15,000 at 3 months was discounted at $4\frac{1}{2}$ per cent., and the proceeds invested in $2\frac{3}{4}$ per cent. Consols at 95, brokerage being $\frac{1}{8}$ per cent., and income tax 5*d.* in the £; determine the net quarterly income.

7. For an annuity at the rate of thirty shillings per week, payable half-yearly, £621. 15*s.* was paid. The first payment was made 6 months after the annuity was bought, and just after the seventh payment the annuitant died. Assuming that the Insurance Company could invest at 4 per cent. per annum, payable half-yearly, determine the profit on the transaction.

8. What rate of exchange is established between London and Paris by bills upon Amsterdam bought in London at 12 $2\frac{1}{4}$, the proceeds being forwarded from Amsterdam to Paris in direct bills at 48 florins per 100 francs?

9. If a merchant has to remit upon Paris at 25·63, and to draw upon Lisbon at 56, at what rate may he draw upon Lisbon if the Paris rate falls to 25·52?

ANSWERS TO THE EXAMPLES.

I. A. Pages 8—9.

1. 5 *units*, 5 *tens*, 5 *hundreds*, 5 *tens-of-thousands*, 5 *hundreds-of-thousands*.

2. 7 *tens*, 7 *thousands*, 7 *hundreds-of-thousands*, 7 *tens-of-millions*, 7 *tens-of-thousands-of-millions*.

3. 3, 1, 4, 0. **4.** 2, 6, 0, 5.

5. Two *tens* five *units*, three *hundreds* five *tens* six *units*, four *thousands* two *hundreds* three *tens* six *units*, five *tens-of-thousands* three *thousands* four *hundreds* six *tens* one *unit*, six *hundreds-of-thousands* one *ten-thousand* three *thousands* four *hundreds* two *tens* five *units*.

6. Three *hundreds* four *units*, two *thousands* one *hundred* six *units*, three *hundreds-of-thousands* seven *thousands* five *hundreds* nine *units*, four *thousands-of-millions* three *tens-of-millions* five *hundreds-of-thousands* seven *thousands* nine *tens*.

7. Twenty-seven, three hundred and forty-nine, five hundred and sixty, three hundred and six, one thousand two hundred and four, five thousand and twenty.

8. Two thousand and nine, six thousand and fifty, twelve thousand three hundred and forty-five, ten thousand three hundred and five, forty thousand and fifty, one hundred and twenty thousand four hundred and sixty.

9. Five hundred and eighteen thousand six hundred and eighteen, six hundred and two thousand and ten, one hundred thousand and ten, five hundred and four thousand and seventy-five, four hundred and twenty thousand and forty, one hundred and seven thousand and five.

10. One hundred and eleven million one hundred and eleven thousand one hundred and eleven, one million two hundred and three thousand four hundred and five, two million three hundred and fourteen thousand one hundred, five million forty thousand three hundred and fourteen, twenty million fifty thousand and sixty, thirty million three hundred thousand and seventy-four.

11. Three million twelve thousand and four, one hundred and ten million one hundred and ten thousand one hundred and ten, two hundred and one million two hundred and one thousand two hundred and one, one billion forty million three hundred and five thousand one hundred and one, six hundred and four billion one hundred and two thousand million three hundred thousand and four.

12. 58, 85, 211, 3012, 6040, 9300.

13. 1014, 7036, 12200, 14016, 60071, 85106.

14. 100404, 305305, 620007, 6006060, 5300000, 7011014.

15. 20020000, 17050019, 104602011, 6307002056, 520314105218.

16. 115000240005, 600000300000, 84000000502, 2000006000045, 9050000200500.

17. 1111, 1460, 1201600, 6001216, 11011000001111.

18. (1) 99, 10; (2) 999, 100; (3) 999999, 100000.

I. B. Page 10.

1. I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX, XX.

2. XX, XXX, XL, L, LX, LXX, LXXX, XC, CC, CD, DC, DCCC, CM.

3. XXXIX, XLIX, LIX, LXIX, LXXIX, LXXXIX, XCIX, XCVI, CCCCXLIV, MCCXCIV, MDCCCLXXXIX.

4. 58, 39, 44, 94, 99, 199, 2799.

II. B. Pages 14—16.

1. 10039.	**2.** 21230.	**3.** 16963.	**4.** 4037.
5. 34, 65, 671.	**6.** 123068.	**7.** 737246.	**8.** 1303198.
9. 214266.	**10.** 2437226.	**11.** 2994527.	**12.** 23263.
13. 1552544.	**14.** 1396230.	**15.** 67628.	**16.** 79380374.
17. 1000031276173.	**18.** 7957187.		**19.** 11360577.
20. 2950812.			

II. D. Pages 20—21.

1. 1849, 2185, 3087. 2. 6318, 981765, 11101.
3. 267, 158, 86, 195, 1527, 21808. 4. 186881. 5. 431.
6. 11755. 7. (1) 96, (2) 57, (3) 109, (4) 431, (5) 3302, (6) 1389.
8. 76408. 9. 421406. 10. 26. 11. 44.
12. 94513. 13. 452. 14. 406. 15. 1828.
16. 1863. 17. 1438. 18. 1878. 19. 758.

II. E. Pages 27—29.

1. 111. 2. 325. 3. 2277. 4. 1576.
5. 2688. 6. 5454. 7. 11913. 8. 40896.
9. 6567. 10. 999999. 11. 111111111. 12. 500.
13. 810. 14. 2520. 15. 15900. 16. 31620.
17. 71520. 18. 87500. 19. 233400. 20. 215100.
21. 86680000. 22. 46320000. 23. 49400000. 24. 4107000000.
25. 625. 26. 1053. 27. 7663. 28. 2128.
29. 6873. 30. 3822. 31. 5875. 32. 24960.
33. 66284. 34. 222222222. 35. 777777777.
36. 85731. 37. 210146. 38. 233814. 39. 1971918.
40. 1783592. 41. 2499279. 42. 6517140. 43. 11992068.
44. 15643010. 45. 7053760. 46. 11635329. 47. 16965032.
48. 294596405. 49. 722646078. 50. 28207026232.
51. 22207400281. 52. 620411030888. 53. 80779853376.
54. 1247048602525. 55. 35520007008168.
56. 645450239625516. 57. 202851555856040.
58. 2500. 59. 17500. 60. 100000. 61. 6300.
62. 50000. 63. 5247000. 64. 10000000. 65. 733248000.
66. 347652000. 67. 3240. 68. 5814.
69. 181440. 70. 240240. 71. 333333333.
72. 999999. 73. 28900. 74. 48400. 75. 129600.
76. 184900. 77. 15625. 78. 288369. 79. 21996100.
80. 4186090000. 81. 306075025. 82. 1502647696.
83. 46539001441. 84. 967191604681. 85. 512000.
86. 4096000. 87. 512000000. 88. 4096000000.
89. 76765625. 90. 4157747712. 91. 43132764843.
92. 4682608217000. 93. 740. 94. 1158080.
95. 85008. 96. 251520. 97. 31536000.
98. 41464280. 99. 14592. 100. 419328.

II. G. Page 33.

1. 14. 2. 12. 3. 13. 4. 21. 5. 25.
6. 31. 7. 27. 8. 31. 9. 135. 10. 43.
11. 64. 12. 8161. 13. 6699. 14. 6066. 15. 7038.
16. 9079. 17. 10607. 18. 10909. 19. 12093.
20. 10890. 21. 31470. 22. 104. 23. 109.
24. 509. 25. 321. 26. 203. 27. 609.
28. 1414. 29. 3005. 30. 6008. 31. 60606.
32. 6234. 33. 594712, rem. 1050. 34. 3009.
35. 6304, rem. 9786. 36. 40809. 37. 1563.
38. 70824. 39. 32409. 40. 470596. 41. 437962.
42. 24681024. 43. 881, rem. 55. 44. 1177, rem. 270.
45. 2325, rem. 999. 46. 2807, rem. 558. 47. 914, rem. 3312.
48. 1858. 49. 107. 50. 157.

II. H. Page 34.

1. 23. 2. 29. 3. 15. 4. 39. 5. 157.
6. 102. 7. 1443. 8. 1067. 9. 396.
10. 568, rem. 6. 11. 688. 12. 2969. 13. 8017, rem. 3.
14. 49441, rem. 2. 15. 102938, rem. 3. 16. 350719, rem. 2.
17. 17636, rem. 4. 18. 109739, rem. 3. 19. 51250336.
20. 114568313.

II. I. Page 37.

1. 22. 2. 34. 3. 25. 4. 139. 5. 245.
6. 375. 7. 30602. 8. 729405. 9. 123516.
10. 80712645. 11. 7039, rem. 39. 12. 205606, rem. 33.
13. 58525, rem. 35. 14. 127390, rem. 63. 15. 224466, rem. 88.
16. 935278, rem. 93. 17. 1608, rem. 3; 1072, rem. 3; 804, rem. 3.
18. 1078, rem. 14; 539, rem. 14; 359, rem. 34.
19. 6172, rem. 16; 4115, rem. 6; 3086, rem. 16.
20. 7946, rem. 17; 3178, rem. 37; 2270, rem. 37.
21. 72, rem. 7; 43, rem. 17. 22. 560, rem. 19; 210, rem. 19.
23. 448, rem. 23; 299, rem. 3. 24. 439, rem. 35; 244, rem. 25.
25. 435, rem. 443. 26. 1017, rem. 515. 27. 1007, rem. 131.
28. 5101, rem. 125. 29. 1864050. 30. 104050.
31. 1584750. 32. 484250. 33. 7882776.
34. 1895103. 35. 29016850. 36. 7918162448.
37. 1213107. 38. 155399517. 39. 14158435272.
40. 11541149100. 41. 1189024668. 42. 57258512010.

MISCELLANEOUS EXAMPLES. I. Pages 37—39.

1. Three millions fifteen thousand six hundred and two. 11500214.
2. 111111. 3. 21. 4. 10000000, 39504000.
5. 4 times, 121. 6. 1879, MCCCCXLIX.
7. 94587. 8. 33. 9. 5332114, 284342058.
10. 263, rem. 3. 11. One million six hundred and thirty-two thousand and fifty-seven; three billions four thousand one hundred and sixty-seven millions two hundred and one thousand five hundred.
12. 794. 13. 84173434504. 14. 788. 15. 902, rem. 8.
16. 992138. 17. 1609. 18. 708. 19. 540000.
20. 3145. 21. 600. 22. 131241. 23. 900.
24. 23, rem. 13. 25. 46. 26. 1894, MMDCCCLXXV.
27. 62. 28. 957. 30. 487.
31. One hundred and ninety-four million nine hundred and ninety-three thousand three hundred and thirty-five.
32. 23715. 33. 6069. 34. Two billion one hundred and ninety-nine thousand and twenty-three million two hundred and fifty-five thousand five hundred and fifty-two; one hundred and twenty-eight. 35. 268.
36. XXIX, XLVII, CLVIII, DCLXXIX, MCCCCLXIV, $\overline{\text{X}}$CCCLXXXV. 37. 192, 2895. 38. 11658794. 39. 333.
40. 105214 and 210428.

III. A. Pages 44—45.

1.	18.	2.	21.	3.	31.	4.	42.	5.	39.	6.	44.
7.	35.	8.	46.	9.	28.	10.	38.	11.	23.	12.	49.
13.	67.	14.	79.	15.	93.	16.	98.	17.	113.	18.	131.
19.	118.	20.	138.	21.	50.	22.	76.	23.	118.	24.	92.
25.	154.			26.	139.	27.	174.			28.	192.
29.	774.			30.	1353.	31.	1856.			32.	1655.
33.	2350.			34.	2145.	35.	3057.			36.	6215.
37.	7906.			38.	14375.	39.	37822.			40.	52319.
41.	1073.			42.	5118.	43.	4628.			44.	7575.
45.	17087.			46.	26730.	47.	110965.			48.	209263.
49.	153213.			50.	367920.	51.	23280.			52.	226920.

III. B. Page 45.

1. $3\frac{1}{4}d.$, $3\frac{2}{4}d.$, $4\frac{3}{4}d.$, $5\frac{3}{4}d.$, $6\frac{3}{4}d.$, $8d.$
2. $9\frac{1}{4}d.$, $9\frac{3}{4}d.$, $10\frac{1}{4}d.$, $1s.\ 0\frac{1}{4}d.$, $1s.\ 1\frac{1}{4}d.$, $1s.\ 3\frac{3}{4}d.$

3. 1*s.* 5*d.*, 1*s.* 11*d.*, 2*s.* 5*d.*, 2*s.* 11*d.*, 3*s.* 1*d.*, 3*s.* 7*d.*
4. 4*s.* 9*d.*, 5*s.* 5*d.*, 5*s.* 10*d.*, 6*s.* 7*d.*, 6*s.* 10*d.*, 7*s.* 5*d.*
5. 7*s.* 9*d.*, 8*s.* 3*d.*, 8*s.* 9*d.*, 10*s.* 7*d.*, 11*s.* 3*d.*, 12*s.* 5*d.*
6. £1. 15*s.*, £2. 7*s.*, £2. 14*s.*, £2. 19*s.*, £3. 7*s.*, £3. 13*s.*
7. £3. 19*s.*, £4. 2*s.*, £4. 8*s.*, £4. 13*s.*, £5. 5*s.*, £6.
8. £6. 16*s.*, £7. 9*s.*, £7. 17*s.*, £8. 4*s.*, £8. 13*s.*, £9. 15*s.*

9.	3*s.* 5¾*d.*	10.	11*s.* 11½*d.*	11.	£1. 0*s.* 10*d.*
12.	£1041. 13*s.* 4*d.*	13.	£4. 3*s.* 4*d.*	14.	£20. 16*s.* 8*d.*
15.	£17. 4*s.* 0½*d.*	16.	£114. 1*s.* 3*d.*	17.	£141. 1*s.*
18.	£45. 17*s.* 6*d.*	19.	£196. 12*s.* 6*d.*	20.	£3314. 17*s.*

III. C. Pages 46—47.

1.	1*s.* 8¾*d.*	2.	2*s.* 4¾*d.*	3.	2*s.* 2¾*d.*	4.	2*s.* 5*d.*
5.	£1. 9*s.* 1*d.*	6.	£1. 16*s.* 7*d.*	7.	£2. 0*s.* 8*d.*	8.	£2. 3*s.* 7*d.*
9.	£22. 4*s.* 9*d.*	10.	£67. 5*s.* 10*d.*	11.	£85. 19*s.* 8*d.*		
12.	£47. 6*s.* 0*d.*	13.	£62. 18*s.* 0*d.*	14.	£43. 5*s.* 10½*d.*		
15.	£112. 15*s.* 10¾*d.*	16.	£189. 10*s.* 10*d.*	17.	£33. 6*s.* 8*d.*		
18.	£3076. 5*s.* 2¼*d.*	19.	£3092. 14*s.* 4¼*d.*	20.	£5704. 5*s.* 9¾*d.*		
21.	£1944. 3*s.* 4½*d.*	22.	£606. 17*s.* 5¼*d.*	23.	£1901. 6*s.* 9½*d.*		
24.	£3094. 6*s.* 7¼*d.*	25.	£7085. 5*s.* 3¾*d.*	26.	£6122. 13*s.* 4¾*d.*		

III. D. Page 49.

1.	£41. 2*s.* 5*d.*	2.	£9. 2*s.* 11*d.*	3.	£3. 5*s.* 8¼*d.*
4.	£13. 2*s.* 8¾*d.*	5.	£19. 8*s.* 3½*d.*	6.	£31. 10*s.* 8½*d.*
7.	£3. 12*s.* 4½*d.*	8.	£69. 13*s.* 10½*d.*	9.	£199. 11*s.* 9¾*d.*
10.	£62. 16*s.* 9¾*d.*	11.	£3454. 12*s.* 8½*d.*	12.	£3254. 7*s.* 7¼*d.*
13.	£2. 7*s.* 2*d.*	14.	£47. 16*s.* 11¾*d.*	15.	£49. 3*s.* 7¾*d.*
16.	£36. 17*s.* 2*d.*	17.	£30. 14*s.* 10*d.*	18.	£7. 18*s.*
19.	1*s.*	20.	3*s.* 9*d.*	21.	£3. 13*s.* 8¼*d.*
22.	£45. 19*s.* 1¾*d.*	23.	£265. 2*s.* 11*d.*		

III. E. Pages 50—51.

1.	9*s.* 4*d.*	2.	£1. 1*s.* 6*d.*	3.	£2. 5*s.* 4*d.*
4.	£11. 3*s.* 4*d.*	5.	£21. 14*s.*	6.	£39. 2*s.* 10*d.*
7.	£113. 5*s.*	8.	£147. 17*s.* 6*d.*	9.	£95. 12*s.* 2*d.*
10.	£8. 14*s.* 2*d.*	11.	£20. 6*s.* 7½*d.*	12.	£67. 19*s.* 2*d.*
13.	£152. 15*s.* 6*d.*	14.	£223. 19*s.* 11¼*d.*		
15.	£113. 18*s.* 7½*d.*,	£189. 17*s.* 8½*d.*,	£265. 16*s.* 9½*d.*,	£417. 14*s.* 11½*d.*	
16.	5*s.* 4¾*d.*	17.	14*s.* 9¾*d.*	18.	£4. 2*s.* 6*d.*

19. £17. 16*s*. 9¾*d*. 20. £58. 18*s*. 6½*d*. 21. £71. 10*s*. 9¾*d*.
22. £32. 11*s*. 8*d*. 23. £37. 3*s*. 9*d*. 24. £114. 0*s*. 3*d*.
25. £185. 17*s*. 26. £703. 5*s*. 3¾*d*. 27. £354. 14*s*. 2*d*.
28. £684. 14*s*. 7½*d*. 29. £1195. 13*s*. 4*d*. 30. £597. 14*s*. 2*d*.
31. £1773. 12*s*. 11*d*. 32. £2573. 10*s*. 10*d*. 33. £2384. 3*s*. 4*d*.
34. £1867. 14*s*. 2*d*. 35. £7771. 17*s*. 6*d*.

III. F. Page 52.

1. £5. 8*s*. 9*d*. 2. £10. 6*s*. 7*d*. 3. £19. 3*s*. 7½*d*.
4. £37. 0*s*. 2½*d*. 5. £82. 2*s*. 6*d*. 6. £153. 5*s*. 3*d*.
7. £519. 0*s*. 8*d*. 8. £452. 8*s*. 9*d*. 9. £1388. 10*s*. 5*d*.
10. £2676. 15*s*. 11*d*. 11. £332. 3*s*. 5¾*d*. 12. £458. 6*s*. 7½*d*.
13. £6476. 18*s*. 7¼*d*. 14. £13723. 6*s*. 10½*d*. 15. £1517. 5*s*. 1½*d*.
16. £943. 14*s*. 7¼*d*. 17. £17375. 17*s*. 2¼*d*. 18. £31300. 19*s*. 4½*d*.
19. £3701. 16*s*. 7½*d*. 20. £3121. 15*s*. 7½*d*. 21. £3640. 14*s*. 7¾*d*.
22. £4469. 7*s*. 1¾*d*. 23. £33516. 5*s*. 2¼*d*. 24. £60398. 7*s*. 9*d*.
25. £10769. 4*s*. 5*d*. 26. £16109. 4*s*. 6¾*d*. 27. £65476. 13*s*. 1*d*.
28. £50611. 7*s*. 1½*d*.

III. G. Page 53.

1. 10*d*. 2. 3*d*. 3. 3*s*. 4*d*. 4. 4*s*. 2*d*. 5. 15*s*. 10*d*.
6. £1. 14*s*. 2*d*. 7. 6*s*. 6¾*d*. 8. 12*s*. 5¾*d*. 9. 14*s*. 0¾*d*.
10. £7. 18*s*. 2¼*d*. 11. £6. 7*s*. 3¼*d*. 12. £16. 3*s*. 4¼*d*.
13. £15. 15*s*. 6¼*d*. 14. £8. 16*s*. 9¼*d*. 15. £149. 17*s*. 10*d*.
16. £16. 16*s*. 7¾*d*. 17. £18. 5*s*. 9¾*d*. 18. £215. 17*s*. 4¼*d*.
19. £147. 12*s*. 4½*d*. 20. £35. 18*s*. 9¾*d*. 21. £5. 16*s*. 7¼*d*.
22. £15. 19*s*. 10½*d*. 23. £13. 16*s*. 10½*d*. 24. £4. 0*s*. 5¼*d*.
25. £16. 17*s*. 7¼*d*. 26. £5. 9*s*. 10¼*d*. 27. £26. 13*s*. 4¼*d*.
28. £432. 6*s*. 2½*d*. 29. £16716. 12*s*. 7¾*d*. 30. £1100. 11*s*. 9¼*d*.
31. £13. 14*s*. 2½*d*. 32. £16. 17*s*. 4¾*d*. 33. £172. 1*s*. 0½*d*.
34. £1. 6*s*. 7¼*d*.

III. H. Pages 54—55.

1. 12. 2. 10. 3. 11. 4. 7. 5. 25.
6. 40. 7. 127. 8. 357. 9. 125. 10. 97.
11. 98. 12. 60. 13. 84. 14. 356. 15. 565.
16. 156. 17. 49, rem. 1*s*. 18. 1752, rem. £1. 7*s*. 11¼*d*.
19. 36. 20. 322. 21. 111, rem. 4½*d*. 22. 44.
23. 607. 24. 19. 25. 29.

III. I. Page 56.

1. 2334. 2. 8108. 3. 22330. 4. 43524.
5. 503. 6. 643. 7. 3970. 8. 4386.
9. 71. 10. 47. 11. 947. 12. 2835.
13. 17812. 14. 11466. 15. 38175. 16. 108380.
17. 636. 18. 1464035. 19. 454807. 20. 210862.
21. 12782. 22. 803539. 23. 13 cwt. 6 lbs.
24. 6 tons 1 cwt. 22 lbs. 25. 5 tons 3 cwt. 2 qrs. 5 lbs.
26. 1 ton 11 cwt. 1 qr. 13 lbs. 6 oz.
27. 1 ton 12 cwt. 1 qr. 1 lb. 4 oz.
28. 2 cwt. 3 qrs. 12 lbs. 3 oz. 6 drs.
29. 1 a. 3 r. 35 p. 30. 32 a. 0 r. 22 p. 31. 9 a. 3 r. 14 p.
32. 23 a. 1 r. 5 p. 33. 4 yds. 1 ft. 34. 9 yds. 1 ft. 6 in.
35. 11 yds. 1 ft. 9 in. 36. 32 yds. 2 ft. 3 in.
37. 2 lbs. 1 oz. 17 dwts. 38. 2 lbs. 4 oz. 14 dwts.
39. 6 oz. 11 dwts. 12 grs. 40. 8 oz. 15 dwts. 15 grs.
41. 2 lbs. 3 oz. 18 dwts. 15 grs. 42. 4 lbs. 3 oz. 9 dwts. 13 grs.
43. 6 lbs. 2 oz. 17 dwts. 9 grs. 44. 9 lbs. 4 dwts. 4 oz.
45. 2 bus. 1 pk. 1 gal. 2 qts. 46. 23 bus. 47. 13 bus.
48. 29 bus. 1 pk. 1 qt. 1 pt. 49. 1 sq. yd. 1 sq. ft. 22 sq. in.
50. 1 sq. yd. 5 sq. ft. 140 sq. in. 51. 2 sq. yd. 6 sq. ft. 108 sq. in.
52. 8 hrs. 46 min. 12 secs. 53. 2 d. 23 hrs. 34 min. 32 secs.
54. 82 d. 15 hrs. 56 min. 9 secs.

III. J. Pages 57—58.

1. 172. 2. 1122. 3. 2531. 4. 11828.
5. 1142784. 6. 1764877. 7. 11272. 8. 1185875.
9. 17424. 10. 76956. 11. 22171860. 12. 651412.
13. 3 mi. 7 fur. 28 p. 14. 12 mi. 2 fur. 2 p.
15. 8 mi. 7 fur. 21 p. 1 yd. 1 ft. 6 in.
16. 1 mi. 1 fur. 22 p. 4 yds. 17. 6 fur. 20 p. 3 yds. 2 ft. 9 in.
18. 2 mi. 4 fur. 1 p. 4 yds. 1 in. 19. 4 a. 1 r. 8 p.
20. 7 a. 3 r. 36 p. 21. 3 r. 11 p. 28 sq. yds. 2 sq. ft. 100 sq. in.
22. 1 a. 1 r. 8 p. 1 sq. yd. 3 sq. ft. 19 sq. in.

III. K. Page 59.

1. £403. 4*s*. 2. 7000. 3. 170. 4. 735.
5. 960. 6. 524. 7. 43 lbs. 9 oz. 8. 875 lbs.
9. 170 lbs. 1 oz. 13 dwts. 8 grs. 10. 13 lbs. 11 oz. 14 dwts. 4 grs.
11. 24 lbs. 12. 3 lbs. 8 oz.

III. L. Pages 59—60.

1. 28 d. 8 hrs. 33 min.
2. 20 hrs. 50 min. 49 secs.
3. 36 d. 14 hrs. 8 min. 52 secs.
4. 18 cwt. 1 qr. 11 lbs.
5. 22 lbs. 0 oz. 1 dr.
6. 26 tons 15 cwt. 1 qr. 9 lbs.
7. 1 qr. 7 lbs. 10 oz. 13 drs.
8. 275 tons 2 cwt. 2 qrs. 11 lbs.
9. 2 tons 10 cwt. 2 qrs. 24 lbs. 6 oz.
10. 31 lbs. 0 oz. 16 dwts.
11. 1 lb. 5 oz. 6 dwts. 14 grs.
12. 20 lbs. 3 oz. 13 dwts. 20 grs.
13. 30 yds. 1 ft. 9 in.
14. 63 yds. 2 ft. 1 in.
15. 105 mi. 26 p.
16. 91 sq. yds. 6 sq. ft. 36 sq. in.
17. 221 sq. yds. 1 sq. ft. 65 sq. in.
18. 258 sq. yds. 4 sq. ft. 5 sq. in.
19. 135 a. 3 r. 37 p.
20. 121 a. 3 r. 17 p.
21. 154 a. 1 r. 23 p.
22. 72 gals. 1 qt. 1 pt.
23. 452 qrs. 1 pk.
24. 278 qrs. 5 bus. 1 pk. 1 gal.
25. 60 mi. 2 fur. 7 p. 3 yds. 2 ft. 4 in.
26. 340 a. 2 r. 14 p. $21\frac{1}{2}$ sq. yds. [$21\frac{1}{2}$ sq. yds. = 21 sq. yds. 4 ft. 72 in.]

III. M. Pages 60—61.

1. 6 d. 1 hr. 48 min.
2. 15 d. 15 hrs. 57 min. 58 secs.
3. 2 cwt. 26 lbs.
4. 9 lbs. 14 oz. 10 drs.
5. 2 tons 18 cwt. 19 lbs. 14 oz.
6. 5 lbs. 8 oz. 15 dwts.
7. 12 lbs. 7 oz. 13 dwts. 3 grs.
8. 4 yds. 1 ft. 2 in.
9. 11 yds. 1 ft. 5 in.
10. 10 mi. 5 fur. 206 yds.
11. 11 sq. yds. 4 sq. ft. 61 sq. in.
12. 96 sq. yds. 5 sq. ft. 116 sq. in.
13. 13 a. 2 r. 35 p.
14. 97 a. 2 r. 34 p.
15. 3 gals. 2 qts. 1 pt.
16. 47 qrs. 6 bus. 3 pks.
17. 10 mi. 3 fur. 26 p. 3 yds. 1 ft. 6 in.
18. 4 mi. 6 fur. 27 p. 2 yds. 6 in.
19. 12 a. 2 r. 32 p. $22\frac{1}{4}$ sq. yds. [$22\frac{1}{4}$ sq. yds. = 22 sq. yds. 2 ft. 36 in.]
20. 14 a. 0 r. 23 p. $15\frac{1}{4}$ sq. yds. [$15\frac{1}{4}$ sq. yds. = 15 sq. yds. 2 ft. 36 in.]

III. N. Pages 61—62.

1. 25 hrs. 52 min. 45 secs., 36 hrs. 13 min. 51 secs., 46 hrs. 34 min. 57 secs.

2. 37 cwt. 2 qrs. 14 lbs., 43 cwt., 48 cwt. 1 qr. 14 lbs.

3. 27 tons 10 cwt. 1 qr. 5 lbs. 9 oz. 4 drs., 41 tons 5 cwt. 1 qr. 22 lbs. 5 oz. 14 drs., 61 tons 18 cwt. 0 qr. 19 lbs. 8 oz. 13 drs.

4. 44 lbs. 5 oz. 19 dwts. 4 grs., 62 lbs. 3 oz. 10 dwts. 20 grs., 106 lbs. 9 oz. 10 dwts.

5. 84 yds. 8 in., 115 yds. 2 ft. 5 in., 126 yds. 1 ft.

6. 42 mi. 3 fur. 195 yds., 67 mi. 7 fur. 180 yds., 101 mi. 7 fur. 160 yds.

7. 95 sq. yds. 1 sq. ft. 24 sq. in., 174 sq. yds. 3 sq. ft. 92 sq. in.

8. 136 a. 10 p., 163 a. 1 r. 4 p.

9. 82 qrs. 1 bus. 2 pks., 180 qrs. 6 bus. 2 pks.

10. 18 gals. 1 pt., 25 gals. 1 qt. 1 pt.

11. 130 d. 58 min. 45 secs., 167 d. 4 hrs. 41 min. 15 secs.

12. 656 tons 0 cwt. 2 qrs., 1124 tons 12 cwt. 1 qr. 4 lbs.

13. 440 lbs. 1 oz. 11 dwts. 18 grs., 754 lbs. 6 oz. 3 dwts.

14. 173 yds. 1 ft. 8 in., 520 yds. 2 ft.

15. 1957 a. 3 r. 10 p.

16. 3648 qrs. 6 bus.

17. 815 d. 20 hrs. 23 min.

18. 504 tons 14 cwt. 10 lbs.

19. 419 lbs. 9 oz. 10 dwts.

20. 2891 a. 3 r. 25 p.

21. 19199 sq. yds. 3 sq. ft. 24 sq. in.

22. 15838 tons 10 cwt. 1 qr. 25 lbs. 15 oz.

23. 7507 lbs. 4 oz. 7 dwts.

24. 6 mi. 3 fur. 38 p. 3 yds. 1 ft. 6 in., 11 mi. 5 fur. 21 p. 3 yds.

25. 29 a. 1 r. 4 p. 7 sq. yds., 43 a. 3 r. 26 p. $10\frac{1}{2}$ sq. yds.

26. 92 mi. 3 fur. 25 p. 3 yds. 1 ft., 203 mi. 3 fur. 8 p. 1 yd. 2 ft. 6 in.

27. 107 a. 3 r. 30 p. 8 yds. 3 ft.

28. 7121 mi. 5 fur. 33 p. 4 yds. 1 ft. 6 in.

III. O. PAGES 62—63.

1. 3 d. 16 hrs. 12 min.

2. 5 cwt. 1 qr. 17 lbs.

3. 5 lbs. 8 oz. 5 drs.

4. 5 lbs. 10 oz. 10 dwts. 8 grs.

5. 13 yds. 2 ft. 11 in.

6. 7 a. 3 r. 32 p.

7. 3 lbs. 5 oz. 5 dwts. 12 grs.

8. 2 d. 11 hrs. 15 min. 18 secs.

9. 7 yds. 2 ft. 11 in.

10. 2 a. 0 r. 13 p.

11. 3 qrs. 4 lbs. 4 oz. 15 drs.

12. 1 ton 1 cwt. 3 qrs. 16 lbs. 8 oz.

13. 6 cwt. 2 qrs. 9 lbs.

14. 6 tons 11 cwt. 3 qrs. 1 lb.

15. 3 cwt. 2 qrs. 14 lbs. 7 oz.

16. 9 in.

17. 2 cwt. 1 qr. 20 lbs. 2 oz.

18. 1 qr. 3 pks. 3 qts.

19. 1 qr. 19 lbs. 4 oz.

20. 34 yards 10 inches.

21. 3 mi. 5 fur. 10 p. 2 yds.

22. 17 a. 1 r. 23 p. 11 yds.

23. 1 mi. 2 fur. 16 p. 2 yds. 1 ft.

24. 4 a. 0 r. 14 p. 20 yds. 5 ft.

25. £39. 11*s.* 6*d.*; £6. 11*s.* 11*d.*

26. 5 ft. 1 in.

27. 11 st. 4 lbs.

III. P. Page 63.

1. 5. 2. 10. 3. 30. 4. 9. 5. 54.
6. 48. 7. 4400. 8. 37. 9. 6336. 10. 16.
11. 279, £3. 1*s*. 3*d*. 12. 248. 13. 135. 14. 15.
15. 5280. 16. 160.

MISCELLANEOUS EXAMPLES. II. Pages 64—66.

1. 79. 2. 1014. 4. £11. 6*s*. 5. 64. 6. 909.
7. 178014648555. 8. 52. 9. 175. 10. 119.
11. 7544676, 2038. 12. 2552. 13. £8. 6*s*. 10*d*.
15. £11. 7*s*. 3*d*. 16. 317450. 17. 992, 124.
18. The 201th. 19. 576 glasses, 30*s*. profit. 20. 17.
21. 3174, 3746. 22. 50. 23. 71 p.
24. £19. 2*s*. 6*d*. 25. 124.
26. XCIX, DLXIV, MDCCXLIX, IↃↃCIↃCCLIX or $\overline{\text{V}}$ICCLIX.
27. 8151. 28. 999663, 100203. 29. 400, rem. $8\frac{1}{2}d$.
30. 37. 31. 5925, 3344. 32. 1925. 33. 9.
34. £3. 8*s*. $11\frac{1}{2}d$. 35. 36 yards, 8*s*. 36. 24022745.
37. 270. 38. 44. 39. 24 lbs. 40. 77. 41. 6119.
42. 258. 43. 22. 44. £44. 14*s*. 10*d*. 45. 35.

IV. A. Page 68.

1. 2, 4. 2. 3, 9, 5, 25. 3. 5, 25, 11. 4. 2, 3, 9, 11.
5. 3, 9, 5, 25. 6. 3, 5, 25, 125, 11. 7. 2, 4, 8, 3, 9, 5.
8. 2, 4, 8, 3, 9, 11. 9. 2, 4, 3, 11. 10. 3, 5, 25, 125.
11. 2, 3, 9, 5, 11. 12. 2, 4, 8, 3, 11.

IV. B. Page 72.

1. 2×3, 3×3, 2×5, 3×5, $3\times2\times2\times2$, $2\times3\times5$, $2\times2\times3\times3$, 3×13, $3\times3\times5$, $2\times2\times2\times2\times3$.
2. 7×7, $2\times5\times5$, $2\times3\times3\times3$, $2\times2\times3\times5$, $2\times2\times17$, $2\times5\times7$, $3\times5\times5$, 7×11, $3\times3\times3\times3$, 2×43.
3. $2\times2\times5\times5$, $3\times5\times7$, $2\times2\times2\times3\times5$, $2\times5\times13$, $2\times2\times2\times17$, $2\times2\times37$, $2\times7\times11$, $2\times2\times3\times13$.
4. $2^3\times23$, $2^2\times7^2$, $3\times7\times13$, $5^2\times11$, $2^2\times7\times13$, 17×23, $3\times5^2\times7$, $5\times3^2\times13$.

5. $2 \times 5^2 \times 7$, $3^3 \times 17$, $5 \times 11 \times 13$, $2^3 \times 7 \times 13$, $2 \times 3 \times 5^3$, $2^3 \times 3^2 \times 11$, $3 \times 5^2 \times 11$, $3^3 \times 37$.

6. $2^2 \times 3 \times 7 \times 13$, $2 \times 7 \times 11 \times 17$, 11×601, $3 \times 5 \times 7 \times 37$, $7 \times 13 \times 41$, $3 \times 7 \times 11 \times 13 \times 17$, $3 \times 7 \times 11 \times 17 \times 19$, $2 \times 3 \times 7^2 \times 11^2 \times 13$.

IV. C. Page 73.

1.	6.	2.	5.	3.	6.	4.	6.	5.	12.	6.	18.
7.	15.	8.	18.	9.	63.	10.	21.	11.	12.	12.	21.
13.	63.		14.	132.		15.	33.		16.	12.	
17.	36.		18.	24.		19.	21.				

IV. D. Page 76.

1.	13.	2.	17.	3.	29.	4.	13.	5.	41.	6.	35.
7.	1.	8.	71.	9.	73.	10.	107.	11.	131.	12.	13.
13.	6214.			14.	214.			15.	112.		

IV. E. Page 76.

1.	11.	2.	13.	3.	2.	4.	31.	5.	41.	6.	91.

IV. F. Page 78.

1.	48.	2.	100.	3.	150.	4.	120.	5.	24.
6.	60.	7.	270.	8.	70.	9.	252.	10.	1620.
11.	396.	12.	7425.	13.	7920.	14.	5040.	15.	168.
16.	720.	17.	6300.	18.	5292.	19.	2520.	20.	36720.

IV. G. Page 78.

1.	8211.	2.	34891.	3.	52455.	4.	406617.
5.	10890.	6.	3432.	7.	54000.	8.	30030.
9.	44100.	10.	240240.				

IV. H. Page 79.

1.	5*d.*, 14*s.* 7*d.*	2.	8*d.*, £1. 6*s.* 8*d.*
3.	1*s.* 2*d.*, £4. 9*s.* 10*d.*	4.	1*s.* 3*d.*, £15. 8*s.* 9*d.*
5.	1 ft. 2 in., 25 yds. 2 ft.	6.	11 in., 30 yds. 9 in.
7.	15 lbs., 27 cwt. 1 qr. 8 lbs.	8.	9 lbs., 27 cwt.
9.	2 qts. 1 pt., 73 gals. 1 pt.	10.	11*d.*, £45. 17*s.* 7*d.*

MISCELLANEOUS EXAMPLES. III. Pages 79—82.

1. 88976. 2. 2629440. 3. £2608. 4. £175, £35.
5. £3150. 6. 231. 7. 2744. 8. 1 m. 4 f. 2 p. 2 yds. 2 ft. 4 in.
9. £19. 16*s*. 8*d*. 10. 20583. 11. £3, 40 boys.
12. 187. 13. 23. 14. £7. 6*s*. 5*d*. 15. 796110.
16. 366. 17. 35. 18. 1 minute. 19. 236912 miles.
20. £3646. 19*s*. 9*d*., £93. 10*s*. 3*d*. 21. 230358.
22. 7207200. 23. 4672 sovereigns 1 half-sovereign.
24. 62, 122, 182. 25. 3170. 26. 673917764.
27. 1 ton 14 cwt. 3 qrs. 14 lbs. 4 oz. 28. 468.
29. $1176 = 2^3 \times 3 \times 7^2$, $19404 = 2^2 \times 3^2 \times 7^2 \times 11$.
G.C.M. $= 2^2 \times 3 \times 7^2$, L.C.M. $= 2^3 \times 3^2 \times 7^2 \times 11$.
30. 7 yds. 2 ft. 31. 1022. 32. 143 lbs. 10 oz. 6 dwts. 1 gr.
33. 1*d*. 34. £486. 4*s*. 35. 960. 36. 1 ft. 4 in.
37. 31300. 38. 772250, 21334976047872. 39. 625, 21999.
40. £7572. 18*s*. $3\frac{1}{4}d$. 41. 7392, 720. 42. 3*s*. 1*d*.
43. 52. 44. 87544, 509040. 45. 100.
46. £420, £520, £620. 47. 1234. 48. 143.
49. 6. 50. 9928, 1022. 51. £4229. 3*s*. 4*d*.
52. 1111. 53. £61. 17*s*. 6*d*. 54. 7560, 15.
55. 225134910, divisible by 2, 3, 5, 6, 7, 9, 10, 11.
56. £7. 1*s*. 9*d*. 57. 10772245. 58. Monday, Tuesday.
59. 223, 2849. 60. 2520 miles.

V. A. Page 84.

1. $\frac{5}{9}, \frac{6}{11}, \frac{11}{23}, \frac{16}{27}, \frac{17}{91}, \frac{95}{104}$.

2. Three-sevenths, five-eighths, two-elevenths, nine twenty-thirds, five thirty-oneths, seventeen fifty-sixths, seven eighty-oneths, ninety one hundred and elevenths, fifteen two hundred and nineteenths, one hundred and nineteen two-thousandths.

3. 8*s*. 4. 9*d*. 5. 6*s*. 8*d*. 6. 4*s*. 8*d*. 7. 1*s*. 8*d*.
8. 1*s*. 9. 15*s*. 10. £1. 11*s*. 6*d*. 11. 2*s*. 4*d*.
12. 14 lbs. 13. 6 lbs. 14. 24 lbs. 15. 240 yds.
16. 1 rood 8 poles. 17. 1 gal. 1 qt. 18. 3 pecks.
19. 6 dwts. 6 grs. 20. 1 hr. 24 min.
21. 11 min. 15 secs. 22. 2 hrs. 12 min.

V. B. Page 85.

1. 3, 4, 5, 6.
2. $3\frac{1}{3}$, $3\frac{1}{4}$, $3\frac{2}{5}$, $4\frac{1}{6}$.
3. $1\frac{6}{11}$, $3\frac{8}{9}$, $8\frac{3}{10}$, $6\frac{7}{12}$.
4. $2\frac{1}{17}$, $2\frac{4}{15}$, $2\frac{8}{21}$, $2\frac{2}{29}$.
5. $4\frac{7}{27}$, $4\frac{19}{29}$, $4\frac{20}{31}$, $4\frac{15}{34}$.
6. $3\frac{89}{111}$, $30\frac{6}{17}$, $8\frac{29}{41}$, $8\frac{11}{101}$.
7. $\frac{21}{7}$, $\frac{35}{7}$, $\frac{63}{7}$, $\frac{91}{7}$.
8. $\frac{3}{2}$, $\frac{7}{4}$, $\frac{7}{3}$, $\frac{16}{5}$.
9. $\frac{37}{7}$, $\frac{8}{5}$, $\frac{31}{9}$, $\frac{47}{11}$.
10. $\frac{78}{10}$, $\frac{77}{12}$, $\frac{93}{13}$, $\frac{48}{31}$.
11. $\frac{79}{19}$, $\frac{91}{17}$, $\frac{338}{113}$, $\frac{1495}{212}$.
12. £4. 10*s*. 8*d*.
13. £48. 3*s*. 4*d*.
14. £7. 11*s*. $0\frac{1}{2}d$.
15. 6 yds. 2 ft. 8 in.
16. 2 cwt. 3 qrs. 12 lbs.
17. 6 lbs. 9 oz. 11 dwts. 15 grs.

V. C. Page 87.

1. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{2}{5}$, $\frac{5}{8}$, $\frac{4}{5}$.
2. $\frac{2}{3}$, $\frac{7}{9}$, $\frac{8}{11}$, $\frac{3}{5}$, $\frac{7}{8}$, $\frac{2}{5}$, $\frac{1}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{2}{3}$.
3. $\frac{12}{13}$.
4. $\frac{8}{9}$.
5. $\frac{3}{7}$.
6. $\frac{11}{13}$.
7. $\frac{8}{17}$.
8. $\frac{3}{11}$.
9. $\frac{13}{15}$.
10. $\frac{10}{11}$.
11. $\frac{5}{9}$.
12. $\frac{13}{15}$.
13. $\frac{9}{11}$.
14. $\frac{13}{38}$.
15. $\frac{311}{410}$.
16. $\frac{2}{3}$.
17. $\frac{101}{111}$.
18. $\frac{3457}{112252}$.

V. D. Pages 88—89.

1. $\frac{4}{8}$, $\frac{6}{8}$, $\frac{5}{8}$.
2. $\frac{18}{27}$, $\frac{12}{27}$, $\frac{8}{27}$.
3. $\frac{4}{20}$, $\frac{6}{20}$, $\frac{7}{20}$.
4. $\frac{30}{36}$, $\frac{21}{36}$, $\frac{22}{36}$.
5. $\frac{160}{360}$, $\frac{168}{360}$, $\frac{81}{360}$.
6. $\frac{75}{210}$, $\frac{110}{210}$, $\frac{144}{210}$.
7. $\frac{245}{280}$, $\frac{240}{280}$, $\frac{224}{280}$.
8. $\frac{140}{240}$, $\frac{165}{240}$, $\frac{156}{240}$.
9. $\frac{70}{105}$, $\frac{63}{105}$, $\frac{75}{105}$.
10. $\frac{165}{198}$, $\frac{176}{198}$, $\frac{162}{198}$.
11. $\frac{99}{144}$, $\frac{104}{144}$, $\frac{114}{144}$.
12. $\frac{176}{216}$, $\frac{171}{216}$, $\frac{174}{216}$.

13. $\frac{96}{180}, \frac{92}{180}, \frac{93}{180}$. 14. $\frac{450}{630}, \frac{140}{630}, \frac{315}{630}, \frac{405}{630}, \frac{336}{630}$.

15. $\frac{50}{240}, \frac{88}{240}, \frac{126}{240}, \frac{145}{240}$. 16. $\frac{240}{420}, \frac{294}{420}, \frac{245}{420}, \frac{228}{420}$.

17. $\frac{135}{360}, \frac{150}{360}, \frac{140}{360}, \frac{132}{360}$. 18. $\frac{216}{252}, \frac{147}{252}, \frac{238}{252}, \frac{228}{252}, \frac{225}{252}$.

19. $\frac{30}{48}, \frac{30}{51}, \frac{30}{46}$. 20. $\frac{144}{204}, \frac{141}{171}, \frac{144}{184}$. 21. $\frac{180}{444}, \frac{180}{430}, \frac{180}{441}$.

V. E. Page 90.

1. 1. 2. $\frac{1}{2}$. 3. $\frac{2}{3}$. 4. $1\frac{2}{11}$. 5. $1\frac{4}{9}$. 6. $\frac{3}{2}$.

7. 3. 8. $3\frac{1}{6}$. 9. $1\frac{1}{4}$. 10. $\frac{7}{8}$. 11. $1\frac{1}{9}$. 12. $1\frac{7}{12}$.

13. $1\frac{17}{24}$. 14. $\frac{25}{42}$. 15. $\frac{53}{80}$. 16. $\frac{59}{90}$. 17. $\frac{47}{84}$.

18. $4\frac{3}{10}$. 19. $7\frac{17}{24}$. 20. $8\frac{5}{18}$. 21. $6\frac{13}{18}$. 22. $4\frac{11}{12}$.

23. $9\frac{131}{216}$. 24. $1\frac{4}{5}$. 25. $1\frac{181}{315}$. 26. $2\frac{29}{60}$. 27. $2\frac{151}{360}$.

28. $1\frac{17}{420}$. 29. $1\frac{37}{108}$. 30. $16\frac{51}{80}$. 31. $19\frac{47}{630}$. 32. $11\frac{7}{30}$.

33. $18\frac{293}{360}$. 34. $14\frac{37}{420}$. 35. $7\frac{866}{8211}$. 36. $14\frac{176588}{953897}$.

37. $20\frac{1134}{5445}$. 38. $29\frac{71}{1144}$.

V. F. Page 91.

1. $\frac{1}{6}$. 2. $\frac{1}{4}$. 3. $\frac{1}{4}$. 4. $\frac{1}{4}$. 5. $\frac{5}{24}$. 6. $\frac{9}{80}$.

7. $\frac{5}{42}$. 8. $\frac{4}{27}$. 9. $\frac{1}{48}$. 10. $\frac{1}{35}$. 11. $\frac{85}{144}$. 12. $\frac{11}{84}$.

13. $\frac{11}{60}$. 14. $\frac{1}{130}$. 15. $\frac{1}{340}$. 16. $\frac{3}{76}$. 17. $1\frac{1}{4}$. 18. $2\frac{9}{14}$.

19. $\frac{61}{180}$. 20. $1\frac{4}{9}$. 21. $2\frac{7}{30}$. 22. $7\frac{1}{6}$. 23. $1\frac{15}{154}$. 24. $1\frac{8}{225}$.

25. $3\frac{5}{16}$. 26. $1\frac{1}{12}$. 27. $10\frac{167}{180}$. 28. $4\frac{8}{45}$. 29. $1\frac{100}{187}$.

30. $\frac{153}{168}$. 31. $8\frac{6888}{6083}$. 32. $1\frac{109}{126}$. 33. $3\frac{199}{240}$. 34. $6\frac{5}{156}$.

35. $16\frac{2}{45}$. 36. $12\frac{3326}{34185}$. 37. $1\frac{5}{8}$. 38. $5\frac{17}{48}$.

39. $\frac{23}{80}$. 40. $8\frac{88}{108}$. 41. $3\frac{239}{636}$. 42. $1\frac{157}{561}$. 43. $8\frac{6239}{10000}$.

V. G. Page 93.

1. $\frac{4}{7}$. 2. $\frac{15}{19}$. 3. $\frac{20}{21}$. 4. $\frac{12}{17}$. 5. $1\frac{5}{7}$. 6. $2\frac{11}{12}$.
7. $2\frac{2}{5}$. 8. $3\frac{9}{11}$. 9. $4\frac{2}{3}$. 10. $2\frac{2}{3}$. 11. $1\frac{7}{8}$. 12. $4\frac{2}{3}$.
13. $16\frac{1}{2}$. 14. $53\frac{3}{4}$. 15. $188\frac{1}{3}$. 16. $147\frac{3}{10}$. 17. $\frac{2}{5}$.
18. $\frac{2}{15}$. 19. $\frac{2}{25}$. 20. $\frac{3}{25}$. 21. $\frac{4}{15}$. 22. $\frac{8}{75}$.
23. $\frac{6}{125}$. 24. $\frac{24}{175}$. 25. $\frac{5}{32}$. 26. $\frac{3}{50}$. 27. $\frac{6}{245}$.
28. $\frac{5}{81}$. 29. $\frac{1}{5}$. 30. $1\frac{1}{5}$. 31. $1\frac{1}{7}$. 32. $1\frac{1}{5}$.
33. $1\frac{3}{40}$. 34. $1\frac{13}{54}$. 35. $\frac{23}{42}$. 36. $\frac{41}{80}$. 37. $9\frac{31}{63}$.
38. $30\frac{7}{12}$. 39. $9\frac{23}{32}$. 40. $10\frac{67}{375}$.

V. H. Page 95.

1. $\frac{1}{6}$. 2. $\frac{10}{21}$. 3. $\frac{9}{40}$. 4. $\frac{20}{63}$. 5. $3\frac{3}{8}$. 6. $7\frac{13}{20}$.
7. $12\frac{1}{32}$. 8. $15\frac{91}{108}$. 9. $\frac{1}{2}$. 10. $\frac{4}{11}$ 11. $\frac{2}{5}$. 12. $\frac{2}{5}$.
13. $\frac{3}{8}$. 14. $\frac{1}{6}$. 15. $\frac{1}{3}$. 16. $\frac{3}{20}$. 17. 4. 18. 4.
19. $2\frac{5}{7}$. 20. 18. 21. $\frac{2}{5}$. 22. $\frac{1}{7}$. 23. $\frac{1}{11}$. 24. $8\frac{3}{4}$.
25. 30. 26. 26. 27. 1. 28. 44. 29. $5\frac{1}{3}$. 30. 6.
31. $\frac{21}{22}$. 32. 1. 33. 18.

V. I. Page 95.

1. $\frac{1}{2}$. 2. $\frac{2}{3}$. 3. $\frac{7}{9}$. 4. $\frac{1}{5}$. 5. 4. 6. 20.
7. $17\frac{1}{2}$. 8. 10. 9. $\frac{1}{6}$. 10. $6\frac{1}{2}$. 11. 20. 12. $\frac{3}{10}$.
13. $5\frac{13}{18}$. 14. 18. 15. 256.

V. J. Page 96.

1. $1\frac{1}{8}$. 2. $1\frac{1}{4}$. 3. $\frac{8}{27}$. 4. $\frac{7}{15}$. 5. $\frac{1}{3}$. 6. $\frac{7}{11}$.

7. $\frac{3}{5}$. 8. $\frac{7}{9}$. 9. 2. 10. $\frac{1}{2}$. 11. 4. 12. $\frac{1}{4}$.

13. $1\frac{2}{3}$. 14. 3. 15. 4. 16. 3. 17. $\frac{3}{5}$. 18. $\frac{5}{7}$.

19. $\frac{3}{11}$. 20. $\frac{5}{12}$. 21. $\frac{20}{63}$. 22. $\frac{17}{48}$. 23. $\frac{44}{75}$. 24. $1\frac{3}{52}$.

25. $2\frac{8}{45}$. 26. $2\frac{38}{85}$. 27. $1\frac{31}{44}$. 28. $\frac{52}{55}$.

29. $\frac{748}{2175}$. 30. $2\frac{34}{77}$. 31. $\frac{21}{23}$. 32. $\frac{104}{209}$.

V. K. Page 98.

1. 6. 2. $9\frac{1}{3}$. 3. $\frac{1}{8}$. 4. $\frac{1}{6}$. 5. $1\frac{1}{2}$. 6. $\frac{2}{3}$.

7. $1\frac{1}{10}$. 8. $1\frac{1}{5}$. 9. $\frac{1}{2}$. 10. $1\frac{1}{4}$. 11. 4. 12. $\frac{9}{14}$.

13. $\frac{20}{21}$. 14. $\frac{13}{31}$. 15. $1\frac{6}{17}$. 16. 1. 17. $\frac{14}{17}$.

18. $1\frac{138}{1067}$. 19. $1\frac{1}{3}$. 20. $1\frac{11}{16}$. 21. 1. 22. $\frac{27}{50}$.

23. 1. 24. $1\frac{2}{3}$. 25. 1. 26. 14. 27. 8.

28. $\frac{15}{32}$. 29. $\frac{18}{31}$. 30. $\frac{100}{101}$. 31. $\frac{21}{31}$. 32. 2. 33. $1\frac{5}{11}$.

V. L. Page 100.

1. 18. 2. 3. 3. 20. 4. 9. 5. 7.

6. 44. 7. 240. 8. 64. 9. 33. 10. 216.

11. 336. 12. 480. 13. 16. 14. 20. 15. 22.

16. 3. 17. 23. 18. 15. 19. $1\frac{7}{12}$. 20. $\frac{23}{24}$.

21. $\frac{7}{9}$. 22. $\frac{29}{49}$.

V. M. Page 101.

1. $\frac{2}{5}$. 2. $\frac{32}{45}$. 3. $1\frac{1}{9}$. 4. $1\frac{1}{2}$. 5. 1.

6. $5\frac{1}{16}$. 7. $\frac{12}{25}$. 8. $\frac{1}{5}$. 9. $2\frac{1}{12}$. 10. $2\frac{2}{3}$.

11. $\frac{5}{6}$. 12. 2. 13. $\frac{1}{2}$. 14. $\frac{1}{2}$. 15. $13\frac{3}{5}$.

16. $\frac{1}{32}$. 17. $\frac{1}{8}$. 18. $\frac{3}{4}$. 19. $\frac{1}{2}$. 20. $4\frac{1}{6}$.

21. $1\frac{1}{6}$. 22. $\frac{3}{8}$. 23. $\frac{13}{24}$. 24. $\frac{7}{60}$. 25. $\frac{1}{20}$.

26. $1\frac{3}{8}$. 27. $11\frac{13}{20}$. 28. 2. 29. $\frac{7}{16}$. 30. $\frac{1}{16}$.

31. 0. 32. 0. 33. $3\frac{3}{8}$. 34. $3\frac{1}{2}$. 35. 1.

36. $\frac{1}{2}$. 37. 2. 38. 3. 39. 4.

V. N. Pages 102—103.

1. £3. 3*s*., £3. 10*s*., £3. 15*s*., £25. 6*s*. 3*d*.
2. 13*s*. $9\frac{3}{4}d$., 19*s*. $5\frac{3}{4}d$., £11. 6*s*. 5*d*., £16. 18*s*. $11\frac{1}{4}d$.
3. 13*s*. 4*d*. 4. £26. 5*s*. 11*d*. 5. 17*s*. $1\frac{1}{2}d$.
6. £10. 8*s*. $8\frac{1}{4}d$. 7. £2. 4*s*. $6\frac{3}{4}d$. 8. £2. 18*s*. $5\frac{1}{4}d$.
9. £37. 5*s*. $9\frac{1}{2}d$. 10. $3\frac{1}{4}d$., 17*s*. $10\frac{3}{4}d$. 11. 5*s*. $11\frac{1}{2}d$.
12. £2. 16*s*. 2*d*. 13. 5 cwt. 1 qr. 10 lbs.
14. 75 cwt. 3 qrs. 27 lbs. 2 oz. 15. 29 feet $\frac{1}{2}$ in.
16. 15 yds. 2 ft. 8 in. 17. 11 m. 4 f. 32 po.
18. 1 a. 9 p. 19. 585 yards. 20. 5 lbs. 4 oz.

V. O. Page 104.

1. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{5}{24}$, $\frac{3}{16}$.
2. $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{2}{15}$, $\frac{4}{15}$, $\frac{1}{12}$, $\frac{1}{16}$.
3. $\frac{1}{6}$. 4. $\frac{13}{55}$. 5. $\frac{4}{9}$. 6. $\frac{1}{10}$. 7. $3\frac{1}{8}$.
8. $\frac{5}{11}$. 9. $3\frac{3}{8}$. 10. $\frac{3}{7}$. 11. (1) $\frac{7}{144}$, $\frac{7}{60}$, $\frac{77}{320}$;
(2) $\frac{1}{25}$, $\frac{12}{125}$, $\frac{99}{500}$. 12. $1\frac{7}{8}$. 13. $\frac{3}{14}$. 14. $\frac{1}{11}$.
15. $\frac{9}{100}$. 16. $\frac{1}{448}$. 17. $\frac{1}{3}$. 18. $1\frac{1}{6}$. 19. $\frac{1}{4}$.
20. $10\frac{3}{80}$.

MISCELLANEOUS EXAMPLES IN FRACTIONS.

PAGES 105—110.

1. $\frac{36}{7}$, $\frac{91}{11}$, $\frac{628}{25}$. 2. $\frac{5}{8}$, $6\frac{2}{2}\frac{3}{4}$. 3. $7\frac{1}{3}$. 4. 7.

5. 13*s*. 9*d*. 6. $\frac{8}{15}$, $\frac{5}{9}$, $\frac{7}{12}$, $\frac{13}{20}$. 7. $\frac{3}{4}$. 8. 54.

9. $\frac{5}{14}$. 10. £1. 7*s*. $4\frac{3}{4}d$. 11. $\frac{35}{64}$, $\frac{2}{7}$.

12. $\frac{13}{6552}$, $\frac{16}{6552}$, $\frac{14}{6552}$, $\frac{27}{6552}$. 13. $2\frac{1}{6}\frac{3}{0}$. 14. $\frac{16}{21}$.

15. £1. 4*s*. $2\frac{1}{4}d$. 16. $\frac{1}{2}$. 17. $1\frac{7}{1}\frac{}{2}$, $4\frac{1}{2}\frac{}{4}$. 18. $2\frac{2}{5}$.

19. $\frac{75}{113}$. 20. $\frac{3}{4}$. 21. $\frac{112}{211}$, $\frac{35}{72}$, $\frac{32}{45}$. 23. 1.

24. $\frac{1}{8}$, $\frac{64}{241}$. 25. $\frac{13}{38}$. 26. $\frac{1}{2}$. 27. $2\frac{9}{1}\frac{7}{0}\frac{}{2}$, $3\frac{1}{1}\frac{3}{8}\frac{7}{0}$.

28. 12. 29. $\frac{1}{8}$, 2. 30. $\frac{127}{168}$. 31. $\frac{13}{120}$. 32. $2\frac{1}{2}$.

33. 9 pieces with $\frac{1}{1\,0}$ of an inch left over. 34. $\frac{73}{1026}$.

35. 1 a. 2 r. 20 po. 36. $\frac{13}{63}$, $\frac{8}{35}$, $\frac{11}{45}$; $\frac{4}{105}$. 37. $1\frac{3}{8}\frac{2}{4}\frac{3}{0}$.

38. 2. 39. $\frac{1}{3}$. 40. $\frac{3}{5}$, $\frac{11}{15}$. 41. $\frac{1}{4}$. 42. $\frac{1}{10}$.

43. 1. 44. $\frac{1}{22}$. 45. £1. 7*s*. 46. 1. 47. 1.

48. $\frac{2}{3}$, $1\frac{1}{1}\frac{}{5}$. 49. $\frac{9}{40}$. 50. 10 a. 3 r. 31 p. 51. $\frac{414}{847}$, $\frac{7}{9}$.

52. $5\frac{1}{2}$. 53. £1155, £280. 54. 1, $\frac{7}{2}\frac{}{5}$. 55. $\frac{7}{8}$.

56. $\frac{37}{1980}$. 57. $3\frac{5}{2}\frac{}{4}$. 58. $\frac{3}{8}$. 59. $\frac{2}{27}$. 60. $\frac{2}{99}$.

61. $\frac{7}{12}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{25}{36}$, $\frac{3}{4}$; $3\frac{2}{7}\frac{3}{2}$. 62. $1\frac{4}{7}\frac{1}{0}$. 63. $5\frac{1}{2}$.

64. $105\frac{1}{3}$. 65. 15*s*., $\frac{2}{5}$. 66. $\frac{27}{71}$, $\frac{17}{19}$. 67. 1.

68. £12. 5*s*. 69. $7\frac{1}{5}$. 70. $\frac{144}{175}$, $\frac{13}{17}$. 71. $4\frac{5}{8}\frac{9}{4}\frac{9}{0}$.

72. 252 miles. 73. $\frac{21}{50}$. 74. $\frac{10}{27}$. 75. $\frac{1}{5}$.

76. $12\frac{13}{15}$, $4\frac{383}{386}$. **77.** £2. 5*s*. **78.** $152\frac{1}{3}$. **79.** $\frac{1}{4}$, 2.

80. £3. 4*s*. $5\frac{1}{2}d$. **81.** $2\frac{21}{40}$. **82.** $3\frac{13}{300}$. **83.** $6\frac{2}{3}$.

84. $\frac{1}{20}$. **85.** £1346. 5*s*. 6*d*. **86.** $\frac{200}{480}$, $\frac{224}{480}$, $\frac{216}{480}$, $\frac{195}{480}$, $\frac{190}{480}$.

87. $\frac{3}{4}$. **88.** $\frac{2}{11}$. **89.** 5*s*. $1\frac{1}{4}d$. **90.** 21 yds.

91. $\frac{7}{13}$, $\frac{44}{45}$, $\frac{137}{731}$. **92.** $13\frac{13}{26}\frac{3}{4}$. **93.** $\frac{7}{9}$, $7\frac{1}{2}$. **94.** $29\frac{37}{42}$.

95. 112, 56, 42, 21. **96.** $\frac{7}{8}$. **97.** $45\frac{1}{6}$. **98.** $4\frac{2}{3}$. **99.** 7.

100. £36.

VI. A. Page 113.

1. Three *units* and five *tenths*. **2.** One *ten* five *units* and seven *tenths*. **3.** One *hundred* two *tens* five *units* three *tenths* and four *hundredths*. **4.** One *ten* two *units* five *tenths* and three *hundredths*. **5.** Eight *hundreds* one *tenth* and seven *hundredths*. **6.** One *thousand* two *hundreds* six *tenths* and three *hundredths*. **7.** Eight *tenths* seven *hundredths* and five *thousandths*. **8.** Three *units* four *tenths* two *hundredths* and seven *thousandths*. **9.** Five *tens* three *hundredths* and seven *thousandths*. **10.** Two *hundreds* five *tens* three *thousandths* four *ten-thousandths*. **11.** Eight *thousands* one *tenth* and nine *ten-thousandths*. **12.** Five *units* one *thousandth* seven *hundred-thousandths*. **13.** Five *hundreds-of-thousands* six *tens-of-thousands* two *units* one *tenth* nine *hundredths* seven *hundred-thousandths*. **14.** Two *tens* seven *units* five *hundredths* six *ten-thousandths* three *millionths*. **15.** 12·3. **16.** 18·7. **17.** 304·9. **18.** 25·34. **19.** 4·607. **20.** 1000000·400003. **21.** Fifteen *hundredths*, one hundred and twenty seven *ten-thousandths* three hundred and fifteen *tenths*, five thousand and sixteen *thousandths*, twenty thousand and eighty-four *ten-thousandths*.

VI. B. Page 114.

1. $\frac{2}{10}=\frac{1}{5}$, $\frac{3}{10}$, $\frac{4}{10}=\frac{2}{5}$, $\frac{5}{10}=\frac{1}{2}$, $\frac{8}{10}=\frac{4}{5}$, $\frac{25}{100}=\frac{1}{4}$, $\frac{35}{100}=\frac{7}{20}$, $\frac{45}{100}=\frac{9}{20}$, $\frac{75}{100}=\frac{3}{4}$, $\frac{88}{100}=\frac{22}{25}$.

2. $\frac{125}{1000}=\frac{1}{8}$, $\frac{375}{1000}=\frac{3}{8}$, $\frac{625}{1000}=\frac{5}{8}$, $1\frac{325}{1000}=1\frac{13}{40}$, $5\frac{424}{1000}=5\frac{53}{125}$, $30\frac{56}{1000}=30\frac{7}{125}$, $15\frac{68}{1000}=15\frac{17}{250}$.

3. $15\frac{3125}{10000}=15\frac{5}{16}$, $17\frac{625}{10000}=17\frac{1}{16}$, $8\frac{125}{100000}=8\frac{1}{800}$, $103\frac{375}{10000}=103\frac{3}{80}$.

4. $\frac{40625}{1000000}=\frac{13}{320}$, $3\frac{3125}{1000000}=3\frac{1}{320}$, $4\frac{9375}{100000}=4\frac{3}{32}$, $1\frac{50128}{100000}=1\frac{3133}{6250}$.

5. ·5, ·17, ·193, 2·14. 6. ·05, ·017, ·009, ·0097, ·000123.

VI. C. Page 115.

1. 23·4, 516·7, 2307, 210·8, 5126, 3145.
2. 71·23, 1234, 5·62, ·034, 5017, ·006.

3.	215·7.	4.	·67.	5.	21035.	6.	1230·8.	7.	2134·5.
8.	3501.	9.	127.	10.	1270.	11.	3150.	12.	3.
13.	70.	14.	3920.	15.	3·57.	16.	2·73.	17.	·516.
18.	2·174.	19.	·0315.	20.	·0637.	21.	·02143.	22.	·0058.
23.	·00067.	24.	·5623.	25.	·3827.	26.	·7108.		
27.	·05813.	28.	·07092.	29.	·03721.	30.	·00416.		
31.	·007102.	32.	·0051.	33.	·00019.	34.	·000084.		
35.	·000718.	36.	6·2341.	37.	·62341.	38.	·0062341.		
39.	·000057.	40.	·0000174.	41.	·0000098.				

VI. D. Page 116.

1.	242·09.	2.	335·21887.	3.	206·7726.
4.	210·09207.	5.	45555·12292.	6.	2·223.
7.	·072.	8.	7·877.	9.	·054.
10.	·112.	11.	19·325.	12.	·00162.
13.	·79477.	14.	2·59657.	15.	8·40922.
16.	1·50557.	17.	6·74566.	18.	·64636.

VI. E. Page 119.

1.	6·4.	2.	45·6.	3.	8·75.	4.	33·48.
5.	213·52.	6.	715·07.	7.	3431·38.	8.	614·376.
9.	29·616.	10.	149·136.	11.	·041496.	12.	·25965.
13.	12·196905.	14.	1·029875.	15.	·8388655.		
16.	·01995551.	17.	·01235718.	18.	·000098363.		
19.	·003340434.	20.	·00006584555.	21.	8·42576610.		
22.	9·4538178.	23.	·00082331456.	24.	·000061163984.		
25.	·001.	26.	·000001.	27.	·216.		
28.	·064821.	29.	·000125.	30.	·015625.		
31.	19·9044.	32.	1·74445788.				

VI. F. Page 121.

1. 8·2.	2. 10·9.	3. 4·3.	4. ·026.
5. ·032.	6. ·015.	7. 6·28.	8. 1·061.
9. ·23443.	10. 2·6925.	11. ·000535.	12. ·00145.
13. ·10875.	14. ·002375.	15. ·0000075.	16. ·0006.
17. ·0314.	18. ·0565625.	19. 3·1.	20. 1·9.
21. 1·4.	22. 30·5.	23. ·00305.	24. 3·0012.
25. ·030012.	26. ·502.	27. ·00000502.	28. ·005008.
29. ·0000207.	30. ·0000005008.		31. ·0002065.
32. ·0000002065.	33. ·00021695.		34. ·0000000012837.
35. ·0000000090706.	36. ·0820.		37. ·0152.
38. ·4691.	39. 7·9411.		40. ·1259.
41. ·7117.	42. ·0000058.		43. ·70955.

VI. G. Page 123.

1. 620.	2. 347.	3. 123000.	4. 7.
5. 15.	6. 250.	7. 25.	8. 25.
9. 250000.	10. 8·8.	11. ·152.	12. 136000.
13. ·625.	14. 1·2.	15. 22·5.	16. 43.
17. 6·4.	18. 8·16.	19. 66·99.	20. 60660.
21. 70380.	22. ·0609.	23. 606·06.	24. ·006008.
25. ·3501, ·0052.	26. 312·5.		27. ·375.
28. ·007875.	29. 46, rem. 2·24 in.		30. 1297, rem. ·55 oz.

VI. H. Page 124.

1. ·5.	2. ·25.	3. ·125.	4. ·0625.
5. ·75.	6. ·375.	7. ·3125.	8. ·5625.
9. ·4.	10. ·44.	11. ·904.	12. ·024.
13. ·275.	14. ·065625.	15. ·128.	16. ·0176.
17. ·052734375.	18. 3·0037109375.		19. 7·1103515625.
20. 13·0004128.			

VI. I. Page 126.

1. $\dot{5}$.	2. 333·$\dot{3}$.	3. 18·$\dot{8}$.	4. ·486$\dot{1}$.
5. ·131$\dot{6}$.	6. ·03$5\dot{4}$.	7. ·$4\dot{4}2857\dot{1}$.	8. 222·$8\dot{5}7142\dot{8}$.
9. ·094$6\dot{9}$.	10. 309·$10\dot{7}14285\dot{7}$.		11. 3·$81\dot{4}93506\dot{8}$.
12. 1169·$2\dot{3}0769\dot{0}$.	13. 1·$2\dot{1}8\dot{9}$.		14. 4·$2\dot{1}77606\dot{8}$.
15. 60820·$\dot{4}63320\dot{8}$.	16. ·$\dot{3}$.	17. ·$\dot{1}42857\dot{1}$.	18. ·$\dot{5}$.
19. ·$6\dot{3}$.	20. ·$\dot{4}61538\dot{4}$.	21. ·$7\dot{8}57142\dot{8}$.	22. ·$5\dot{3}$.

23. $\cdot\dot{2}941176470588235\dot{}$. 24. $2\cdot3\dot{1}\dot{8}$. 25. $3\cdot\dot{5}23809\dot{}$.
26. $5\cdot53\dot{5}7142\dot{8}$. 27. $7\cdot\dot{2}58064516129032\dot{}$.
28. $11\cdot053\dot{5}71428\dot{}$. 29. $13\cdot113\dot{0}95238\dot{}$. 30. $2\cdot998\dot{3}76623\dot{}$.
31. $5\cdot1\dot{3}70656\dot{}$.

VI. J. Page 128.

1. $\frac{1}{3}$. 2. $\frac{1}{11}$. 3. $\frac{3}{11}$. 4. $\frac{5}{33}$. 5. $\frac{1}{37}$.
6. $\frac{1}{27}$. 7. $\frac{5}{27}$. 8. $\frac{44}{111}$. 9. $\frac{1}{7}$. 10. $\frac{2}{7}$.
11. $\frac{3}{7}$. 12. $\frac{1}{77}$. 13. $\frac{2}{41}$. 14. $\frac{3}{41}$. 15. $\frac{7}{30}$.
16. $\frac{4}{5}$. 17. $\frac{11}{30}$. 18. $\frac{52}{165}$. 19. $\frac{7}{60}$. 20. $\frac{7}{275}$.
21. $\frac{1}{60}$. 22. $\frac{3}{4}$. 23. $\frac{223}{1100}$. 24. $\frac{143}{740}$. 25. $\frac{33}{82}$.
26. $\frac{25}{82}$. 27. $\frac{36853}{121950}$. 28. $\frac{19}{35}$. 29. $\frac{1}{81}$. 30. $\frac{11}{81}$.

VI. K. Page 130.

1. 1. 2. $25\cdot8$. 3. $\cdot\dot{2}\dot{3}$. 4. $22\cdot5\dot{8}\dot{3}$.
5. $5\cdot\dot{7}$. 6. $23\cdot\dot{6}$. 7. $\cdot3\dot{5}5828\dot{5}$. 8. $14\cdot8\dot{3}$.
9. $16\cdot69\dot{9}52\dot{6}$. 10. $117\cdot103\dot{0}63791\dot{}$. 11. $17\cdot650\dot{9}77067\dot{}$.
12. $14\cdot04\dot{9}=14\cdot05$. 13. $31\cdot5$. 14. $\cdot40\dot{9}$.
15. $1\cdot4\dot{1}8\dot{5}$. 16. $\cdot81\dot{8}0616\dot{9}$. 17. $\cdot45\dot{1}06399829172\dot{}$.
18. $1\cdot79\dot{5}07780\dot{}$. 19. $2\cdot99\dot{2}94021\dot{}$.

VI. L. Page 131.

1. $4\cdot\dot{3}$. 2. $19\cdot5\dot{3}$. 3. $\cdot37$. 4. $126\cdot\dot{2}$.
5. $14\cdot3\dot{1}$. 6. $\cdot196$. 7. $\cdot40\dot{9}$. 8. $181\cdot0\dot{3}\dot{9}$.
9. $17\cdot085\dot{8}$. 10. $238\cdot9\dot{3}$. 11. $19\cdot25\dot{5}\dot{7}$. 12. $\cdot413684\dot{6}$.
13. $\cdot01656\dot{6}5\dot{7}$. 14. $2\cdot6574850\dot{}$. 15. $1\cdot0\dot{7}$.
16. $\cdot01\dot{8}$. 17. $14\cdot01\dot{7}$. 18. $\cdot04\dot{5}$. 19. $\cdot054\dot{2}2\dot{9}$.
20. $12\cdot\dot{5}2861\dot{9}$. 21. $3\cdot1\dot{8}\dot{5}$.
22. $13\cdot\dot{5}\dot{3}$. 23. $4783\cdot7\dot{8}\dot{3}$. 24. $123\cdot\dot{4}\dot{3}$.

VI. M. Page 132.

1. $\cdot\dot{1}$. 2. $\cdot20\dot{3}\dot{7}$. 3. $\cdot0649\dot{5}$. 4. $\cdot\dot{3}08641975\dot{}$.
5. 180. 6. $\cdot2\dot{2}987654320\dot{}$. 7. $\cdot0\dot{3}$. 8. $\cdot\dot{6}$.

9. $\cdot 27\dot{7}0\dot{2}$. 10. 2. 11. $\cdot 00\dot{3}\dot{6}$. 12. $6\cdot 7\dot{6}$.
13. $\cdot\dot{7}4\dot{0}$. 14. $1\cdot 0\dot{5}$. 15. $71\cdot 5$. 16. 3.
17. $\cdot 2\dot{}$. 18. $\cdot 019\dot{4}$. 19. $\cdot 8$. 20. $7\cdot\dot{3}\dot{0}$.

VI. N. Page 134.

1. $\cdot 08\dot{3}$, $\cdot 1\dot{6}$, $\cdot 25$, $\cdot\dot{3}$, $\cdot 41\dot{6}$, $\cdot 5$. 2. $\cdot 125$, $\cdot 208\dot{3}$, $\cdot 1875$, $\cdot 625$, $\cdot 875$.
3. $\cdot 05$, $\cdot 1$, $\cdot 15$, $\cdot 2$, $\cdot 25$, $\cdot 3$. 4. $\cdot 125$, $\cdot 0625$, $\cdot 1\dot{6}$, $\cdot 375$, $\cdot 58\dot{3}$.
5. £3·86875, £1·978125, £2·048958$\dot{3}$. 6. $\cdot 25\dot{3}$. 7. $\cdot 5208\dot{3}$.
8. $\cdot 857142\dot{}$. 9. 3·6. 10. $\cdot 0875$. 11. $\cdot 5875$.
12. $\cdot 2765625$. 13. $\cdot 212708\dot{3}$. 14. $2\cdot 423295\dot{4}$. 15. 15*s*.
16. 12*s*. 6*d*. 17. 7*s*. 6*d*. 18. $7\frac{1}{2}d$. 19. 17*s*. $7\frac{1}{2}d$.
20. 10*s*. $9\frac{3}{4}d$. 21. £10. 17*s*. 6*d*. 22. $5\frac{1}{4}d$.
23. 5 cwt. 2 qrs. 7 lbs. 7 oz. 24. 11 lbs. 3 oz. 6 dwts. 9 grs.
25. 6 days 5 hrs. 27 min. 26. 3 fur. 23 p. 2 yds. 1 ft. 6 in.
27. £14. 15*s*. $7\frac{1}{2}d$., £6802. 8*s*. 6*d*. 28. £1. 6*s*. 29. $\cdot 3095$.
30. 19*s*. $7\frac{1}{2}d$. 31. £1. 5*s*. $1\frac{1}{2}d$., $\cdot 05025$.

MISCELLANEOUS EXAMPLES IN DECIMALS.

Pages 135—138.

1. 189·691237. 2. 1·05122. 3. $\cdot 9928$, $\cdot 000078125$.
4. $\cdot 401$, 1200. 5. £3. 9*s*. $10\frac{1}{2}d$. 6. $\cdot 001$.
7. $\frac{7}{200}$, $\frac{5}{8}$, $\frac{384}{3125}$. 8. 7890, $\cdot 000789$.
9. $\cdot 6875$, $\cdot 144$, $\cdot 009632$. 10. $\cdot 97875$.
11. $\cdot 00043$, $\cdot 084375$, $\cdot 0546875$. 12. $4\cdot 6225$, $\cdot 00104976$.
13. 3250. 14. 11·25. 15. £3. 7*s*. $1\frac{1}{2}d$. 16. $\cdot 121$.
18. 832. 19. $\cdot 143359375$. 20. £2. 10*s*. 21. $\cdot 1751\ldots$.
22. 1·0504857. 23. 9·8, 40·1. 24. £10.
25. 1·984375. 26. £1. 10*s*. 27. $\frac{1}{32}$, $\frac{496}{625}$, $\frac{448}{3125}$.
28. $\cdot 00144$, 1440. 29. $\cdot 2375$. 30. £5. 12*s*. $10\frac{1}{2}d$.
31. $\cdot 2625$, 26·25. 32. 7890, $\cdot 00789$. 33. 6·25.
34. $\frac{1}{5}$. 35. 1·5. 36. $\cdot\dot{4}2857\dot{1}$, $\cdot 5\dot{3}571428\dot{}$, $\cdot 1\dot{3}4857142\dot{}$.
37. $15\cdot 8\dot{3}$. 38. $7\cdot 9\dot{4}$, $1\cdot 38\dot{1}$. 39. 3*s*. 2*d*., £1. 12*s*. $1\frac{1}{4}d$.
40. $\cdot 002$. 41. 8510. 42. $\cdot 00748$. 43. $\frac{447}{925}$, $\frac{13}{44}$.
44. $1\cdot\dot{7}11285\dot{}$. 45. $\frac{8}{9}$. 46. $\cdot 032$, $\cdot 00000002$.

47. 124, ·01 inches. 48. $\cdot\dot{8}823529411764705\dot{}$, $\cdot48\dot{4}3\dot{2}$.

49. $\frac{6}{13}$. 50. ·475. 51. ·0046, 3162162·$\dot{1}6\dot{2}$.

52. 846, ·004687 inches. 53. $\frac{7}{55}$, $\frac{1279}{370000}$, $\frac{271}{820}$.

54. £6. 19*s*. 6*d*. 55. ·62963 yards. 56. 1·010016, 6700.

57. 8·5$\dot{4}$87$\dot{1}$. 58. 1600. 59. ·009$\dot{9}$99$\dot{1}$80. 60. $1\frac{3}{4}$*d*., ·01.

VII. B. Page 141.

1. £1576. 2. £1878. 10*s*. 3. £578.
4. £234. 7*s*. 6*d*. 5. £452. 10*s*. 6. £325. 11*s*. 8*d*.
7. £912. 10*s*. 8. £1596. 13*s*. 4*d*. 9. £11556. 10*s*.
10. £9789. 15*s*. 11. £7755. 15*s*. 12. £37075. 16*s*.
13. £2988. 15*s*. 14. £29295. 5*s*. 15. £11846. 8*s*.
16. £23388. 15*s*. 17. £16937. 2*s*. 18. £137720. 14*s*.
19. £34987. 14*s*. 20. £47113. 5*s*. 21. 64 tons 18 cwt. 2 qrs.
22. 571 tons 6 cwt. 3 qrs. 14 lbs. 23. 8232 yards.
24. 7365 yards.

VII. C. Pages 142—143.

1. £3431. 16*s*. 8*d*. 2. £23758. 1*s*. 3. £21001. 10*s*.
4. £93235. 4*s*. 5. £21703. 10*s*. 6*d*. 6. £57241. 19*s*. 6*d*.
7. £73113. 12*s*. 8. £22662. 18*s*. 9. £26526. 15*s*.
10. £68690. 14*s*. 11. £34791. 6*s*. 12. £61007. 3*s*.
13. £61472. 16*s*. 3*d*. 14. £29980. 8*s*. 15. £8149. 3*s*.
16. £39105. 6*s*. 17. £96740. 8*s*. $1\frac{1}{2}$*d*. 18. £4173.
19. £7948. 16*s*. $9\frac{1}{2}$*d*. 20. £2084. 18*s*. 11*d*. 21. £8769. 10*s*. $7\frac{1}{2}$*d*.
22. £356. 1*s*. $9\frac{3}{4}$*d*. 23. £28750. 3*s*. 24. £523. 0*s*. $2\frac{1}{2}$*d*.
25. £1650. 5*s*. $10\frac{1}{2}$*d*. 26. £13018. 11*s*. $1\frac{1}{2}$*d*. 27. £3121. 2*s*. $11\frac{1}{4}$*d*.
28. £5818. 10*s*. $7\frac{1}{2}$*d*. 29. £10995. 13*s*. 2*d*. 30. £2832. 11*s*. $2\frac{1}{2}$*d*.
31. £11425. 18*s*. 7*d*. 32. £12231. 8*s*. $1\frac{1}{2}$*d*. 33. £53635. 15*s*. 10*d*.
34. £7464. 19*s*. 7*d*.

VII. D. Pages 144—146.

1. £13. 14*s*. 2*d*. 2. £79. 14*s*. 1*d*. 3. £11. 10*s*.
4. £21. 11*s*. $4\frac{1}{2}$*d*. 5. £302. 8*s*. 6. £241. 10*s*.
7. £15. 12*s*. 6*d*. 8. £5. 18*s*. 1*d*. 9. £10. 16*s*. $1\frac{1}{4}$*d*.
10. £40. 0*s*. 11*d*. 11. £447. 19*s*. $8\frac{1}{4}$*d*. 12. £21. 5*s*. 9*d*.
13. £2571. 3*s*. $4\frac{3}{4}$*d*. 14. £482. 2*s*. $9\frac{3}{4}$*d*. 15. £3135. 6*s*. 3*d*.
16. £69. 4*s*. $8\frac{3}{4}$*d*. 17. £12. 8*s*. $10\frac{1}{4}$*d*. 18. £121. 18*s*. $10\frac{1}{4}$*d*.

19. £9884. 5*s*. 11¼*d*. 20. £96424. 14*s*. 8*d*. 21. £156.
22. £447. 9*s*. 0¼*d*. 23. £573. 10*s*. 10½*d*. 24. £3237. 16*s*. 9¼*d*.
25. £371. 2*s*. 8½*d*. 26. £2560. 9*s*. 6¾*d*. 27. £5846. 18*s*. 6*d*.
28. £32. 18*s*. 10¼*d*. 29. £468. 4*s*. 3*d*. 30. £23. 4*s*. 7½*d*.
31. £180. 13*s*. 6*d*. 32. £60500. 3*s*. 6*d*. 33. £92. 10*s*. 8*d*.
34. £75. 7*s*. 9¾*d*. 35. £71. 1*s*. 5¼*d*. 36. £71. 4*s*. 7*d*.

VII. E. Pages 147—148.

1. 14*s*. 1½*d*. 2. 16*s*. 3*d*. 3. 14*s*. 4¼*d*. 4. £4. 6*s*. 6¾*d*.
5. £1. 2*s*. 1½*d*. 6. £2. 11*s*. 7. £1. 3*s*. 11¾*d*.

MISCELLANEOUS EXAMPLES IV. Pages 148—150.

1. 458. 2. 17078, 80848. 3. 137.
4. 117, 23400. 5. $\frac{3}{4}$. 6. 5·005, ·000001874048.
7. £98. 8*s*. 6½*d*., £3. 2*s*. 6*d*. 8. £322. 3*s*. 10*d*. 9. 20263.
10. £51375. 5*s*. 5*d*., £618. 19*s*. 7*d*. 11. 20064, 2 oz.
12. 0. 13. £67. 10*s*. 14. ·07. 15. 433, ·061 in.
16. £60. 12*s*. 4¾*d*. 17. 47865, rem. 90. 18. £244. 4*s*. 3¾*d*.
19. 14832. 20. 63, 6300. 21. $\frac{4}{83}$, $\frac{3}{62}$, $\frac{7}{144}$.
22. 32·5, $9\cdot\dot{7}2\dot{9}$. 23. $\frac{7}{18}$, $\frac{71}{105}$, $\frac{1}{4}$. 24. £219. 12*s*. 6¾*d*.
25. At least 1764. 26. 159 a. 2 r. 21 p. 26¾ sq. yds. 27. 19.
28. 501, 879746982. 29. $\frac{87}{560}$. 30. 1.
31. 15 hrs. 46 min. 48 secs. 32. £8. 11*s*. 3¼*d*.
33. 53, Wednesday. 34. 5 times, rem. £13. 11*s*. 0½*d*.
35. 4½*d*. 36. 72. 37. $14\frac{6}{11}$, $\frac{1}{10}$. 38. £1. 3*s*. 6*d*.
39. $2\cdot\dot{3}07692\dot{}$. 40. 8*s*. 2½*d*. 41. 5 ft. 11 in.
42. 652014 in., 13707¼ sq. yds. 43. 348. 44. 21 yards.
45. $1\frac{47}{176}$. 46. $\frac{1}{6}$, $\frac{21}{31}$. 47. ·2. 48. £156. 6*s*. 11¼*d*.

VIII. A. Pages 153—154.

1. 168 sq. ft. 2. 374 sq. ft. 3. 425 sq. yds.
4. 13908 sq. in. 5. 7 sq. ft. 12 sq. in. 6. 76 sq. yds. 6 sq. ft.
7. 19 sq. yds. 5 sq. ft. 8. 390 sq. yds.

9. 122 sq. ft. 32 sq. in. 10. 100 sq. ft. 123 sq. in.
11. 18 acres 20 poles. 12. 18 acres.
13. $17\frac{8}{11}$ acres. 14. 6 a. 2 p.
15. 45 a. 1 r. 32 p. 22 sq. yds. 16. 1 a. 1 r. 4 p. $16\frac{1}{2}$ yds.
17. 5 a. 2 r. 32 p. 18. 7 a. 27 p.
19. 37 in. 20. 10 ft. 8 in. 21. 11 ft.
22. £6. 14*s*. 9*d*. 23. 77 yds. 24. £208. 6*s*. 8*d*.
25. £416. 13*s*. 4*d*.

VIII. B. Pages 155—156.

1. £7. 15*s*. 2. $52\frac{1}{2}$ yds., £11. 3*s*. $1\frac{1}{2}d$. 3. £18.
4. £12. 16*s*. 8*d*. 5. £20. 10*s*. 8*d*. 6. £9. 12*s*. 8*d*.
7. 29760. 8. £14. 14*s*. 9. 53 yds. 1 ft.
10. £12. 12*s*. 11. 3·33 inches, 775. 12. £47. 14*s*.
13. 9*d*. 14. 2*s*. 3*d*. 15. 580 sq. feet.
16. 680 sq. feet. 17. $604\frac{4}{9}$ sq. ft. 18. 144 yards, £1. 10*s*.
19. £6. 6*s*. 8*d*. 20. 15 pieces.

VIII. C. Page 159.

1. 40 c. ft. 2. 288 c. ft. 3. 81 c. ft. 4. 16 c. yds. 18 c. ft.
5. 67 c. ft. 504 c. in. 6. 142 c. ft. 702 c. in.
7. 17 c. yds. 25 c. ft. 8. 10 c. yds. 24 c. ft. 1143 c. in.
9. 1 ft. 9 in. 10. 24 ft. 6 in. 11. $11\frac{1}{9}$ loads. 12. 15 ft.
13. 5655·5 gallons, 25 tons 4 cwt. 3 qrs. 23 lbs.
14. 112 cubic feet, 3 tons 2 cwt. 2 qrs.
15. 101 tons 5 cwt. 2 qrs. 19 lbs. 16. $46298057\frac{1}{7}$ tons. 17. 343.
18. 1 ton 1 cwt. 24 lbs. 9 oz. 19. 12464.
20. 3 tons 4 cwt. 2 qrs. 13 lbs. 8 oz.

IX. Page 164.

1. 465·9. 2. 311·8. 3. 27440. 4. 12410.
5. 11·70. 6. ·6289. 7. 79·12. 8. 111·2.
9. 1030·876. 10. ·5190914. 11. 599·5 sq. in.
12. 20740 sq. yards. 13. (i) 2·7183, (ii) ·3679.
14. £407. 3*s*. 15. £55. 2*s*. $3\frac{1}{4}d$. 16. £405. 16*s*. $6\frac{3}{4}d$.
17. £74. 11*s*. 9*d*. 18. 7*s*. $1\frac{1}{4}d$.

X. Page 168.

1. 171·25 fr. 2. £167. 1*s*. $7\frac{3}{4}d$. 4. ·02539...
5. 258·9939. 8. 494·07 lbs. 9. 58·95 fr.

10. 1·1088 Ha., 8316 fr. 11. 34·50 fr. 12. 270 fr.
13. The former is equivalent to 75 c. for 965·6... metres.
14. 33·3557.

XI. A. Page 170.

1. 2 : 3. 2. 13 : 7. 3. 15 : 17. 4. 6 : 7.
5. 2 : 3. 6. 38 : 43. 7. 27 : 37. 8. 13 : 7.
9. 5 : 7. 10. 2 : 3. 11. 4 : 9. 12. 11 : 7.
13. £1. 14. £13. 3*s*. 6*d*. 15. 7 cwt. 2 qrs. 7 lbs.
16. 9 hrs. 51 min. 40 secs.

XI. B. Pages 171—172.

1. 21. 2. $2\frac{2}{3}$. 3. $\frac{5}{9}$. 4. 6. 5. 12. 6. 1. 7. 8*s*. 6*d*.
8. 2*s*. 9. 1 cwt. 2 qrs. 2 lbs. 10. 1 cwt. 18 lbs.

XI. C. Pages 173—175.

1. £120. 2. £12. 15*s*. 3. 6. 4. 99 days.
5. 10 days. 6. 6 hours. 7. £5. 1*s*. 9*d*. 8. 90 miles.
9. 171 days. 10. 28. 11. £93. 12. 15 secs.
13. 12. 14. 8. 15. 8. 16. £11. 13*s*. 9*d*.
17. £37. 6*s*. 8*d*. 18. 151 yards. 19. 2 hrs. 40 min.
20. 70 weeks. 21. £30. 6*s*. 3*d*. 22. £46. 3*s*. 9*d*.
23. 199. 24. £5. 4*s*. 2*d*. 25. £21. 12*s*. 1*d*.
26. £35. 11*s*. 1*d*. 27. £43. 28. £6. 11*s*. 3*d*.
29. £140. 12*s*. 10*d*. 30. £746. 13*s*. 4*d*. 31. £3899. 2*s*.
32. £1275. 19*s*. 10*d*. 33. £3. 12*s*. $11\frac{3}{4}$*d*. 34. 1 cwt. 21 lbs.
35. £500. 36. 16*s*. 8*d*. 37. £266. 15*s*. 38. 5*s*. 3*d*.
39. £4373. 40. £7421. 2*s*. $6\frac{3}{4}$*d*. 41. £76152.
42. £2583. 18*s*. 11*d*. 43. £84. 44. £550. 45. £87. 10*s*.

XI. D. Pages 176—178.

1. 7. 2. 12 days. 3. 160. 4. 5*s*. 3*d*.
5. £182. 6. 32. 7. £131. 5*s*. 8. 11.
9. 189. 10. 21 weeks. 11. 20 days. 12. 18.
13. $13\frac{1}{2}$. 14. £272. 5*s*. 15. £4. 17*s*. 6*d*. 16. 16 days.
17. 1 day. 18. $7\frac{1}{2}$. 19. £303. 15*s*. 20. £11. 13*s*. 4*d*.
21. 8 days. 22. 15. 23. 7 hrs. 48 min. 24. 3 days.
25. 36.

XII. A. Pages 181—182.

1. $\frac{1}{2}$. 2. $\frac{1}{4}$. 3. $\frac{1}{5}$. 4. $\frac{1}{10}$. 5. $\frac{1}{20}$. 6. $\frac{1}{40}$.
7. $\frac{1}{8}$. 8. $\frac{1}{6}$. 9. $\frac{1}{3}$. 10. $\frac{1}{30}$. 11. $\frac{1}{16}$. 12. $\frac{1}{18}$.
13. 50. 14. 75. 15. $66\frac{2}{3}$. 16. $33\frac{1}{3}$. 17. 10. 18. 5.
19. $7\frac{1}{2}$. 20. 44. 21. $26\frac{2}{3}$. 22. $31\frac{1}{4}$. 23. $26\frac{2}{3}$. 24. $91\frac{2}{3}$.
26. 25. 27. $18\frac{3}{4}$. 28. $11\frac{1}{9}$. 29. 30. 30. 5.
31. $6\frac{1}{4}$. 32. 5. 33. 30.
34. 7*s.* 6*d.*, 17*s.* 9*d.*, £1. 3*s.* $10\frac{1}{2}$*d.*, £5. 15*s.* 2*d.*, £6. 7*s.* 7*d.*, £15. 14*s.* $8\frac{1}{2}$*d.*
35. £1. 15*s.* 5*d.* 36. £3. 19*s.* $8\frac{1}{2}$*d.* 37. 3 cwt. 3 qrs. 3 lbs.
38. £97. 5*s.* 39. £15. 15*s.* 40. £1. 14*s.* 6*d.*
41. £450. 42. 23078. 43. ·26.
44. ·29. 45. £584. 46. £517. 12*s.*
47. £1233. 10*s.* 10*d.* 48. 0·2, 2·2, 6·9, 5·5, 5·7, 5·2.
49. 0·4, 0·3, 6·2, 5·5. 50. 2·9, 2·7, 2·8, 2·8.

XII. B. Pages 184—185.

1. 20. 2. 14. 3. 10. 4. 20. 5. $7\frac{1}{7}$. 6. 35.
7. $16\frac{2}{3}$. 8. 15. 9. 5. 10. 35. 11. $17\frac{1}{2}$. 12. 17.
13. $57\frac{1}{7}$. 14. 2*s.* 8*d.* 15. 1*s.* 9*d.* 16. 2*s.* 1*d.* 17. $8\frac{8}{9}$.
18. 20. 19. 10. 20. 10*s.* 6*d.* 21. £112. 10*s.* 22. 5.
23. 65. 24. 50. 25. 10*d.* 26. £75. 27. 20.
28. 10. 29. 21*s.* 8*d.* 30. 36. 31. 12*s.* 6*d.* 32. $16\frac{2}{3}$.
33. 8*s.* 4*d.* 34. £2666. 13*s.* 4*d.* 35. £450.

XIII. A. Pages 187—188.

1. £48. 2. £72. 3. £225. 4. £430. 10*s.*
5. £15. 6. £149. 7. £25. 5*s.* 8. £34. 9*s.*
9. £200. 19*s.* $7\frac{1}{2}$*d.* 10. £185. 7*s.* 11. £55. 15*s.* $5\frac{3}{4}$*d.*
12. £175. 17*s.* 9*d.* 13. £560. 14. £474. 6*s.*
15. £471. 17*s.* $7\frac{1}{4}$*d.* 16. £520. 16*s.* 8*d.*
17. £344. 16*s.* $8\frac{1}{4}$*d.* 18. £1. 19*s.* 4*d.*

XIII. B. Pages 189—190.

1. £27. 1*s.* 11*d.* 2. £187. 8*s.* 9*d.* 3. £166. 10*s.* 10*d.*
4. £1. 15*s.* 5. £116. 14*s.* 8*d.* 6. £36. 4*s.* 6*d.*
7. £12. 2*s.* 1*d.* 8. £5. 12*s.* 6*d.* 9. £7. 19*s.* 11*d.*
10. £3. 17*s.* 1*d.* 11. £4. 10*s.* 3*d.* 12. £192. 1*s.* 8*d.*

XIII. C. Pages 191—192.

1. 3 years.
2. 4 years.
3. 5 years.
4. $1\frac{1}{2}$ years.
5. 4 per cent.
6. 5 per cent.
7. $3\frac{1}{3}$ per cent.
8. $3\frac{3}{4}$ per cent.
9. £370.
10. £213. 6*s*. 8*d*.
11. £144.
12. £772. 10*s*.
13. £450.
14. £1127. 7*s*. 11*d*.
15. £350.
16. $4\frac{1}{4}$ years.
17. 120.
18. $4\frac{1}{2}$.
19. $1\frac{5}{6}$ years.
20. £240.
21. £75.
22. $2\frac{1}{2}$.
23. 4 years.
24. 7 months.
25. £588. 10*s*. 10*d*.
26. £176. 15*s*. 9*d*.
27. £2250.
28. 4, $3\frac{3}{4}$.
29. 4.

XIII. D. Pages 196—197.

1. £15. 7*s*. 6*d*.
2. £102.
3. £78. 0*s*. 10*d*.
4. £75. 13*s*. 2*d*.
5. £139. 1*s*. 10*d*.
6. £110. 3*s*. 6*d*.
7. £43. 1*s*. 11*d*.
8. £148. 6*s*. 4*d*.
9. £180. 1*s*. 6*d*.
10. £134. 19*s*. 2*d*.
11. £325. 5*s*. 4*d*.
12. £31. 13*s*. 9*d*.
13. £23. 19*s*. 8*d*.
14. £55. 11*s*. 9*d*.
15. £169. 9*s*. 3*d*.
16. £51. 18*s*. 2*d*.
17. £52. 0*s*. 6*d*.
18. £61. 2*s*. 8*d*.
19. £25. 18*s*. 3*d*.
20. £110. 4*s*. 6*d*.
21. £10. 5*s*. $10\frac{1}{2}$*d*.
22. £7. 12*s*.
23. £30. 12*s*. 1*d*.
24. 8*s*. 4*d*.
25. £800.
26. £6000.
27. £3125.
28. £8400.
29. £2083. 6*s*. 8*d*.
30. £910. 3*s*. 4*d*.
31. £153. 6*s*. 8*d*.
32. £531. 5*s*., 4 per cent.
33. £1·6288, £2·0789.

XIV. A. Pages 199—200.

1. £500.
2. £400.
3. £250.
4. £90. 8*s*. 4*d*.
5. £1596.
6. £1000.
7. £10.
8. £8.
9. £6. 15*s*.
10. £66. 10*s*.
11. £19. 10*s*.
12. £9. 15*s*.
13. £466. 13*s*. 4*d*.
14. £4166. 13*s*. 4*d*.
15. £50000.
16. £8533. 6*s*. 8*d*.
17. £1811. 18*s*.
18. £279. 2*s*. 6*d*.
19. $4\frac{1}{2}$.
20. £239. 5*s*. 9*d*.

XIV. B. Pages 202—203.

1. £5.
2. £25. 4*s*.
3. £6. 2*s*. 4*d*.
4. £11. 12*s*. 9*d*.
5. (i) £993. 0*s*. 3*d*., (ii) £1491. 12*s*. 4*d*., (iii) £1848. 17*s*. 6*d*., (iv) £2740. 6*s*. 8*d*.
6. £150.
7. £1. 15*s*.
8. £3. 5*s*. 1*d*.

9. £3. 10. 1*s*. 11. 4*s*. 11*d*.
12. 125. 13. £307. 10*s*. 14. £162. 10*s*.
15. $6\frac{2}{3}$. 16. £783. 10*s*. 6*d*. 17. £456. 9*s*.

XV. A. Pages 206—207.

1. £375. 2. £195. 3. £262. 10*s*. 4. 13. 5. 530.
6. £75. 7. £190. 8. £55. 9. £20.
10. £122. 10*s*. 11. 324. 12. 39. 13. £13. 4*s*.
14. £37. 16*s*. 15. 3. 16. 4. 17. £5 more.
18. £26. 13*s*. 4*d*. 19. $5\frac{1}{3}$. 20. £32.

XV. B. Page 209.

1. £800. 2. £650. 3. £2250. 4. £725. 5. £675.
6. £1380. 7. £1056. 8. £2114. 12*s*. 9. £36.
10. £44. 13*s*. 9*d*. 11. £34. 2*s*. 6*d*. 12. £252.
13. £1920. 14. $3\frac{1}{5}$. 15. $3\frac{1}{8}$. 16. 130.
17. 136. 18. £1150 stock, £1121. 5*s*.
19. The percentages are 3·15... and 3·14... respectively.
20. The percentages are 3·100... and 3·103... respectively.

XV. C. Pages 212—213.

1. £400. 2. £550. 3. £725. 4. £525.
5. £312. 10*s*. 6. £513. 9*s*. 6*d*. 7. £480. 8. £436. 10*s*.
9. £270. 3*s*. 9*d*. 10. £316. 17*s*. 6*d*. 11. £505. 6*s*. 2*d*.
12. £1447. 19*s*. 5*d*. 13. £8. 11*s*. 10*d*. 14. £3. 17*s*. 1*d*.
15. £4. 17*s*. 6*d*. 16. £1800. 17. £593. 6*s*. 8*d*.
18. £5425. 19. £2032. 20. £2313.
21. £3200 stock, £3220 money. 22. £10. 10*s*.
23. 132. 24. £11256, £158. 25. £7860.
26. £63. 11*s*. 8*d*. 27. $123\frac{3}{4}$. 28. £105 less.

XVI. A. Page 215.

1. 14. 2. 15. 3. 18. 4. 20. 5. 21. 6. 24.
7. 26. 8. 33. 9. 35. 10. 45. 11. 51. 12. 63.
13. 65. 14. 84. 15. 105. 16. 126. 17. 3. 18. 5.
19. 6. 20. 13. 21. 14. 22. 35. 23. 6. 24. 21.

XVI. B. Pages 218—219.

1. 27. 2. 34. 3. 43. 4. 47. 5. 59. 6. 78.
7. 41. 8. 237. 9. 2039. 10. 7009. 11. 5160.
12. 3057. 13. 42032. 14. 40795. 15. 82929.

16. 3·7. 17. ·53. 18. 2·14. 19. 20·02.
20. 140·18. 21. 13·509. 22. ·0149. 23. ·04078.
24. 2·2360.... 25. 4·3588.... 26. 4·6368. 27. 11·1982.
28. 5·5718. 29. ·6324. 30. ·2846. 31. ·1317.

XVI. C. Pages 220—221.

1. $\frac{5}{12}$. 2. $1\frac{1}{12}$. 3. $\frac{13}{17}$. 4. $3\frac{2}{5}$. 5. $6\frac{1}{4}$. 6. $\dot{3}$.
7. $\cdot 0\dot{6}$. 8. $\cdot 3\dot{6}$. 9. $1\cdot 1\dot{6}$. 10. $2\cdot 09\dot{3}$. 11. ·6614.
12. ·6030. 13. 1·8027. 14. 2·8784. 15. 1·5634.
16. ·2035. 17. ·2895. 18. 1·8751. 19. ·704.
20. 10·54. 21. 1·211. 22. 1·278. 23. $2\frac{1}{4}$ ft.
24. 220 yards. 25. 98·38 yards. 26. £73. 6*s*. 8*d*.
27. $148\frac{1}{2}$ yards. 28. 249 yards. 29. 220 yards.
30. 55 yards. 31. 12 ft. 32. 12 ft. 6 in.
33. 87 yards by $43\frac{1}{2}$ yards. 34. 14 yards 5 inches.
35. 9·3 inches. 36. £32. 14*s*. 37. 3 per cent.

XVI. D. Page 222.

1. 22. 2. 15. 3. 33. 4. 24. 5. 27. 6. 35.
7. 2. 8. 14. 9. 1764. 10. 21. 11. 147. 12. 84.

XVI. E. Page 225.

1. 11. 2. 15. 3. 17. 4. 23.
5. 31. 6. 41. 7. 430. 8. 530.
9. 207. 10. 902. 11. 133. 12. 257.
13. 1·3. 14. ·17. 15. ·62. 16. 50·07.
17. $1\frac{1}{3}$. 18. $3\frac{2}{5}$. 19. $23\frac{1}{4}$. 20. $6\frac{2}{3}$.
21. 2·15. 22. 1·14. 23. 1·55. 24. ·407.
25. 18 in. 26. 49 in. 27. 2 sq. yds. 8 ft. 100 in.
28. 6·52 inches. 29. 19·97 inches. 30. $2\frac{1}{2}$.

XVII. A. Page 228.

1. £105, £140. 2. £75, £90. 3. £15. 12*s*., £17. 11*s*.
4. £9, £18, £27. 5. £23. 12*s*. $9\frac{1}{2}d$., £30. 7*s*. $10\frac{1}{2}d$., £43. 18*s*. $0\frac{1}{2}d$.
6. £4. 17*s*. 6*d*., £5. 7*s*. 3*d*., £7. 6*s*. 3*d*. 7. $\frac{80}{281}$.
8. £3118. 10*s*., £1871. 2*s*., £1247. 8*s*.
9. £1054. 7*s*. 6*d*., £753. 2*s*. 6*d*., £451. 17*s*. 6*d*., £150. 12*s*. 6*d*.
10. £300, £180, £75. 11. £1666. 13*s*. 4*d*., £1200, £533. 6*s*. 8*d*.

12. £1216 *B*, £1317. 6*s*. 8*d*. *C*. 13. £528, £594, £396.
14. £47. 5*s*., £63, £15. 15*s*., £31. 10*s*.
15. Each man 24*s*., each woman 16*s*., each boy 8*s*.
16. Each man £2. 12*s*. 6*d*., each woman £1. 10*s*., each child 16*s*. 8*d*.
17. £138. 2*s*., £552. 8*s*., £69. 1*s*. 18. £8750 *A*, £6250 *B*.
19. *A* £264, *B* £360. 20. One ounce.

XVII. B. Page 230.

1. 1*s*. 7*d*. 2. 14*s*. 6*d*. 3. $3\frac{3}{4}d$.
4. 44. 5. 2 to 1. 6. 3 to 1.
7. 4 gallons. 8. 320 lbs. 9. 2 to 1.
10. 1 to 2. 11. $6\frac{2}{3}$. 12. 150 lbs.

XVII. C. Page 232.

1. 2 days. 2. 10 minutes. 3. 15 days.
4. 5 days. 5. 60 days. 6. 15 minutes.
7. 50 days each. 8. *A* 30 days, *B* 90 days. 9. 1 day.
10. $14\frac{2}{7}$ min. 11. 5 hrs. $38\frac{2}{11}$ min. 12. 2 days.
13. *A* 15 days, *B* 10 days, *C* 12 days.
14. *A* 40 days, *B* 60 days, *C* 120 days.

XVII. D. Page 234.

1. $9\frac{3}{4}$ yds. 2. 22. 3. 51 yds. 4. 20 yds.
5. 25. 6. 340. 7. 40. 8. 230.
9. 51. 10. $54\frac{9}{22}$ secs., 57 secs., 60 secs.

XVII. E. Page 236.

1. In 24 secs. 2. In 2 hours.
3. $27\frac{3}{11}$ minutes past 5. 4. $27\frac{3}{11}$ minutes past 2.
5. 198 yds. 6. 220 yds., 45 miles an hour.
7. 80 yds. 2 ft., 161 yds. 1 ft. 8. At 20 minutes past 5 o'clock.
9. 5*d*. 10. £1. 1*s*. 3*d*.

XVIII. A. Page 237.

1. 2645. 2. 22030. 3. 472, 17. 4. 524.
5. 12003. 6. 20251452. 7. 234. 8. 651.

XVIII. B. Page 239.

1. 22343. 2. 12770. 3. 131211. 4. 111022.
5. 2*t*94. 6. 7189. 7. 718. 8. 10525.
9. 2445. 10. 1009*e*.

XVIII. C. Page 240.

1. 640·2. 2. 11020·02. 3. 3123·11.
4. 451·0213. 5. 225·16. 6. $1336\cdot\dot{3}6\dot{3}0\dot{}$.
7. $126\cdot\dot{2}59148036\dot{t}$. 8. $19\cdot80\dot{5}$. 9. 103·3125.
11. $56\cdot7\dot{2}$. 12. $3000\cdot\dot{1}2346\dot{0}$.

XIX. Page 245.

1. £1976. 5*s*. 8*d*. 2. £60. 16*s*. 7*d*. 3. £124. 4*s*. 6*d*.
4. £276. 5*s*. 5. £85. 10*s*. 10*d*. 6. £139. 17*s*. 10*d*.
7. £180. 3*s*. $6\frac{1}{2}d$. 8. £68. 0*s*. 11*d*. 9. £2083. 6*s*. 8*d*.
10. £111. 11*s*. 3*d*. 11. (i) 25·52, (ii) 12, $3\frac{9}{10}$, (iii) $42\frac{3}{40}$, (iv) $24\frac{1}{2}$
12. (i) £61. 3*s*. 6*d*., (ii) £648. 11*s*. 2*d*., 14354 Roubles, 8459$.
13. London on Petersburg at 3 mos. 22·17... [pence for 1 R.].

MISCELLANEOUS EXAMPLES. Pages 247—284.

1. Five million six thousand and seventeen, 13025011.
2. 1000769. 3. 204. 4. 192283. 5. 143, 85800.
6. $3\frac{13}{24}$. 7. $\frac{185}{1456}$. 8. $\cdot0\dot{4}16$. 9. 6*d*.
10. £494. 13*s*. $1\frac{1}{2}d$. 11. 470046066583. 12. 10624.
13. 969318. 14. 108 lbs.
15. $2^2\times3^3\times7$, $3\times5\times7\times11$, $2^3\times3\times7^2$. 16. $\frac{13}{48}$.
17. $2\frac{1}{4}$. 18. 8·51. 19. £1. 20. £419. 13*s*. 21. 762632.
22. 117. 23. 85 m. 4 fur. 6 p. 3 yds. 25. 1008000, 16128000.
26. $\frac{1}{2}$. 27. £1435. 28. ·0000025, 2500. 29. ·08125.
30. 9*s*. 3*d*. 31. 1899, MCCCCLXXXIX.
32. 11 miles 640 yards. 33. 22 tons 5 cwt. 2 qrs. 34. 227.
35. $3\frac{2}{5}$. 36. $\frac{10}{441}$. 37. £161. 10*s*. $6\frac{3}{4}d$.
38. $\frac{7}{11}, \frac{447}{925}, \frac{551}{33300}$. 39. ·57875, 14*s*. $10\frac{1}{2}d$. 40. £10. 19*s*. 1*d*.
41. £2. 13*s*. $3\frac{1}{2}d$. 42. 5 ft. $\frac{1}{2}$ in. 43. 11 m. 3 f. 28 p. 1 yd. 1 ft. 2 in.
44. In $1\frac{1}{2}$ min., 462 and 440 yards from starting. 45. 3 yds. 9 in.
46. $\frac{1873}{3920}$. 47. $\frac{5}{88}$. 48. $\frac{49}{50}$, $6\cdot\dot{1}6\dot{}$. 49. 8*s*. 5*d*.
50. £1. 1*s*. $1\frac{1}{2}d$. 51. 1196. 52. £253. 5*s*. 8*d*.
53. 4 f. 4 p. 3 yds. 2 ft. 7 in. 54. 360. 55. $\frac{27}{28}$.
56. $\frac{79}{279}$. 57. $\frac{1}{10}$. 58. 30·0736, 30073600.

59. $\frac{181}{240}$. 60. £273. 2*s*. 1*d*. 61. £1.

62. 1578 m. 2 f. 10 p. 2 yds. 2 ft. 4 in. 63. £1. 18*s*. 6*d*.

64. 23. 65. $15\frac{35}{48}$. 66. $1\frac{1}{13}$, 25.

67. $3882\cdot\dot{5}\dot{7}$, $\cdot00330578512396694214876\dot{}$. 68. 9·875 yds.

69. £23. 4*s*. $3\frac{3}{4}d$. 70. £11. 13*s*. 4*d*. 71. £1158. 6*s*. 9*d*., 4*d*.

72. 104. 73. 94 lbs. 74. $4\frac{9}{10}$. 75. £22.

76. 60 years, 28 years. 77. £81. 10*s*. $11\frac{1}{4}d$.

78. £20092. 10*s*. 79. $2\frac{1}{2}$ acres. 80. 64, 128.

81. 4, 1, 3. 82. 137.

83. 3 m. 3 fur. 15 p. 1 yd. 2 ft. 5 in., 11 a. 3 r. 25 p. $10\frac{3}{4}$ yds. 84. 5.

85. 15925, $\cdot\dot{3}$. 86. £1001. 17*s*. 6*d*. 87. 9 cwt. 2 qrs. 21 lbs.

88. 74 yds. 2 ft. 89. 4·54 chains. 90. 3·872, ·1936, ·2846.

91. Saturday. 92. 332. 93. $52\frac{1}{2}$ yards. 94. 9, $68\frac{76}{225}$.

95. $11\cdot\dot{3}\dot{1}$. 96. £1260. 10*s*. 97. 10 days.

98. £6. 9*s*. 2*d*. 99. $8\frac{1}{3}$. 100. £507. 5*s*. 101. £261. 11*s*. $4\frac{1}{2}d$.

102. 34538724, 15 a. 3 r. 30 p. 22 yds. 8 ft. 136 ins.

103. £26. 5*s*. 104. 18. 105. 1 qr. $8\frac{2}{5}$ lbs.

106. £3796. 7*s*. 6*d*. 107. 200 ft. 108. 40 a. 35 p.

109. 1296. 110. 30 per cent. 111. 484.

112. 488. 113. 48. 114. (i) $\frac{1}{8}$. (ii) $2\frac{9}{77}$.

115. £1. 15*s*. 8*d*., $\cdot178\dot{3}$. 116. £1642. 16*s*. $4\frac{1}{2}d$. 117. 5.

118. 2400, £8. 8*s*. 119. $21\frac{7}{33}$. 120. £33. 12*s*. $7\frac{1}{2}d$.

121. 9052, r. 50. 122. £5. 10*s*. 3*d*., £6. 2*s*. 9*d*.

123. 2. 124. 2·0634, 1530000, $\cdot000\dot{8}06451612903225\dot{}$.

125. £301. 6*s*. $1\frac{3}{4}d$. 126. 258·989504. 127. 15*s*.

128. 448·7... gals. 129. £1644. 2*s*. 130. 2*s*.

131. 72361, 54984. 132. £300, £250, £325. 133. 517.

134. £3. 3*s*. 135. 33, rem. ·0036 yds. 136. £3. 3*s*. $11\frac{1}{2}d$.

137. 13 pieces. 138. 45. 139. £2. 8*s*.

140. £2500. 141. 80. 142. 8 m. 7 fur. 4 p. 2 ft. 11 in.

143. $2^2\times3^3\times7^2\times11$, $2^2\times3^2\times7^2$. 144. $9\frac{401}{615}$. 145. 5*d*.

146. £86. 16*s*. $0\frac{1}{4}d$. 147. 22 a. 2r. 12 p., £56. 8*s*. 9*d*.

148. 220·09... gals. 149. 5*d*., £2500. 150. £425.

151. $2\frac{1}{2}d$. 152. $22\frac{1}{2}$. 153. $2431=11\times13\times17$. 154. $\frac{4}{11}$.

155. $6\cdot80388\dot{6}6\dot{3}$, $\cdot1\dot{0}\dot{9}$. 156. 47*s*. 3*d*. 157. 120.

158. $4\frac{1}{2}$ in. 159. 1702·88.... 160. £7. 3*s*. 4*d*. 161. 134903.

162. £33852. 8*s*. $4\frac{3}{7}d$. 163. 102 yds. 2 ft. 9 in. 164. $1\frac{2}{3}\frac{9}{1}\frac{1}{6}\frac{9}{3}$.

166. 8. 167. 11. 168. £1053. 169. $14\frac{2}{7}$.
170. 8 months. 171. 50. 172. $1\frac{3}{16}$ miles.
173. £4000, £2500, £2000. 174. 20. 175. 1.
176. 19*s*. $7\frac{1}{2}d$. 177. 195 miles. 178. £9. 3*s*. 9*d*
179. 1035. 180. £650. 181. 27. 182. Tuesday.
183. 9*s*. 10*d*. 184. 1. 185. £414. 1*s*. 186. $\frac{1}{14}$.
187. 250 a. 2 r. 188. £230. 189. £233. 6*s*. 8*d*. 190. 16523
191. MDCCCLXXXVII. 192. £1050. 193. $5\frac{2}{5}\frac{3}{1}\frac{3}{2}$ tons
194. $\frac{1}{20}$. 195. $\frac{7}{32}$. 196. 6. 197. £11. 10*s*. 1*d*
198. 4 a. 1 r. 199. 4401·57... oz. 200. £2. 19*s*. 7*d*.
201. 40002. 202. $6\frac{2}{3}$. 203. $8{\cdot}19\dot{4}\dot{0}$. 204. £60. 12*s*. $4\frac{3}{4}d$
205. £2. 10*s*. $1\frac{1}{4}d$. 206. 1*s*. $3\frac{1}{4}d$. 207. 4.
208. £4000. 209. £79. 2*s*. 11*d*. 210. 15. 212. 37.
213. ·375. 214. £45. 8*s*. $6\frac{1}{2}d$. 215. £7. 10*s*.
216. £1. 217. £10000. 218. $2\frac{2}{3}$.
219. £854. 16*s*. 220. $126\frac{1}{2}$. 221. $51\frac{1}{3}$.
222. £1. 9*s*. 2*d*., $1\frac{7}{18}$. 223. 352. 224. £1001. 6*s*.
225. $16\frac{2}{3}$ loads, £2. 18*s*. 4*d*. 226. £21. 13*s*. 6*d*.
227. £37. 10*s*., £18. 15*s*. 228. £403. 6*s*. 5*d*. 229. £48. 8*s*
230. £246. 15*s*. 231. 627; 3, 11, 19, 33, 57, 209, 627.
232. $\frac{241}{259}$, $\frac{290}{659}$. 233. 3*s*. $8\frac{1}{4}d$. 234. £91749. 3*s*. 9*d*.
235. 1447842 gallons, 6463 tons. 236. 1*s*. 1*d*.
237. 1 pint. 238. 18·08 per cent. 239. £515.
240. £28. 12*s*. 11*d*. 241. 193. 242. £360.
243. $56{\cdot}\dot{1}75786047073\dot{}$. 244. 8978 gals. 1 pt. 245. 1*s*. 6*d*.
246. 2663323 acres, £14821030. 247. £2. 10*s*.
248. $1\frac{1}{2}$ years. 249. £13. 12*s*. 11*d*. 250. £5383.
251. £18. 14*s*. $6\frac{1}{2}d$. 252. $1\frac{1}{9}$. 253. 4*s*., £5. 16*s*
254. £1924. 18*s*. $7\frac{1}{2}d$. 255. £34. 7*s*. 4*d*. 256. 36 days.
257. £5750. 258. $2\frac{1}{2}$ years. 259. £444. 10*s*.
260. 175. 261. 3 a. 1 r. 10 p. 5 yds. 262. £675.
263. $\cdot 000\dot{4}1\dot{6}$. 264. £11. 1*s*. 8*d*. 265. $49\frac{1}{2}$ yds.
266. 937·4. 267. 40. 268. 4, £135. 8*s*. 4*d*.
269. £25. 0*s*. $2\frac{1}{2}d$. 270. £220. 271. 15 a. 3 r. 30 p. 22 yds. 8 ft. 136 in.
272. $\frac{13}{320}$. 273. $\cdot 13\dot{8}$. 274. 44 lbs. 275. 99·7... minutes.
276. £14. 18*s*. $7\frac{1}{4}d$. 277. $13\frac{3}{4}$. 278. £12. 6*s*. 3*d*.

279. £2100. 280. £41. 6*s*. 8*d*. 281. 12 lbs. 282. $\frac{29}{280}$.
283. 40, 10·325. 284. 13 pieces.
285. 1 in. = 2·53995 c.m., 1 sq. in. = 6·45136 sq. centimetres.
286. £11. 8*s*. 287. 4 roods. 288. $4\frac{1}{2}$ years.
289. £496. 16*s*. 9*d*. 290. £350. 291. 11. 292. £2. 3*s*. 4*d*.
293. $11\cdot\dot{3}\dot{1}$ lbs. 294. £1667. 14*s*. 295. £2. 2*s*.
296. £8400. 297. $28\frac{4}{7}$. 298. £87. 18*s*. 2*d*.
299. £834. 8*s*. 2*d*. 300. £202. 10*s*. 301. £40.
302. $18\frac{9}{10}$. 303. £249. 6*s*. 304. 60. 305. 10 ft.
306. 12. 307. £750. 308. £3850, £3630.
309. 6·9, 26·6133. 310. 14 miles at 40, and $82\frac{1}{2}$ at 50.
311. £25. 13*s*. 4*d*. 312. $\frac{174}{4025}$. 313. $\frac{187}{185}$.
314. £667. 18*s*. 6*d*. 315. £5. 16*s*. 8*d*. 316. $3\frac{1}{3}$.
317. 130. 318. $21\frac{7}{8}$. 319. 130452. 320. 6 days.
321. 292. 322. $1\frac{1}{2}$. 323. $8\frac{1}{4}d$. 324. £1. 4*s*. 6*d*.
325. 125000. 326. $28\frac{8}{39}$. 327. £315. 328. 127 yds.
329. £8822. 330. £1380, £1242, £276. 331. 2 m. 3 fur.
332. 3. 333. £11. 8*s*. 9*d*. 334. 10 days. 335. 92.
336. £96. 17*s*. 5*d*. 337. 5 per cent. 338. 98.
339. $393\frac{5}{13}$, 1061. 340. £7. 13*s*. 3*d*. 341. £468. 4*s*. 3*d*.
342. 1. 343. ·06494 days = 1 hr. 33 min. 39·816 secs.
344. 4*s*. 6*d*. 345. 21 ft. by 16 ft. 4 in. 346. 10·16.
347. £3311. 348. 4 days. 349. $27\frac{3}{11}$ min. past 5.
350. 251968·5 fr., 48656·86 dollars, 204255·31 m.
351. Saturday. 352. 63. 353. 45 days.
354. £1019086. 16*s*. 355. $18\frac{3}{4}$.
356. £501. 16*s*. 6*d*., £250. 18*s*. 3*d*., £83. 12*s*. 9*d*., £20. 18*s*. $2\frac{1}{4}d$.
357. $22\frac{11}{18}$, 234. 358. $4\frac{1}{2}$ per cent.
359. £1939. 13*s*. 2*d*., £13. 6*s*. 8*d*.
360. *A* at scratch, *B* 15 ft., *C* 29 ft., *D* 42 ft. 361. $18\frac{54}{77}$.
362. $\frac{39}{50}$. 363. 204. 364. 16 days. 365. £264.
366. £1190, £610. 367. $\cdot\dot{1}$, $4\cdot4\dot{6}$, 8 ft. 368. 60, 30, 12.
369. 3 to 4. 370. 20*s*., 18*s*., 15*s*. 371. £33.
372. $\frac{45}{76}$. 373. $1\cdot\dot{9}8\dot{1}$, $\cdot5\dot{3}$. 374. 1543·16.
375. £1. 8*s*. [12 pieces]. 376. 4800 eggs, $9\frac{1}{11}$ per cent.
377. $3\frac{3}{4}$ minutes. 378. 3 per cent. 379. 12 min., 24 min.
380. £152. 381. £2. 9*s*. $11\frac{1}{2}d$. 382. 66 yds., 15 miles an hour.

383. £7. 8*s*. 384. £29081. 8*s*. $11\frac{1}{4}d$. 385. £16000.
386. 31 ft. 6 in., 10 ft. 6 in., 12 ft. 387. $33\frac{1}{3}$. 388. £10562. 10*s*.
389. $\cdot\dot{0}0\dot{1}$, $\cdot\dot{0}10212\dot{}$, $\cdot\dot{0}2\dot{1}$, $\cdot\dot{0}32412\dot{}$, $\cdot\dot{0}\dot{5}$, $\cdot 1$, $\cdot\dot{1}2\dot{5}$, $\cdot\dot{1}42857\dot{}$.
390. 25·52 fr. 391. £1050. 12*s*. $6\frac{1}{2}d$. 392. $\cdot 013\dot{}$.
393. 140 miles. 394. £720. 395. 7 to 5.
396. 1 ft. 6 in. 397. £901. 6*s*. 8*d*. 398. 732.
399. £165. 400. £1 = 25·23 fr. 401. 10 ft.
402. £10489. 10*s*. 403. £1200. 404. 6*s*. 2*d*.
405. 1440, 144, 1017. 406. 417 yds., 973 yds. 407. 12 o'clock.
408. £1575. 410. One has 4 half-crowns 1 fl. and 15 sixpences.
411. 132 yds. 412. £7765. 6*s*. 8*d*. 413. $4\frac{1}{2}$ per cent.
414. 44000. 415. 10 ft. 416. £600.
417. $1\frac{1}{2}$ cubic feet. 418. $\frac{505}{189618}$.
419. Consols, *A* £3860, *B* £3877. 10*s*. 420. 20.
421. $15\frac{5}{29}$ yds. 422. $32\frac{8}{11}$ min. 423. 37730000.
424. 20 per cent. 425. £5. 18*s*. 426. £11040.
427. (i) $42\frac{1}{2}$ sq. ft., (ii) 99 ft. 52 in., (iii) 88136. 428. £2625.
429. 61 tons 12 cwt. 430. £517. 10*s*., £828, £1012. 10*s*.
432. 315. 433. The latter, $\frac{3}{4}$ per cent. nearly. 434. $78\frac{2}{3}$.
435. 12 per cent. 436. 21 ft. by 18 ft. 437. £30. 8*s*.
438. £54. 12*s*. 439. 2*s*. 6*d*., 3*s*. 440. £7392, £5733.
442. Half a pint. 443. 10 miles an hour.
444. 13 weeks, 13 shillings. 445. 250. 446. 60.
447. 10 per cent. 448. £367. 4*s*. 2*d*. 449. 72.
450. 4808. 451. £843. 15*s*. 452. 15*s*. 6*d*.
453. £124. 2*s*. 7*d*. 454. $8\frac{1}{2}$ per cent., £971. 5*s*. carried forward.
455. $12\frac{1}{4}$ days. 456. $11\frac{9}{21}$ minutes. 457. 2 days.
458. 38. 459. 3232. 460. 216, 135, 81, 189.
461. 50 miles. 462. *A* 5 min. 20 secs., *B* 5 min. 52 secs.
463. *A* wins by $\frac{1}{7}$ mile. 464. £300.
465. £65. 8*s*. 9*d*., £196. 6*s*. 3*d*., £261. 15*s*. 466. £3. 16*s*.
467. 15. 468. 15, 60, 75. 469. 11 hrs. $54\frac{6}{11}$ min.
470. 1 hr. $5\frac{5}{11}$ min., 6 o'clock. 471. 42 times an hour, $\frac{3}{8}$ of a mile.
472. 6·907...per cent. 473. 30*s*. 474. 40. 475. £3750.
476. 5 hours. 477. 23·37, 25·87, 25·7, 28·54, 33·34, 26·98.
478. At 6 P.M., $13\frac{1}{2}$ miles from *C*. 479. Each man £4, each woman £3, each child £2. 480. 15. 481. £124. 19*s*. 9*d*.
482. 100 lbs. 483. 12*s*. 484. £605. 485. 11040808.
486. 12 min., $13\frac{1}{2}$ min. 487. 457×38. 488. $4\frac{1}{5}$ acres, $146\frac{2}{3}$ yds.

489. A by 18 yards. **490.** 3·2808. **491.** 2*s.* 6*d.*, 4*s.*, 2*s.*
492. 243. **493.** 24 min.
494. £179. 5*s.*, £59. 15*s.*, £96, £48. **495.** 168 lbs.
496. 126 yards. **497.** 20 miles out, 16 miles an hour.
498. 672 oz. of gold and 64 of alloy.
499. 19 sheep, 80 geese and 101 pigs; or 38 sheep, 160 geese and 2 pigs.
500. £91395. 4*s.*

EXAMINATION PAPERS. PAGES 284—300.

I. A 1. 689902. A 2. £1344. 4*s.*, 25 p. 3 yds. 1 ft. 7 in.
A 3. 2 tons 7 cwt. 2 qrs. 23 lbs. 8 oz. 5 drs. A 4. £12. 0*s.* $6\frac{1}{2}d.$
B 1. £7. 9*s.* $7\frac{1}{2}d.$, $\frac{1}{5}$. B 2. ·0302, $\cdot 4036458\dot{3}$. B 3. 14*s.* 9*d.*
B 4. £261. 15*s.* $7\frac{3}{4}d.$ C 1. $\frac{1}{6}$ gallon. C 2. £552. 14*s.* 3*d.*
C 3. £937. 4*s.* C 4. 8 seconds, 154 feet.

II. A 1. £26. 6*s.* 8*d.* A 2. $\frac{7}{12}$. A 3. 15125.
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